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Comparative study of artificial neural network and physics-informed neural network application in sheet metal forming

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Abstract. Accurate prediction of the resultant geometry in sheet metal forming simulation is necessary to achieve zero-defect production. To quantify the effect of process parameters on the final geometry, numerical methods are used to simulate the process outputs for a given set of process variables. Finite element methods are employed in process optimization and design exploration. However, these computationally expensive models are unhelpful for process control applications. Surrogate models allowing fast prediction of resultant geometry or stress distribution can be plausible solutions. In the current study, we propose a sequential surrogate model to fit the stress field as a function of the process variable and the initial spatial coordinates. The framework is composed of two surrogate models. First, an artificial neural network (ANN) evaluates the displacement and the strain. Then, a second surrogate is employed to fit the stress using input strain and displacement. Here, ANN and physics-informed neural networks (PINN) are compared concerning prediction accuracy for the second surrogate model. The PINN is enhanced with the equilibrium equations. The developed method is demonstrated using a v-bending process. The results show that both surrogate models return good approximations, with ANN showing slightly better results.

Introduction

Computer-based simulations are widely employed in many engineering fields to characterize and design a process. Numerical solvers such as finite element (FE) simulations, computational fluid dynamics (CFD), and Monte-Carlo simulations are utilized to perform preliminary design exploration and subsequent reliability and maintenance operations during the process life. However, these tasks are computationally expensive and sometimes prohibitive due to the complexity of the problems. An alternative solution involves employing surrogate models to replace these methods. Surrogate models mimic the relation between inputs and outputs data collected from computer-based simulations. The built relation is then used to obtain fast and cheap predictions of unseen data.

Surrogate models are also widely employed in manufacturing for process optimization. In process optimization, surrogate models deliver fast predictions to easily explore the design space and search for optimal process conditions. Similarly, in robust optimization, the features of the final product have to be guaranteed under uncertainties [8]. In global metamodeling, the aim is to build a globally accurate surrogate model within a reasonable computational time [9]. Global metamodels are challenging to achieve due to the curse of dimensionality, i.e., the number of sample points in the design space grows exponentially with the number of parameters. Therefore,

selecting the most suited surrogate models, together with a proper sampling plan, is essential to reach appropriate levels of accuracy.

Artificial neural networks (ANN) are surrogate models categorized as regression models. In recent years, scientific interest in ANN has increased due to the rise of machine learning in many engineering fields. Several studies have utilized ANN in elastoplastic deformation problems such as sheet metal forming to obtain accurate predictions. In the work of El Mrabti *et al.* [1], the authors compare radial basis function (RBF), response surface methodology (RSM), Kriging, and ANN for failure predictions in a deep drawing process. Their results show that ANN performs better than other models in predicting thinning and rupture. Abbassi *et al.* [2] employed an ANN to predict the material parameters of the Gurson–Tvergaard–Needleman damage model for sheet metal forming. Ghaisari *et al.* [3] predicted the mechanical properties of ST14 steel using an ANN with nineteen inputs. The model evaluates acceptable mechanical properties on a test dataset. More generally, Gorji *et al*. [4] used a recurrent neural network (RNN), a class of neural networks that works with time series, to predict anisotropic plane stress plasticity for arbitrary loading paths. The authors proved that RNNs are well-suited for modeling plastic response through supervised learning.

Physics-informed neural network (PINN) is a recently developed surrogate model class. PINNs are neural networks in which the loss function is augmented with governing equations of the system. In the article by Hoffer *et al.* [5], the authors compare several surrogate models (ANN, PINN, simple Kriging, support vector regression (SVR), K-nearest neighbor regressor (KNNR), and gradient boosting decision tree regressor (GBDTR)) for predicting the outputs of three different use cases using a perfect plasticity model. Haghighat *et al.* [6] proposed a PINN architecture applied to a linear elastic problem and a nonlinear perfectly plastic problem under the assumption of small deformations. Niu *et al*. [7] present a novel PINN framework for finite deformation elastoplasticity. A loading, unloading, and reloading cycle is applied to a plate with a hole in the center. The PINN accurately predicts displacement, plastic deformation, and stress state.

In this article, we propose a surrogate model framework to predict the final (deformed) configuration and stress field under plastic deformation from the initial (undeformed) configuration. A comparison between ANN and PINN accuracy is carried out to describe the relationship between total strain and the stress field while satisfying the equilibrium equations in PINN. The data are obtained from the FE simulations of a V-bending process. The sample points are selected using an optimized Latin hypercube (OLH) to achieve a space-filling design of experiments (DOE). The hyperparameters of the surrogate models within the framework are tuned through an automatic tuner. Finally, we tested the proposed framework considering a third of the loading step.

