Theoretical Evaluation of Capsule Material Strain Hardening on the Deformation of Long Cylindrical Blanks During HIP Process

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Abstract. The paper presents the results of the theoretical study of the effect of strain rate hardening (viscosity) of the capsule material on the process of hot isostatic pressing of a long cylindrical can with powder (so that the influence of lids can be neglected). Relations are obtained that make it possible to evaluate the qualitative estimations of the processes taking place at different stages of the HIP process. Calculations are made for the capsule shrinkage values.

Introduction

Powder Metallurgy and HIP have been established as a promising method of manufacturing large complex shape parts operating at harsh environment and loads. For such parts made of expensive materials high dimensional precision in production is needed. This is especially true for small-scale production of large-sized products, when the cost of trial samples can be very high.

The HIP process that is conducted at high temperature and pressure causes substantial (30-35%) volumetric changes of a capsule filled with powder material and, if necessary, with inserts embedded in certain places, which are subsequently removed from the finished product. During such HIP process, this capsule takes large non-uniform linear deformations, up to 20%. This deformation occurs under changing temperature and with non-uniform temperature field inside the capsule and the capsule itself exhibits plastic and viscous deformations. It is the complexity of modeling the process of distortion of the workpiece that makes it difficult to obtain the product of the required geometry from the "first shot".

Therefore adequate modeling of the geometrical changes during HIP of the resulting product becomes a mandatory technological tool, which can be carried out by mathematical modeling methods, some aspects of which are considered in [1, 2]. In the most general form, the formulation of the problem of mathematical modeling of the process of HIP can be formulated as follows: it is required to design the capsule in such a way that the final shape and dimensions of the powder part obtained after removal of the capsule satisfies the required geometry. Note that due to the specifics of the use of such products, these requirements are quite stringent.

For a qualitative understanding of various aspects of the process of HIP and further comparison of the experimental data with the results of numerical experiments, a theoretical study of the effect of various factors on the process of HIP is necessary. In this paper, we investigate the effect of viscosity of the capsule material on the process of HIP of a long cylindrical billet, which is one of the typical problems to be solved in the production.

Mathematical Statement of the Problem of Modeling of the HIP Process

The powder material is considered as a single plastically compressible medium. The problem of HIPing a plastically compressible material in a capsule made of a non-compressible material with

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strain hardening under conditions of an inhomogeneous nonstationary temperature field is considered.

The general mathematical statement of the problem of modeling the process of HIP includes the following:

Equilibrium equation:

$$div\overline{\sigma} = 0 . \tag{1}$$

where $\overline{\sigma}$ – is the stress tensor.

The equation of the yield surface in the general case is given in the form:

$$\Phi\left(\sigma_{ij}\right) = 0 . \tag{2}$$

The relationship between the stress tensor σ_{ij} and the strain rate tensor ε_{ij} is determined by the associated flow law:

$$\varepsilon_{ij} = \omega \frac{\partial \Phi}{\partial \sigma_{ij}} . \tag{3}$$

where ε_{ij} – is the strain rate tensor.

To describe the behavior of the capsule material and the embedded inserts, the ideal plasticity condition and incompressibility condition are used:

$$S^2 = T^2 . \tag{4}$$

$$div\overline{u} = 0 . \tag{5}$$

where T – is the yield strength, S^2 – the intensity of the deviator of the stress tensor.

To determine the density, the continuity equation is used:

$$\frac{d\rho}{dt} + \rho div\overline{u} = 0 . \tag{6}$$

where $\overline{u}$ – is the displacement speed.

The problem is considered in a quasi-static formulation, the process of deformation is considered to be rather slow, therefore, the acceleration in the equilibrium equations can be neglected. The current formulation of this does not take into account the temperature expansion of the materials.

HIP of a long Cylinder

Let us consider the process of hot isostatic pressing of a cylinder, assuming that its height greatly exceeds the radius. The influence of the capsule top and bottom caps can be then neglected.

Suppose that the axis Z is directed along the axis of the cylinder. The problem is considered in an axisymmetric formulation in a cylindrical coordinate system. We will assume that the region

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$0 < r < R, 0 < z < H$ is occupied by a powder material, and the region $R < r < R+h, 0 < z < H$ – by a capsule.

