A New Constitutive Modeling for Hot Isostatic Pressing of Powders

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Abstract. Regarding the fact that a major deformation and thus densification of powder takes place during both ramp and holding stages in the process of hot isostatic pressing, a more precise model must be applied to describe the inelastic behavior of powder. For this aim, a plastic-viscoplastic modeling is developed in this work. This modeling is based on considering two logical phenomena, one is the strain hardening of powder particles during isostatic pressing and the other one is the increase in relative density following the particle deformation. Based on these facts, the plastic-viscoplastic model is formulated for the isostatic pressing. Finally, the numerical and experimental results will be compared and discussed.

Introduction
Hot Isostatic Pressing (HIP) is a sustainable manufacturing process to produce near net/net shape products in small to large sizes. The powder poured in a thin metallic container is compacted by isostatic pressure of an inert gas through applying some cycles of temperature and pressure. Fig. 1 is a schematic demonstration of the process.

In a general consideration, material inelastic deformation and thus densification take place at both ramp and holding stages shown in Fig. 1. From a phenomenological point of view, a pure plastic model is not able to predict any densification at the holding stage because pressure is held constant at this stage. On the other hand, an ideal viscoplastic model is not capable to describe material deformation at the ramp stage if the pressure ramp occurs at low temperatures. Therefore, a more complete modeling other than pure plastic or ideal viscoplastic model is needed. To achieve such a model, there are two solutions, either a unified viscoplastic or a plastic-viscoplastic model. In this research, we select the plastic-viscoplastic approach due to this fact that the parameters introduced in this modeling are relatively easy to be identified by some classical experiments.

Figure 1. Schematic illustration of HIP process and example of temperature and pressure cycles.

Theoretical Basis of the Modeling
In a powder compaction process, the inelastic deformation of particles is responsible for material densification. Evidently, density or relative density (density of porous material to the density of the same material without porosity) is an isotropic hardening parameter. However, along with
densification, particle strain hardening should be considered as another isotropic hardening parameter if compaction is assumed in a general condition of temperature and stress. Before presenting the final equation of the model, the material strain hardening is clarified in the following.

Effect of Strain Hardening. Classical theory for material deformation in a uniaxial test considers an initial elastic domain before achieving an elastic limit beyond which a permanent deformation begins to happen. However, according to the microplastic phenomena that may occur during deformation, one can consider a plastic deformation from the beginning of loading so that the yield stress \( \sigma_y \) can be expressed as the following:

\[
\sigma_y = k(e^p)^m. \tag{1}
\]

Where \( e^p \) is the axial plastic strain and \( k \) and \( m \) are two material parameters. For a multiaxial loading, the material yield point is defined by \( k(e_{eq}^p)^m \) where \( e_{eq}^p \) is the equivalent plastic strain defined by the following:

\[
e_{eq}^p = \int \frac{1}{2} \varepsilon_{ij} \dot{\varepsilon}_{ij} \, dt. \tag{2}
\]

Plasticity of Powder and Yield Function. As a powder is porous, the yield function is dependent on the first invariant of stress in addition to the second invariant of deviatoric stress. Therefore, regarding Eq. 1, the yield function given by Green [1] can be extended to the following form:

\[
\left( h S_1^2 + \frac{3}{2} g S_2^2 \right)^{\frac{1}{2}} - k e_{eq}^p m = 0. \tag{3}
\]

Where \( S_1 \) is the first invariant of stress tensor and \( S_2 \) equal to \( [\bar{\sigma}_{ij} \bar{\sigma}_{ij}]^{\frac{1}{2}} \). \( \bar{\sigma}_{ij} \) is the component of deviatoric stress. Function \( h \) can be defined as a function of relative density \( (\rho) \) with \( \alpha \) and \( \beta \) as two constants.

\[
h = \alpha \left( \frac{1-\rho}{\rho-\rho_0} \right)^{\beta}. \tag{4}
\]

The parameter \( \rho_0 \) is the relative density in a powder situation with point-like contacts between particles. It should be pointed out that Eq. 3 defines a loading surface characterized by a given pair of \((e_{eq}^p, \rho)\) in the stress space. In the case of isostatic pressing, the yield function is simplified to the following equation with having \( S_2 = 0 \):

\[
\sqrt{h} S_1 - k e_{eq}^p m = 0. \tag{5}
\]