Artificial and Physics-Informed Neural Networks

Artificial neural networks (ANN) are surrogate models used for classification and regression. Their core structure consists of one or more hidden layers and neurons. The neurons, i.e., the basic units of the layer, receive the outputs of the previous layer as inputs [10]. The activation function φ applied to the neurons transforms the linear relation between layers into a nonlinear relation. Therefore, activation functions are essential to enforce the nonlinear relationship between inputs and outputs. The node responses $\boldsymbol{z}_l$ for the layer l with $l = \{1,2, \dots, L\}$ can be written as:

$$\boldsymbol{z}_l = \varphi_l(\boldsymbol{b}_l + \boldsymbol{W}_l \boldsymbol{z}_{l-1}) \tag{1}$$

with $\boldsymbol{W}_l$ and $\boldsymbol{b}_l$ the weights matrix and bias vector of the lth layer, respectively, which are usually called trainable parameters $\boldsymbol{\theta}$ (Eq. 3). Assuming that for $l = 0$ the layer $\boldsymbol{z}_0$ is the input vector and

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for $l = L$ the layer $\boldsymbol{z}_L$ is the output vector, the output layer can be written as the following composition of function:

$$\boldsymbol{z}_L = \hat{\boldsymbol{y}} = \boldsymbol{z}_{L-1}(\boldsymbol{z}_{L-2}(\dots(\boldsymbol{z_1}(\boldsymbol{z_0})\dots) \tag{2}$$

A cost function $\mathcal{L}$, as the average value of a loss ℓ, defines the performance of a surrogate model. Generally, regression models aim to minimize the cost function via iterative improvement of the trainable parameters (Eq. 3). In ANN, the derivative of the outputs with respect to the inputs is evaluated through the chain rule through the so-called backpropagation algorithm. The ANN exploits the backpropagation algorithm to search iteratively for the trainable parameter that minimizes the cost function.

$$\min_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{\theta}) = \frac{1}{N}\sum_{j=1}^{N} \ell(\boldsymbol{y}, \hat{\boldsymbol{y}}) \tag{3}$$

A common loss function is the quadratic loss ℓ_2 which, substituted in Eq. 3, returns the mean squared error.

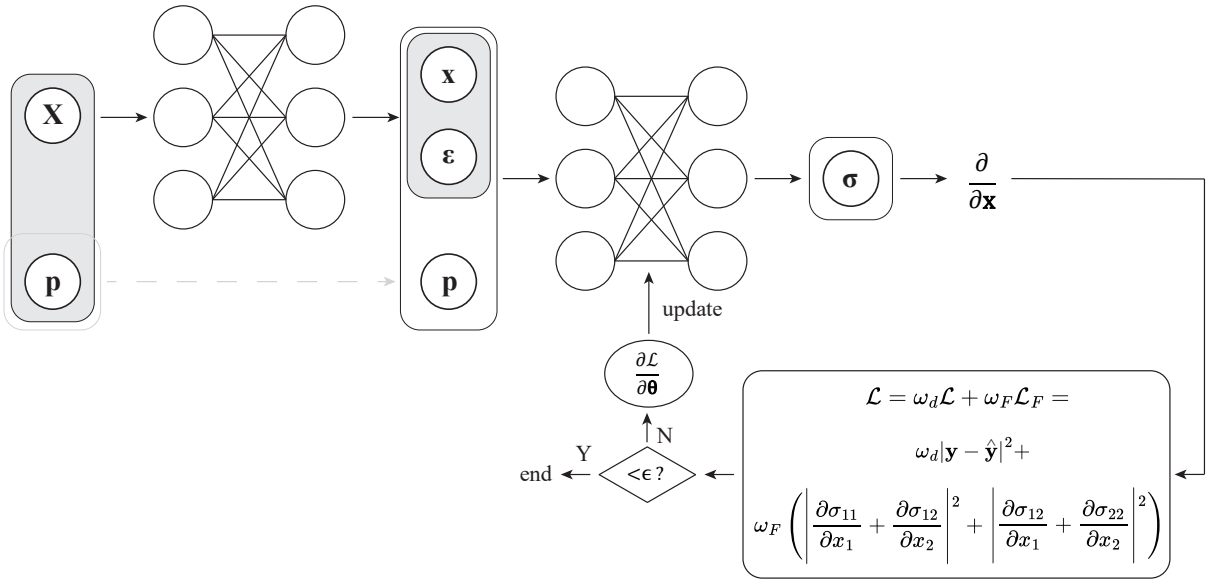

Fig. 1. Schematic representation of the sequential surrogate model; the $\boldsymbol{p}$ vector contains the process variables. In the grey boxes are the inputs and outputs of the first surrogate model, while in the white boxes are the inputs and outputs of the second surrogate model.