To describe the mechanical properties of the powder material, we will use the Green's yield criterion [3,4]:

$$\frac{\sigma^2}{f_2^2} + \frac{S^2}{f_1^2} = Y^2 . \tag{7}$$

where $\sigma = \frac{1}{3}\left(\sigma_r + \sigma_\varphi + \sigma_z\right)$ – the average stress; $S^2 = \frac{3}{2}\left(\sigma_{ij} - \sigma\delta_{ij}\right)\left(\sigma_{ij} - \sigma\delta_{ij}\right)$ –intensity of stress tensor deviator; Y – yield stress of a fully dense powder material; f_1 and f_2 – experimental functions of relative density ρ.

We assume that the rate of deformation along the Z symmetry axis is constant throughout the volume of the powder material and the capsule. In view of the incompressibility condition (5), in the capsule material, and the condition of continuity of the radial velocity $r = R$, we obtain the strain rates are:

1) at $0 < r < R, 0 < z < H$ (in the powder material)

$$\varepsilon_z = -\varepsilon\,,\ \varepsilon_r = -A\,,\ \varepsilon_\varphi = -A\,. \tag{8}$$

2) at $R < r < R+h, 0 < z < H$ (in the capsule material)

$$\varepsilon_z = -\varepsilon\,,\ \varepsilon_r = \frac{1}{2}\varepsilon + \frac{1}{2}\varepsilon\frac{R^2}{r^2} + A\frac{R^2}{r^2}\,,\ \varepsilon_\varphi = \frac{1}{2}\varepsilon - \frac{1}{2}\varepsilon\frac{R^2}{r^2} - A\frac{R^2}{r^2}\,. \tag{9}$$

In this case, the total power of the internal forces in the powder material is:

$$W_p = \pi R^2 H \frac{Y}{3}\sqrt{\left(9f_2^2 - 2f_1^2\right)\left(2A+\varepsilon\right)^2 + 6f_1^2\left(2A^2+\varepsilon^2\right)}. \tag{10}$$

In addition, the total power of the internal forces in the capsule:

$$W_c = 2\pi HT\sqrt{\frac{2}{3}}\int_R^{R+h}\sqrt{\frac{3}{2}\varepsilon^2 + 2\left(A+\frac{1}{2}\varepsilon\right)^2\frac{R^4}{r^4}}\,rdr\,. \tag{11}$$

The external HIP pressure is assumed to be equal P. In this case, the total power of the external forces at the surface of the capsule can be expressed as follows:

$$N = P\pi HR^2\left(\varepsilon + 2A\right). \tag{12}$$

Assuming $\left(\varepsilon + 2A\right) = 1$ and denoting H_0 the value H and R_0 the value R for $\rho = \rho_0$, we get:

$$H(\rho) = H_0 \exp\left\{ -\int_{\rho_0}^{\rho} \frac{Y}{Y \cdot \frac{3f_1^2}{\sqrt{\left(9f_2^2 - 2f_1^2\right)}} + 2\sqrt{3}\frac{h}{R}T} \cdot \frac{f_1^2}{\sqrt{\left(9f_2^2 - 2f_1^2\right)}} \frac{d\rho}{\rho} \right\}. \quad (13)$$

The value $R(\rho)$ is given by:

$$R^2(\rho) = R_0^2 \frac{\rho_0}{\rho} \cdot \frac{H_0}{H}. \quad (14)$$

Note that the final geometry depends significantly on the ratio of the yield strength of the powder material and the yield point of the capsule material. In actual processes with relatively fast ramp of temperature and pressure during HIP, the yield strength of the capsule depends essentially on the rate of deformation. Let us examine the influence of this factor on a specific example.

Let the pressure grow with time as $P = P_0 t$. Suppose that the yield strength of the capsule material is dependent on the deformation rate and has the form:

$$T = B\left(\frac{3\sqrt{2}}{2}\right)^{\alpha} \left(\varepsilon^2 + \varepsilon A + A^2\right)^{\frac{\alpha}{2}} = B\left(\frac{3\sqrt{2}}{2}\right)^{\alpha} A^{\alpha} \left(x^2 + x + 1\right)^{\frac{\alpha}{2}}. \quad (15)$$

where $x = \frac{\varepsilon}{A} \le 1$.

Considering dimensionless time τ according to relation $t = \frac{Y}{P_0}\tau$ and dimensionless $\overline{A}$ according to $A = \frac{P_0}{Y}\overline{A}$ (do not write line further), we get system of equations determine the solution of the entire Task – functions $A(\tau), \rho(\tau), x(\tau)$.

$$\frac{1}{3}\frac{\left[9f_2^2(2+x) + 2f_1^2(1-x)\right]}{\sqrt{9f_2^2(2+x)^2 + 2f_1^2(1-x)^2}} + \left[\frac{2f_1^2}{3\gamma}\frac{(1-x)(x+2)}{x\sqrt{\left(1+x+x^2\right)}}\frac{1}{\sqrt{9f_2^2(2+x)^2 + 2f_1^2(1-x)^2}}\right] = \tau. \quad (16)$$

where $\gamma = \frac{B}{Y}\left(\frac{3\sqrt{2}}{2}\frac{P_0}{Y}\right)^{\alpha}$ characterizes the pressure loading rate and α determines the relation of the capsule yield strength from the strain rate (strain hardening).

