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The Influence of Variable Cross-Section of System Modeling Mining Support Structure on the Value of Critical Load under Active and Passive Specific Load

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Abstract. Theoretical considerations and results of numerical computations regarding a slender column modeling mining supporting structure were presented in this paper. The system was subjected to a selected case of a specific load (follower force directed towards the positive pole), active and passive. The numerical simulations presented in this article refer to the influence of a cross-section variable along the axis and the geometry of a structure implementing an external load on the value of critical force.

Introduction

The widespread use of slender elements (beams, rods and columns) in mechanical engineering, mining industry or civil engineering leads to the necessity of paying special attention to the issue of their vibrations and stability. A significant problem of the exploitation of such structures is a search for and implementation of new structural solutions, relating both to models of systems generating a conservative external load and to the appropriate shaping of rods in order to increase their critical load. The works [1, 2] propose design solutions of loading heads that can generate active and passive (as a supporting structure) external loads. The article [2] where the schemes of a loading structure implementing a specific load (L. Tomski's load, [3]) are presented is of particular significance. The described structures under such an external loading are characterized by much greater critical forces compared to the classic Euler's load.

A separate group of publications is devoted to the issue of stability and free vibrations of systems of a variable cross-section. The paper [4] presents a scientific problem of shape optimization of a slender column subjected to an axial load with a concentrated force and continuous load using the Rayleigh – Ritz method. The study of the influence of size and location of a crack in a beam with a variable cross-section on its eigenvalue and stability constitutes the subject of the publication [5]. The issue of the stability of columns with a variable cross-section was also discussed in the works [6, 7]. This paper presents a model of a non-prismatic column modeling a mining support structure subjected to a selected case of a specific load, active and passive.

Physical Model

A slender column of a cross-section variable along an axis modeling mining support structure presented in Fig. 1 is taken into consideration in this work. In order to model the variable cross-section, the column is considered as a set of prismatic segments of constant length l and thickness h, whose width is described by approximation functions, while maintaining the conditions of a constant total volume and total length of the system. The performed analysis includes a shape approximation using:

- a linear function;
- a polynomial of degree 2.

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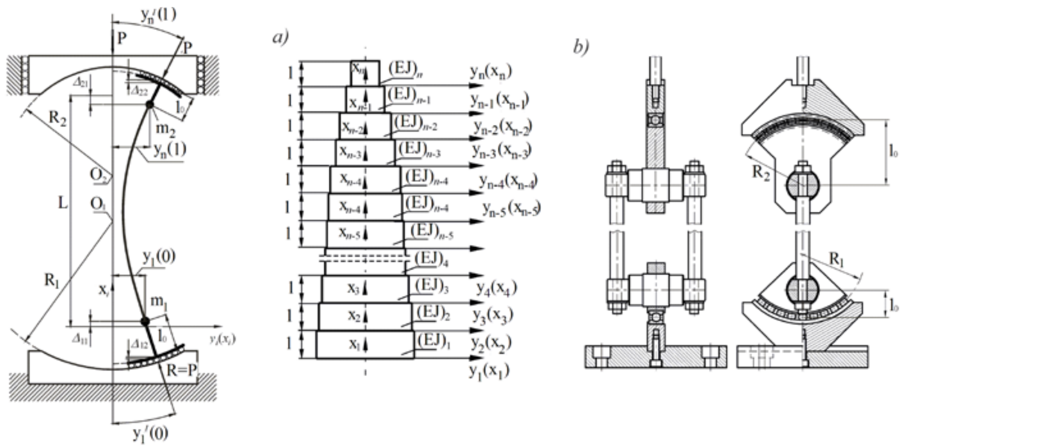

Fig. 1. Physical model of a non-prismatic column subjected to a passive and active specific load: a) division into segments, b) real construction solutions.

The analyzed column is subjected to the follower force directed towards the positive pole (the case of the specific load defined by L. Tomski), active and passive. The active load is implemented through the upper structure realizing the load, built of two elements of a circular outline (constant curvature, a slice of a rolling bearing) – loading and receiving heads. The supporting structure constructed in the same way generates the passive load. Both constructions are real objects whose validity has been confirmed experimentally. For structural reasons, theoretical considerations and numerical calculations include also a rigid element of a length of l_0, to ensure a proper mounting of the column.