Physics-informed neural networks (PINN) are a class of neural networks recently developed by Raissi *et al.* [11]. PINNs are addressed as surrogate models capable of solving partial differential equations, embedded in the loss function terms. These partial differential equations may involve initial conditions, boundary conditions, and governing equations. Thus, ANN and PINN share almost the same architecture and objective, i.e., minimizing the cost function.

$$\boldsymbol{\theta} = \underset{\boldsymbol{\theta}}{arg\,min}\,[\omega_F \mathcal{L}_F(\boldsymbol{\theta}) + \omega_B \mathcal{L}_B(\boldsymbol{\theta}) + \omega_d \mathcal{L}_d(\boldsymbol{\theta})] \tag{4}$$

where $\mathcal{L}_F$, $\mathcal{L}_B$, and $\mathcal{L}_d$ are the cost function concerning the governing equations, the boundary and/or initial conditions, and the target data respectively. The coefficients ω_F, ω_B, ω_d are corresponding multiplied weights.

In general, neural networks have a flexible architecture. Hyperparameters, such as the number of hidden layers, number of neurons, activation functions, optimizers, and so on, can be chosen arbitrarily. Nevertheless, the possible combinations of hyperparameters entail many appropriate architectures for one problem. Indeed, the hyperparameter tuning procedure involves time-consuming trial and error methods and the operator experience [12]. Despite this, neural networks are widely studied in literature due to their ability to perform implicit sensitivity analysis, dimensionality reduction, and their universal approximator's property [13, 14].

Sequential surrogate models

The proposed sequential surrogate (SS) model consists of two sequentially connected surrogate models for approximating the response of a finite-deformation elastoplastic problem (Fig. 1). The SS model takes the process variables and the reference configuration coordinates as inputs to generate a stress distribution as a response. Furthermore, the goal is to construct a global metamodel capable of delivering good approximation for unknown samples in the design space. To this end, picking an appropriate design of experiments (DOE) is crucial. The desired DOE must be spread uniformly over the entire domain utilizing the least amount of samples. Indeed, the last condition is necessary when the problem has many input variables to avoid the curse of dimensionality. Optimal Latin hypercube merges a Latin hypercube sampling with the maximin Morris-Mitchell criteria to achieve a space-filling DOE that involves a relatively small number of sample points [15]. Following the guidelines of Jin et al. [16], given six variables, the number of samples for a LH necessary to represent the design space is 112, i.e., 112 FE simulations.

The first surrogate model is an ANN, entitled to evaluate the total strain and displacement. From now on, this surrogate will be referred to as ANNε. The ANNε takes as inputs a set of process variables $\boldsymbol{p}$ and the finite element integration point coordinates of the reference configuration. The latter are used to evaluate the total strain tensor and the displacement at the integration points at the end of the process. In the second segment of the SS model, a comparative study is conducted between ANN and PINN. The goal is to state which surrogate best imitates the relationship relating total strain and the stress tensor while respecting the strong form of the equilibrium equations. We will refer to the models with ANNσ and PINNσ. The second surrogate model uses as inputs the same set of process variables used for the ANNε, extended with the total strain components and the displacement at the integration points at the last increment. Furthermore, from our findings and as demonstrated by Haghighat *et al.* [6], better fitting of the stress components is achieved if the outputs have dedicated neural networks and, therefore, dedicated trainable parameters. From now on, we will refer to it as a multiple-network structure.