It is worthy to mention that \( S_1 = 3P \) in isostatic pressing where \( P \) is the applied pressure. And also:

\[
e_{eq}^p = \sqrt{2} e_1^p. \tag{6}
\]
The mass conservation implies that:

\[ \dot{\rho}^p = 3 \rho \dot{\epsilon}_1^p. \]  

(7)

We finally have:

\[ \dot{\epsilon}_1^p = \frac{2hS_1}{2\sqrt{2}mk^2 \epsilon_{eq}^{2m-1} - 3 \rho S_1^2 \frac{d\rho}{d\rho}} \dot{\hat{S}}_1. \]  

(8)

As a pair of \( \rho, \epsilon_{eq} \) is obtained uniquely by plastic mechanisms of deformation in the above formulation, we reshow \( \dot{\epsilon}_1^p \), \( S_1 \) and \( \hat{S}_1 \) as \( \dot{\epsilon}_1^{p*} \), \( S_1^* \) and \( \hat{S}_1^* \), respectively and rewrite Eq. 8 as the following equation:

\[ \dot{\epsilon}_1^{p*} = \frac{2hS_1^*}{2\sqrt{2}mk^2 \epsilon_{eq}^{2m-1} - 3 \rho S_1^* \frac{d\rho}{d\rho}} \dot{\hat{S}}_1^*. \]  

(9)

Ideal Viscoplasticity of Powder. The intention from ideal viscoplasticity is in fact the permanent deformation resulted from the secondary creep mechanisms of deformation. For this aim, the viscoplastic dissipation potential of the Odqvist’s type (\( \phi \)) is used [2]:

\[ \phi = A \sigma_{eq}^{n+1}. \]  

(10)

Where \( A \) and \( n \) are two material’s parameters depending on temperature. The equivalent stress (\( \sigma_{eq} \)) is selected as the one suggested by Abouaf [3]:

\[ \sigma_{eq} = \left( fS_1^2 + \frac{3}{2}cS_2^2 \right)^{\frac{1}{2}}. \]  

(11)

Where \( f \) and \( c \) are two functions of relative density. In the viscoplastic case, we have:

\[ \dot{\epsilon}_{ij}^{vp} = \frac{\partial \phi}{\partial \sigma_{ij}}. \]  

(12)

We can therefore obtain the following equation in the case of isostatic pressing:

\[ \dot{\epsilon}_1^{vp} = A(3P)^n f^{\frac{n+1}{2}}. \]  

(13)

Function \( f \) is considered as the following equation in this study.

\[ f = a \left( \frac{\rho - \rho_0}{\rho_0} \right)^b. \]  

(14)

Plastic-Viscoplastic Modeling. In a material loading process, one can generally assume two simultaneous deformations:

- An ideal viscoplastic deformation at each stress state,
- A plastic deformation due to the stress increment at each stress state.
In the case of isostatic pressing of a powder, the axial viscoplastic strain rate can simply be calculated by Eq. 13. This viscoplastic strain partially contribute to the densification. However, by assumption of ideal viscoplasticity, no material hardening happens by viscoplastic mechanisms of deformation. Concerning the plastic part of deformation, it contributes not only to densification but also to the increase of $\varepsilon_{eq}^p$. Now, imagine a pair of $\rho$ (achieved by both plastic and viscoplastic mechanisms) and $\varepsilon_{eq}^p$. It is evident that since $\rho$ is obtained by both mechanisms, this pair would be achieved at a value of $S_1$ inferior to the value of $S_1^*$ that is required to obtain the same pair of $(\varepsilon_{eq}^p, \rho)$ by uniquely plastic mechanism (Fig. 2):

\[
\begin{array}{c}
0 \\
M \\
M^* \\
S_1 (\varepsilon_{eq}^p - \rho) \\
S_1^* (\varepsilon_{eq}^p - \rho) \\
S_1
\end{array}
\]

**Figure 2. Comparison of $S_1$ and $S_1^*$ to give a given pair of ($\varepsilon_{eq}^p, \rho$).**

It is therefore necessary to do some logical assumption to calculate $\dot{\varepsilon}_1^p$ at point $M$ in Fig. 2 specified with $S_1$. As point $M^*$ is the radial projection of $M$ on the loading surface characterized by $(\varepsilon_{eq}^p, \rho)$, we make the following hypothesis:

$\dot{\varepsilon}_1^p$ at point $M$ is proportional to $\dot{\varepsilon}_1^{p*}$ at $M^*$ when the same value is selected for $S_1$ and $S_1^*$ ($S_1 = S_1^*$). The proportionality factor ($\omega$) is considered to be equal to $S_1 / S_1^*$. Therefore, regarding Eq. 9, we have:

\[
\dot{\varepsilon}_1^p = \omega \frac{2hS_1}{2\sqrt{2mk^2\varepsilon_{eq}^p \frac{2^{m-1}}{3\rho S_1^2\frac{d}{d\rho}}}} S_1^*.
\]  \hspace{1cm} (15)

Or:

\[
\dot{\varepsilon}_1^p = \frac{2hS_1}{2\sqrt{2mk^2\varepsilon_{eq}^p \frac{2^{m-1}}{3\rho S_1^2\frac{d}{d\rho}}}} S_1.
\]  \hspace{1cm} (16)

Therefore, by a plastic-viscoplastic mechanism of deformation, the viscoplastic and plastic strain rates defined by Eqs. 13 and 16, respectively, happen during isostatic pressing.

**Materials and Experiment**

Some samples were prepared at Bodycote Inc.. The samples were some cylindrical containers including stainless steel 316L powder. The container was from low carbon steel. The container thickness, internal diameter and internal height were 0.89, 17.3 and 76.2 mm, respectively [4]. The value of $\rho_0$ was measured equal to 0.6684. The applied isostatic pressing included two steps. First, the sample was heated to 1115°C at a very low pressure. The obtained relative density at this stage is 0.71. Then at 1115°C, different pressure ramps were applied to the sample to achieve a final pressure. Then the heating was cut and the sample was cooled. The relative density attributed to the selected pressure ramp and final pressure was measured. The applied pressure ramp and the final pressure are cited in Table 1. It should be mentioned that each test was repeated three times and a good stability in the results were observed.
Table. 1. Values of pressure ramp and the final pressure achieved in HIP experiments

<table>
<thead>
<tr>
<th>P (MPa)</th>
<th>2</th>
<th>7</th>
<th>18</th>
<th>24</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure rate (MPa/min.)</td>
<td>1.8</td>
<td>3.3</td>
<td>1</td>
<td>0.9</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Materials Identification

Despite the presence of container, it is approximated that the densification of stainless steel 316L powder occurs without a major influence of container on the pressure transfer to the powder at 1115°C. Therefore, it is necessary to identify the stainless steel 316L parameters. The following parameters are obtained from literature [4]:

\[
\ln A = -\frac{44229}{T} + 17.125. \tag{17}
\]
\[
 n = -0.0057T + 9.926. \tag{18}
\]
\[
 m = -0.00017T + 1655. \tag{19}
\]
\[
 k = -0.8886T + 1655. \tag{20}
\]
\[
 a = 0.23 \tag{21}
\]
\[
 b = 1.1 \tag{22}
\]

Where \( T \) is temperature in Kelvin. It is also necessary to identify function \( h \).

Numerical Simulation

A Fortran program based on Eqs. 13 and 16 was written and run to simulate the powder densification under applying different pressure ramps (Table 1) at 1115°C. The function \( h \) was adjusted so that the relative densities at the final pressure, given by experiment and by simulation, were comparable. Therefore, the values of \( \alpha \) and \( \beta \) were approximated as 15 and 0.7. The simulation results are shown and compared with the experimental ones in Fig. 3.

![Figure 3. Comparison of plastic-viscoplastic simulation results with the experimental ones.](image)

Conclusion

A plastic-viscoplastic model was introduced for inelastic deformation and thus densification of powders in the HIP process. While relative density is an isotropic hardening parameter for the plastic and viscoplastic behaviors, it is considered that the plastic behavior is also influenced by the equivalent plastic strain as an additional isotropic hardening parameter. With this consideration, the plastic and viscoplastic components of the axial strain in isostatic pressing were derived in this work. The model was applied to the HIP of stainless steel 316L powder at 1115°C while different pressure ramps were applied to the powder. The experimental and simulation...
results were compared. Satisfactory results were obtained. The model must be generalized to the deviatoric stress loading in the next step.

References