Problem Formulation

On the basis of the presented description of a physical model of a slender column of a variable cross-section modeling a mining support structure, taking into account the Bernoulli – Euler's theory, the potential energy of the analyzed system was defined. The potential energy V is a sum of the energy of the bending elasticity of the particular steps of the column V_1 and the potential energy derived from the vertical (V_2) and horizontal (V_3) components of the active and passive external load:

$$V_1 = \frac{1}{2}\sum_{i=1}^{n}(EJ)_i \int_0^1 \left[y_i^{II}(x_i) \right]^2 dx_i; \tag{1}$$

$$V_2 = -\frac{P}{2}\sum_{i=1}^{n}\int_0^1 \left[y_i^{II}(x_i) \right] dx_i + P\frac{l_0}{2}\left[\left(y_1^{I}(0) \right)^2 + \left(y_n^{I}(1) \right)^2 \right], \tag{2}$$

$$V_3 = \frac{P}{2}\left[R_1 \left(y_1^{I}(0) \right)^2 + R_2 \left(y_n^{I}(1) \right)^2 \right]; \tag{3}$$

$$V = V_1 + V_2 + V_3. \quad (4)$$

where: EJ – flexural stiffness, $y_i(x_i)$ – function of the deflection of the i-th segment, R_1, R_2 – radii of the heads implementing the passive and active load, l_0 – length of the rigid element.
The boundary problem was formulated on the basis of the minimum of the potential energy principle (static criterion of the stability loss):

$$\delta V = 0. \quad (5)$$

The geometric boundary conditions and the continuity conditions referring to the deflection and the angles of the deflection in the mounting of the system and in the points of joining individual segments are expresses by the following formulas:

$$y_1(0) = -(R_1 - l_0)\, y_1^{I}(0), \qquad y_n(1) = (R_2 - l_0)\, y_n^{I}(1), \quad (6\text{-}7)$$

$$y_j(1) = y_{j+1}(0), \qquad y_j^{I}(1) = y_{j+1}^{I}(0). \quad (8\text{-}9)$$

Formulas (Eq. 6-7) result directly from the design assumptions of the loading structure. Performing the variational calculus of the potential energy (Eq. 4), substituting to the equation which defines the principle of the minimum of the potential energy (Eq. 5) and then taking into account the geometric boundary conditions (Eq. 6-7) and the continuity conditions (Eq. 8-9) makes it possible to determine:

- the differential equations of displacements:

$$(EJ)_i\, y_i^{IV}(x_i) + P y_i^{II}(x_i) = 0; \quad (10)$$

- the natural boundary conditions:

$$(R_1 - l_0)\, y_1^{III}(0) + y_1^{II}(0) = 0, \qquad (R_2 - l_0)\, y_n^{III}(1) + y_n^{II}(1) = 0; \quad (11\text{-}12)$$

- the natural continuity conditions:

$$(EJ)_j\, y_j^{II}(1) = (EJ)_{j+1}\, y_{j+1}^{II}(0), \qquad (EJ)_j\, y_j^{III}(1) = (EJ)_{j+1}\, y_{j+1}^{III}(0). \quad (13\text{-}14)$$

The solution of the differential equations of the displacements is written in the following form:

$$y_i(x_i) = A_i \sin(k_i x_i) + B_i \cos(k_i x_i) + C_i x_i + D_i. \quad (15)$$

After substituting the solution (Eq. 15) with the geometrical and natural boundary conditions as well as continuity conditions, a system of homogeneous equations is obtained. Equating zero to the characteristic determinant of the system of equations defines a transcendental equation for the value of the critical force of the considered column.

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Results of Numerical Simulations

The results of the numerical calculations are presented in the dimensionless form, introducing the following parameters:

- the parameters describing the approximating functions:

$$p^* = \frac{p}{L}, \quad q^* = \frac{q}{L}, \quad Z^* = \frac{b_1 - b_n}{L} \cdot 100\%; \qquad (17\text{ a-c})$$

- the parameter of the critical load:

$$\lambda_{kr} = \frac{P_{kr} L^2}{(EJ)_{pr}}; \qquad (18)$$

- the parameter of the increase of the critical load:

$$\delta_{\lambda_{kr}} = \frac{\lambda_{kr} - (\lambda_{kr})_{pr}}{(\lambda_{kr})_{pr}} \cdot 100\%. \qquad (19)$$

The values with the subscript "pr" in the formulas (Eq. 18, 19) refer to the prismatic (of constant cross-section) comparative system of the same volume.