As mentioned, the second surrogate evaluates the stress components through a relation that includes total strain and displacement. In PINNσ the objective is to minimize the cost function as the sum of $\mathcal{L}_F$ and $\mathcal{L}_d$. The former takes into account the equilibrium equations, which, for a 2D case and under the assumption of plane strain, can be written as follows.

$$\mathcal{L} = \omega_d |\boldsymbol{y} - \hat{\boldsymbol{y}}|^2 + \omega_F \left(\left| \frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} \right|^2 + \left| \frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} \right|^2 \right) \tag{5}$$

For this study, we use a k-fold cross-validation resampling technique to judge the quality of the surrogate models dictated by their accuracy. The dataset used to fit the surrogate is initially shuffled and then divided into k subsets of equal dimensions, in which $k-1$ are used for training, and the remaining is used for validation. This procedure is performed for each possible and unique combination of the k groups. Common values used for k are 5, 10, and n, where the latter case represents the leave-one-out cross-validation strategy. A frequent mistake in ANNs is to assess the model generalization ability by monitoring the validation loss. Feature selection should not be performed on data outside the training set for model training, otherwise, it causes a leakage of

information and leads to excessively optimistic predictions. Therefore, training loss and validation loss are utilized to observe if the model is learning in training and if it is flexible enough to predict unseen data. In this regard, a test dataset, i.e., data never seen by the model during training, is built to evaluate the model's accuracy and generalization ability. It should be noted that the overall accuracy of the SS model is estimated through the accuracy in predicting the stress state given the prediction of the ANNε as input. Thus, in testing, the SS model predicts the strain and the stress components given the process parameters and the undeformed configuration.

V-Bending process

V- bending of metal sheets is characterized by large strain and plastic deformation. The proposed SS model is applied to a V-bending process inspired by the work of Wiebenga et al. [17]. In the article, the authors perform a sensitivity analysis to reduce the number of process variables and select those with the highest influence on the primary angle (α). Consequentially, we choose thickness (t), punch radius (R_p), angle of the die (α), and depth (D) as input variables (see Fig. 2). In addition, we include the yield stress (σ_y) and the Young modulus (E) of the sheet material as inputs due to their possible effect on the stress state. In Table 1, the process variables and the respective ranges are listed. It is worth mentioning that the angle α is the same for both the punch and the die, and variation in yield stress will correspond to a shift of the stress-strain curve.

Table 1. Process variables used as inputs for the SS model

Process variables	Range
Thickness [mm]	$0.49 \leq t \leq 0.51$
Radius die [mm]	$0.8 \leq R_d \leq 1.2$
Angle [deg]	$42.5 \leq \alpha \leq 47.25$
Depth [mm]	$0.4 \leq D \leq 0.6$
Young modulus [GPa]	$190 \leq E \leq 210$
Yield stress [MPa]	$280 \leq \sigma_y \leq 300$

To run the FE simulations, the V-bending process is built in Marc Mentat 2022.1, assuming plane strain condition, isotropic hardening, and half of the geometry due to symmetry. The mesh counts 1800 quadrilateral linear elements, split 10 on the thickness and 180 along the length of the sheet.

Hyperparameters tuning

Tuning the hyperparameters of neural networks is an expensive task typically performed using trial and error methods, relying on the operator's expertise. Another approach involves automatic tuners to optimize an objective function based on testing several sets of hyperparameters. Thus, the hyperparameter searching algorithm is responsible for hyperparameter space exploration and computational efficiency. For example, a random search algorithm will test different hyperparameter sets randomly taken from the hyperparameter space. This method is highly inefficient when many hyperparameters are considered in the optimization process (curse of dimensionality). Bayesian optimization algorithms instead use a Gaussian process to sample successive points in the hyperparameter space.

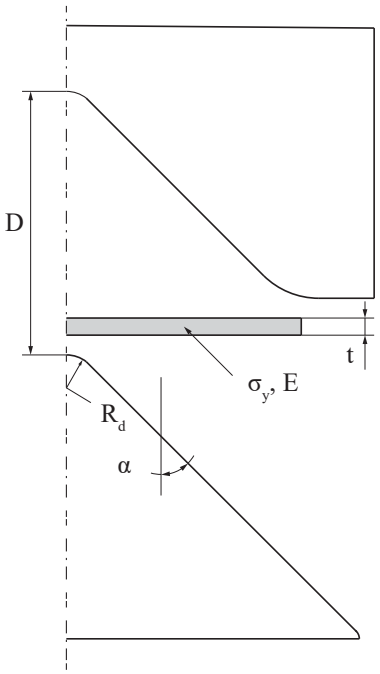

Fig. 2. 2D representation of the V-bending process implemented in the FE solver.