The equation for the powder density:

$$\frac{d\rho}{d\tau} = \rho A\left(2+x\right). \tag{17}$$

The reduction of the radius is determined from the relation $\frac{dR}{d\tau} = -AR$, then:

$$R = R_0 \exp\left(-\int_0^{\tau} A(\tau) d\tau\right). \tag{18}$$

where R_0 – the initial radius.

From the mass conservation law for powder volume we get:

$\pi\rho_0 R_0{}^2 H_0 = \pi\rho R^2 H$

Accounting (18) it is possible to write:

$$H = H_0 \frac{\rho_0}{\rho} \exp\left(2\int_0^{\tau} A(\tau) d\tau\right). \tag{19}$$

For the non-compressible capsule material we can write:

$2\pi R_0 h_0 H_0 = 2\pi R h H$

where h_0 – the initial capsule wall thickness.

Using (18, 19), we get:

$$h = h_0 \frac{\rho}{\rho_0} \exp\left(-\int_0^{\tau} A(\tau) d\tau\right). \tag{20}$$

According to (17-20), we get:

$$A = \frac{1}{\sqrt{\left(1+x+x^2\right)}} \left[\frac{2 f_1^2}{\gamma\sqrt{3}} \cdot \frac{R_0}{h_0} \cdot \frac{\rho_0}{\rho} \cdot \frac{1-x}{x} \frac{1}{\sqrt{9 f_2^2 \left(2+x\right)^2 + 2 f_1^2 \left(1-x\right)^2}} \right]^{\frac{1}{\alpha}}. \tag{21}$$

The system of equations of (16-21) with the corresponding initial conditions defines the final solution of the Task.

For the qualitative analysis, let us consider functions $f_1(\rho)$ и $f_2(\rho)$ as follows:

$$f_1{}^2(\rho) = \frac{3}{2+\rho^2} \cdot \left(\frac{\rho-\rho_p}{1-\rho_p}\right)^m, \qquad f_2{}^2(\rho) = \frac{1}{3\left(1-\rho^2\right)} \cdot \left(\frac{\rho-\rho_p}{1-\rho_p}\right)^m$$

Below we can see in Table 1 the shrinkage values – the ratios of the initial and final dimension of the HIPed capsule as a function of the material and process parameters: R_f – is the final radius value at full densification, H_f – is the final height value. $e_r = \frac{R_0}{R_f}$, $e_z = \frac{H_0}{H_f}$

Calculations have been done at $m = 2$, $\frac{h_0}{R_0} = 0,025$.

Table 1. Radial and axial shrinkage values.

№	γ	α	e_r	e_z	$\frac{e_z}{e_f}$
1	10	0.1	1.199	1.043	0.870
2		0.2	1.192	1.055	0.885
3		0.3	1.189	1.061	0.892
4	5	0.1	1.184	1.071	0.904
5		0.2	1.180	1.078	0.914
6		0.3	1.176	1.084	0.922
7	2	0.1	1.167	1.102	0.945
8		0.2	1.164	1.107	0.951
9		0.3	1.161	1.112	0.957
10	1	0.1	1.157	1.120	0.967
11		0.2	1.156	1.123	0.972
12		0.3	1.154	1.127	0.976
13	15	0.1	1.214	1.017	0.838
14		0.2	1.201	1.041	0.866
15		0.3	1.196	1.048	0.876
16	20	0.1	1.211	1.023	0.845
17		0.2	1.203	1.037	0.863
18		0.3	1.200	1.042	0.868
19	25	0.1	1.220	1.008	0.827
20		0.2	1.206	1.032	0.856
21		0.3	1.203	1.036	0.862

Conclusions

Analytic relationships have been obtained within the framework of the analysis of the process of the HIP of a long cylindrical billet, making it possible to evaluate the qualitative picture of the processes occurring at different stages of deformation of the cylinder. The effect of the viscous properties (strain hardening) of the capsule material on the final geometry of the product was studied. Calculations have been made for the ratio of the initial cylinder sizes to the final ones (HIP shrinkages), as a function of the process and material parameters.

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