Figure 2 shows the influence of the shape approximation of the non-prismatic rod by means of the approximating functions on the value of the critical load increment in relation to the equal values of the radii of the heads generating the active and passive load. On the basis of the presented graphs, it is possible to determine the ranges of the values of the parameter of the convergence of the system (in the case of the shape approximation by the linear function, Fig. 2a) or the parameters of the position of the vertex of the parabola (in the case of the shape approximation by the polynomial of degree 2, Fig. 2b), for which there is an increase in the value of the critical force of the structure compared to the reference system of the constant cross-section. The maximum increase of the critical load achieved was about 10%.

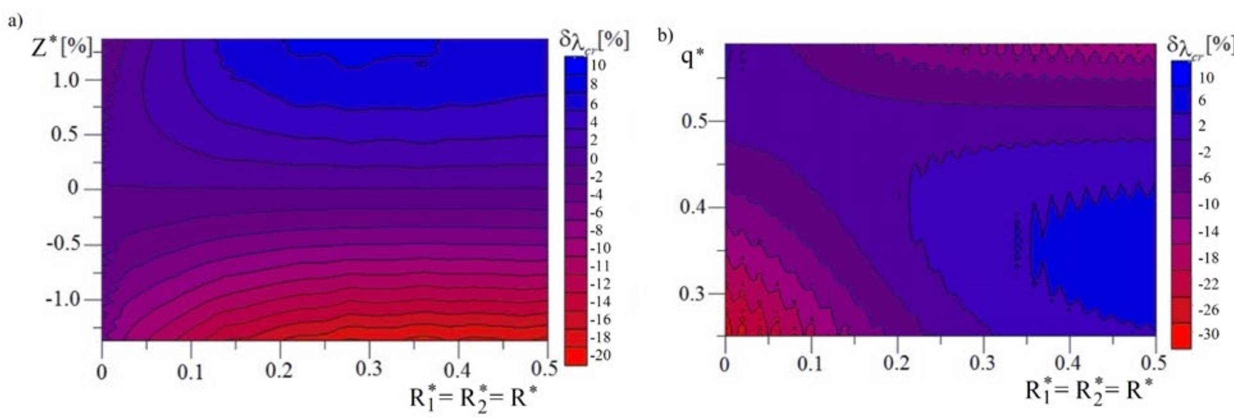

Fig. 2. Value of the critical load increase of the non-prismatic column as a function of the variable geometry of the structures performing the active and passive load: a) when the outline is approximated by the linear function; b) when the outline is approximated by the parabolic function.

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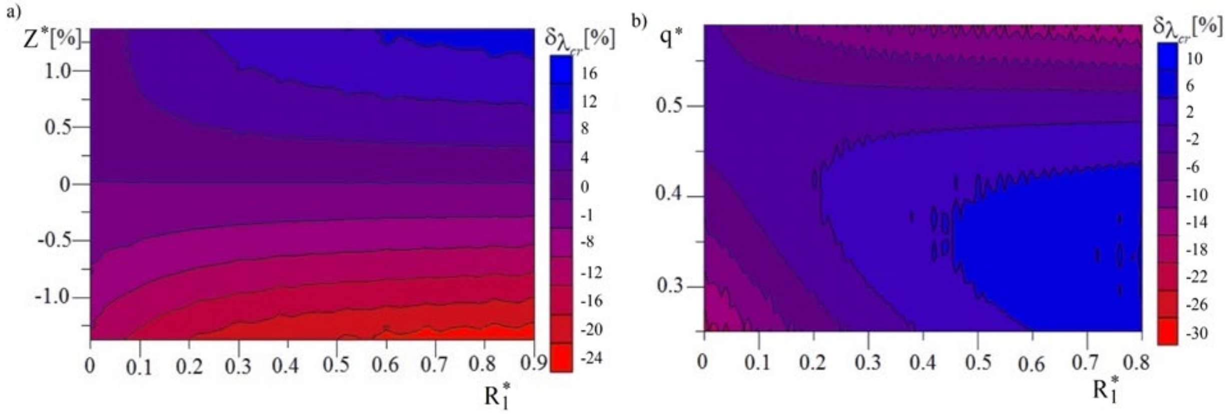