The Bayesian optimization searching algorithm (built in the KerasTuner library and available for Keras and Tensorflow) uses the upper confidence bound as an acquisition function. The hyperparameters selected to tune are the number of hidden layers between 2 and 5, the number of nodes 10 to 50, and the activation functions between rectifier (relu), hyperbolic tangent (tanh), and softplus. The automatic tuner runs the training process for 300 epochs for each hyperparameter set. To prevent overfitting, the searching algorithm aims to minimize the validation loss.

In Fig. 3, five validation losses and the corresponding hyperparameter set, ranked according to the objective function, are depicted. It can be noticed that the three neural networks tend to prefer complex models (higher number of trainable parameters). Concerning the activation function, the ANNε shows better validation using the softplus, while both ANNσ and PINNσ prefer tanh. Based on Fig. 3a, the Bayesian optimization tuner ranks properly the hyperparameter sets for displacement and strain. None of the curves show that the model is overfitting and the lowest validation loss value is reached with hyperparameters set ranked as first. Conversely, in Fig. 3b, the automatic tuner suggests for ANNσ the hyperparameter sets in which the validation loss starts overfitting (around 200 epochs), providing a ranking based only on the minimum value. Hence, the best result is the third option (4 layers, 42 nodes, and tanh), in which the validation loss smoothly decreases over the epochs. Analogously to ANNσ, the first-ranked validation loss of the PINNσ is overfitting due to the model complexity (Fig. 3c). The second set in Fig. 3c, instead, is slowly decreasing and reaches a minimum comparable to the first. It is worth noticing that both chosen hyperparameter sets for the surrogates are similar. Therefore, for the comparative study, we set the number of layers to 4, the number of nodes to 40, and the activation function to tanh.

Results and discussion

The surrogate models are trained using the hyperparameter found with the Bayesian optimization tuner for 3000 epochs per 5 times due to the 5-fold cross-validation. The OLH is used to generate 112 training simulations and 20 testing simulations, being careful not to have repeated sample points. To compare the results with the FE simulations, a mean field is evaluated.

$$\overline{\boldsymbol{y}} = \frac{1}{N_{test}} \sum_{i=1}^{N_{test}} \boldsymbol{y}_i \tag{6}$$

The error measure chosen is the mean absolute error (MAE) to exhibit the error in terms of MPa for stress. As expected, the ANNε approximates both displacement and strain (Fig. 4) well enough. Regarding the strain components, the highest error is located in the bending region under compression, which corresponds to the region in contact with the die, and in proximity of $x = 0$ (symmetry line).

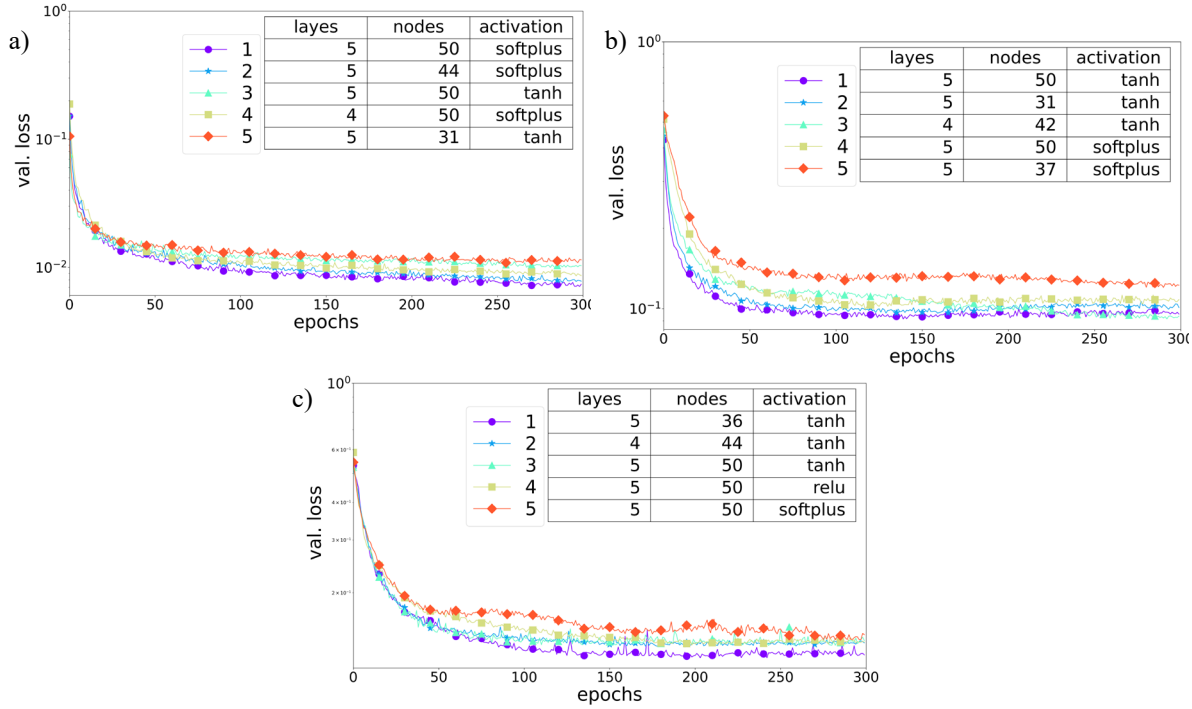

Fig. 3. Validation losses of the best five hyperparameter sets ranked by the automatic tuner: a) ANNε with single network structure, b) ANNσ with multiple network structure, c) PINNσ with multiple network structure.

Fig. 5b and 5c depict the MAE of the stress components for ANNσ and PINNσ concerning the complete loading-unloading cycle. In the bending region and on the tip, the ANNσ fits better the results than PINNσ. In particular close to the tip of the sheet is subject to forging, i.e., the tip is extruded onto the outside, due to extreme cases in which the distance between the die and punch walls is relatively small. Moreover, the predicted σ_{12} using PINNσ shows an error zone in the middle of the sheet in correspondence with the high gradient region observed in Fig. 5a. It is highly likely that the error in these areas is more emphasized due to the computation of the stress divergence. For example, the stress component σ_{12} is derived two times with respect to x_1 and x_2 for both equilibrium equations. Thus, the trainable parameters are forced to satisfy both relations. It is worth noting that σ_{33} error is almost identical for both models as a consequence of the assumption that $\sigma_{13} = \sigma_{23} = 0$. In conclusion, ANN$\varepsilon$, ANN$\sigma$, and PINN$\sigma$ reproduce a distribution of the quantities similar to FEM, but the last two models are not capable of returning a good fitting of stress components, with MAE in the order of $30 - 50\ MPa$.

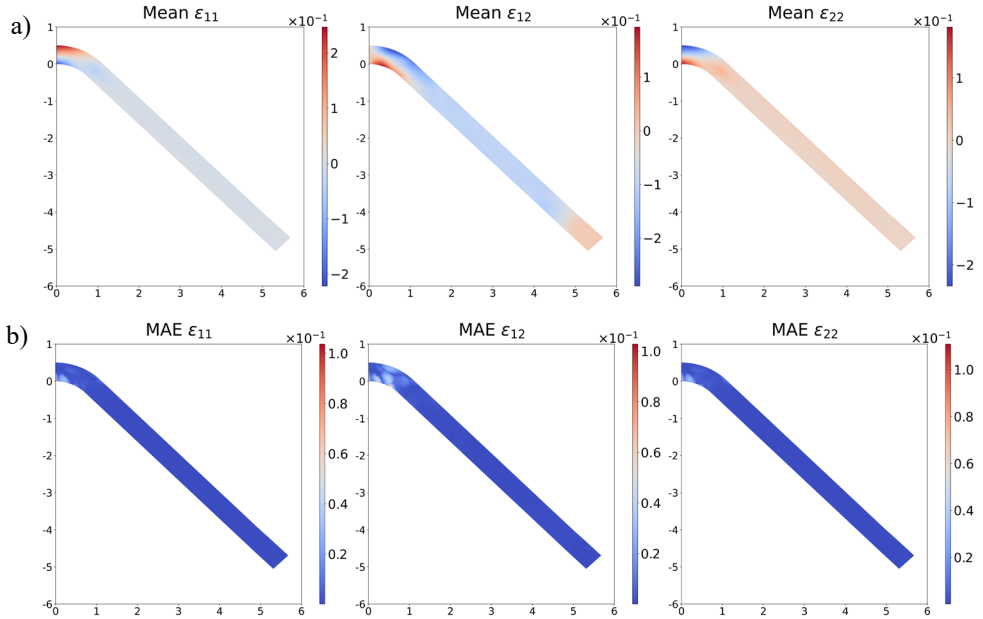

Fig. 4. a) Mean field of the strain and b) mean absolute error of the strain for the complete loading-unloading cycle.