Fig. 3. Value of the increment of the critical load of the non-prismatic column depending on the variable geometry of the support structure realizing the passive load (when $R_2^* = 0.4$): a) when the outline is approximated by the linear function; b) when the outline is approximated by the polynomial of degree 2.

Similar data is presented in Figure 3 that illustrates a detailed analysis of the influence of the cross-section variable along the axis on the increment if the critical load value as a function of the variable geometry of the support structure implementing the passive load, at a specified radius of the heads performing the active load. The distribution of critical load increments is similar to that shown in Figure 2. In the case of the linear approximation function, it has been shown that in comparison with the comparative system of the constant cross-section, the critical load can be increased to 16%, while the approximation with parabolic function to 10%.

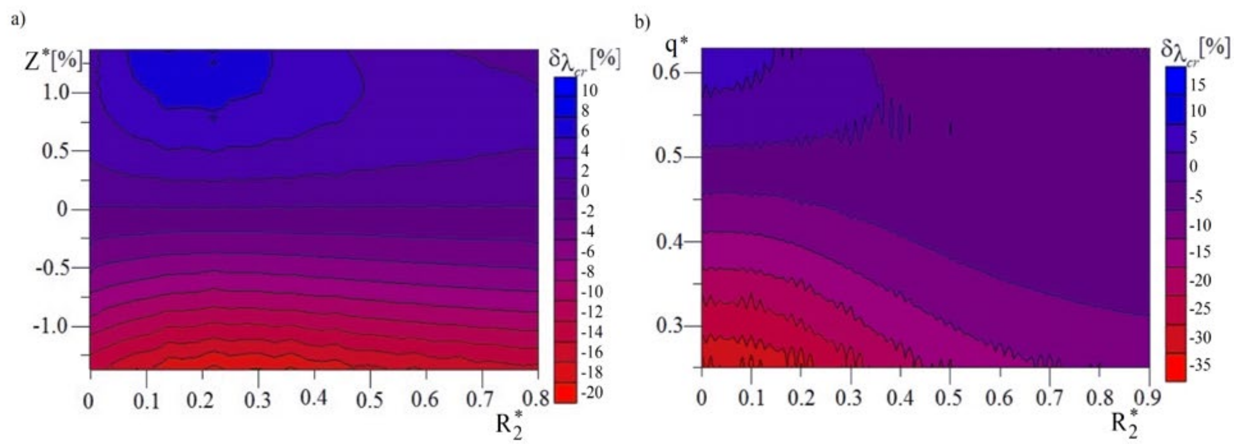

Fig. 4. The value of the increase of the critical load of the non-prismatic column depending on the variable geometry of the structure implementing active load (while $R_1^*=0.3$): a) when the outline is approximated by the linear function; b) when the outline is approximated by the parabolic function.

Figure 4 shows the increase of the critical load depending on the coefficients of the approximating functions and the variable radius of the upper heads realizing the active load, with the given geometry of the support structure. In the discussed case there was a slightly different distribution of critical load increments with maximum values of 10% in the approximation of the outline by a linear function and 15% when considering the polynomial of degree 2.

Summary
The work aimed at performing a numerical analysis of the issue of the stability of a slender column modeling a mining support structure subjected to the follower force directed towards the positive pole, active and passive. Based on the performed computations, the influence of the variable geometry of the structures implementing the selected case of the specific load and the parameters defining approximate functions on the value of the increase of the critical load in comparison with the comparative system of the constant cross-section and the same volume. The ranges of the convergence parameter and the coordinates of the vertex of the parabola parameters, for which the column with variable cross-section is characterized by a higher critical load than the reference system have been defined and the maximum recorded increase of the critical load value was about 16%. The experimental verification of the assumptions made in the mathematical model should be the next stage of the work.

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