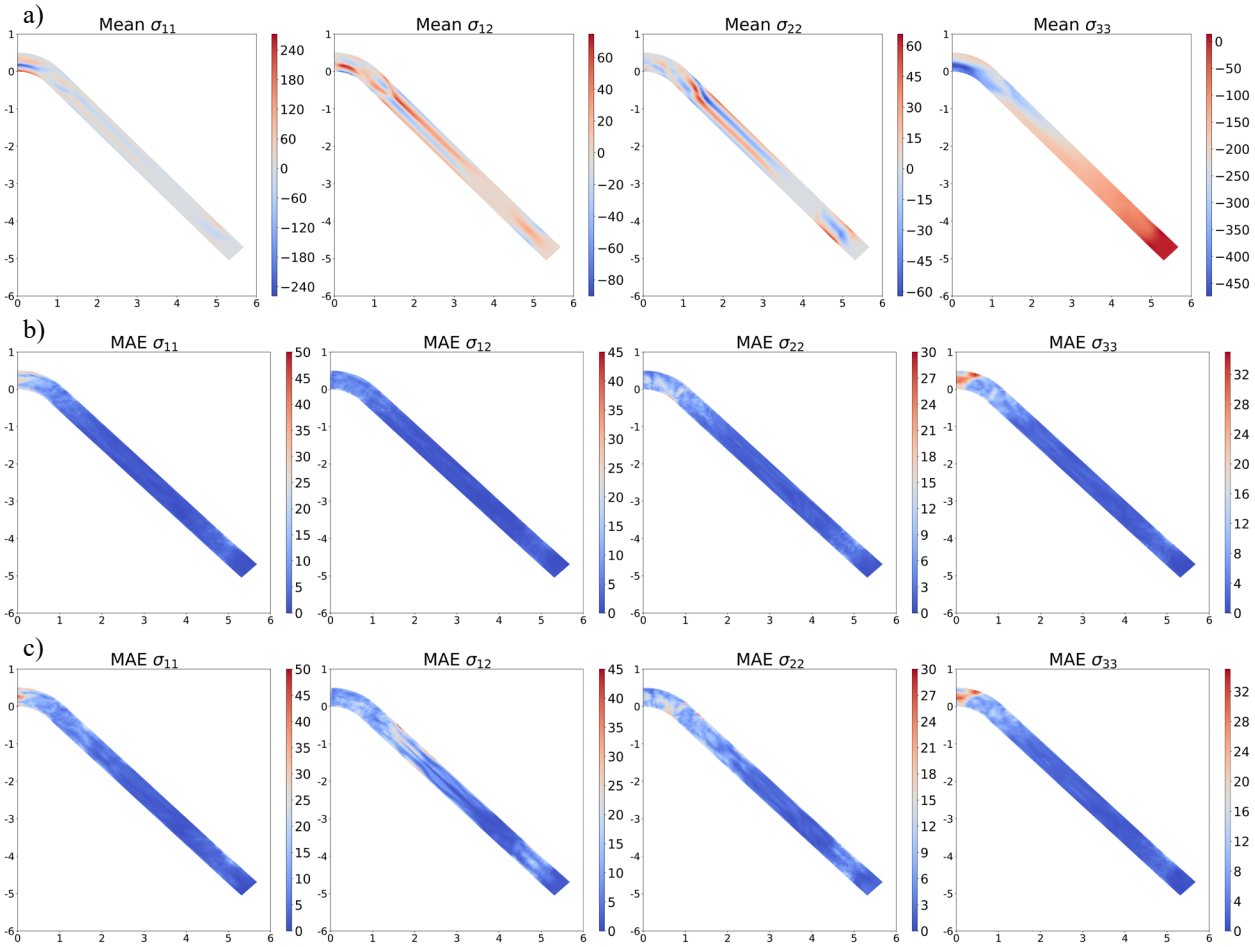

Fig. 5. a) Mean field of the stress, b) mean absolute error of ANNσ, and c) mean absolute error PINNσ for the loading-unloading cycle.

Processes subject to large and plastic deformation are known to be path-dependent. Predicting the stress field of a complete loading-unloading cycle means that information related to the history of the process is lost. Hence, it can be supposed that the surrogate models should lead to better fitting for small-time steps. Employing the same hyperparameters found with automatic tuners, the surrogates are trained using the information collected at a third of the loading step. Once more, a mean stress field is calculated to compare the predicted errors (Fig. 6a). In this case the predictions are plotted with respect to the maximum errors of ANNσ due to a similar error range. This helps emphasize the areas prone to return worse predictions in PINNσ than ANNσ. The evaluated MAE is almost ten to twenty times lower than the one calculated for the previous case. Both models show good generalization ability on the testing dataset, probably due to the low standard deviation among the test simulations. As expected, both models predict σ_{33} similarly since it does not contribute to the equilibrium equation. Regarding the other components, the models exhibit an error concentration in correspondence of the contact point of the sheet with the punch and the die. The inferior performance of PINNσ can be ascribed to the combination of high nonlinearity regions and the constraint on the model parameters enforced by the equilibrium equation. Nevertheless, the model returns a satisfactory fit if compared to ANNσ.

Concerning the computational efficiency, the time needed to train the two surrogate models for the same number of epochs is almost the same. In PINNσ the evaluation of the derivatives implies additional computations, which have a negligible impact on the overall efficiency. If time increments are taken into account, it is expected that the computational efficiency of the PINNσ will drop significantly.

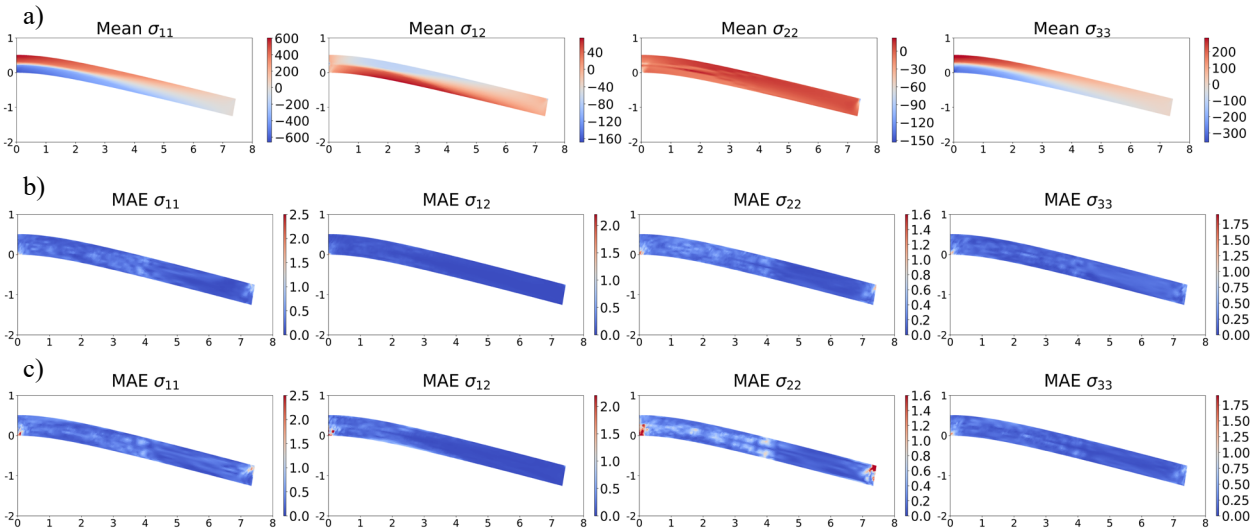

Fig. 6. a) Mean field of the stress, b) mean absolute error of ANNσ, and c) PINNσ for a third of the loading step.

Summary

In this work, we proposed a sequential surrogate model framework to predict the stress field using process variables and initial spatial coordinates as inputs for the surrogate models. Simulations of a V-bending process are run via FE software to collect the necessary data to fit the surrogate models. We have shown the benefit of using automatic tuners as tools for the rapid evaluation of hyperparameters. A comparative study of ANN and PINN has been performed to evaluate their accuracy in predicting stress for the complete loading step and a third of it. In the former case, both surrogates did not fit satisfactorily through the data due to the loss of information related to the path dependency of the plasticity problem. Conversely, the surrogate models delivered better results for a smaller time step. ANN and PINN predictions have comparable results, despite the

latter showing an inappreciable higher error in the contact regions. Not to mention that PINN fits very well through data while satisfying the system governing equations. In conclusion, the subdivision of the loading step in smaller time steps should lead to better predictions. Therefore, adopting recurrent neural networks (RNN) for sequential data could help to consider path dependency and the evolution in time of the stress field. Besides, improving the PDE loss by taking into account time derivatives and constitutive equations may enhance the predictions of PINN. The former is the subject of our future work.

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