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Virtual element method for damage modelling of two-dimensional metallic lattice materials

Marco Lo Cascio[1,a *], Ivano Benedetti[1,b] and Alberto Milazzo[1,c]

[1]Dipartimento di Ingegneria, Università degli Studi di Palermo, Viale delle Scienze, Ed. 8, Palermo, 90128, Italy

[a]marco.locascio01@unipa.it, [b]ivano.benedetti@unipa.it, [c]alberto.milazzo@unipa.it

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Abstract. Additively-manufactured metallic lattice materials are a class of architectured solids that is becoming increasingly popular due to their unique cellular structure, which can be engineered to meet specific design requirements. Understanding and modelling the damage in these innovative materials is a significant challenge that must be addressed for their effective use in aerospace applications. The Virtual Element Method (VEM) is a numerical technique recently introduced as a generalisation of the FEM capable of handling meshes comprising an assemblage of generic polytopes. This advantage in creating domain discretisation has already been used to model the behaviour of materials with complex microstructures. This work employs a numerical framework based on a nonlinear VEM formulation combined with a continuum damage model to study the fracture behaviour of two-dimensional metallic lattice material under static loading. VEM's effectiveness in modelling lattice failure behaviour is assessed through several numerical tests. The influence of micro-architecture on the material's failure behaviour and macroscopic mechanical performance is discussed.

Introduction

The computational modelling of the behaviour of lattice materials is an active field of research aimed at complementing the experimental activity in the quest for a better understanding of the potential of these materials in engineering applications. The Virtual Element Method (VEM) [1] is a recent generalisation of the Finite Element Method (FEM) for the treatment of general polygonal/polyhedral mesh elements that has been already used for several problems in structural mechanics [2,3,4,5,6] applications.

Formulation

For the lowest-order VEM formulation herein adopted, for a general polygonal virtual element E, the element degrees of freedom are the values of the components of the displacement at each of its n vertex, collected into the vector $\boldsymbol{u}_E$. The displacements field is expressed as $\boldsymbol{u} = \boldsymbol{N}(x,y)\,\boldsymbol{u_E}$, where $\boldsymbol{N}(x,y)$ is the matrix containing the virtual shape functions $\boldsymbol{N_v}(x,y)$ associated with each vertex $\boldsymbol{v}$. Shape functions are known only on the element edges of $\boldsymbol{E}$, where they are globally continuous linear polynomials. An explicit expression for the strains is unavailable because the shape functions $\boldsymbol{N_v}$ are not explicitly known within the polygonal element. An approximated constant strain field $\boldsymbol{\varepsilon_\Pi}$ is assumed within each element, which can be computed from the degrees of freedom $\boldsymbol{u_E}$ as $\boldsymbol{\varepsilon}_\Pi = \boldsymbol{\Pi}_E\,\boldsymbol{u}_E$, where $\boldsymbol{\Pi}_E \in R^{3\times 2n}$ is the matrix representation of a projection operator defined as

$$\boldsymbol{\Pi}_E = \frac{1}{A_E}\sum_{v=1}^{n}\int_{e_v} \boldsymbol{N}_v^E\ \boldsymbol{N}(x,y)\,ds \tag{1}$$

where A_E is the area of the polygonal element E, bounded by its n edges e_v and $\boldsymbol{N}_v^E$ is the matrix containing the components n_x and n_y of the outward unit normal vector over each edge. Since the

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virtual shape functions $\boldsymbol{N}_v$ on the element edges are known polynomials, the integrals appearing at the right-hand side of previous equation are exactly computable.

The tangent stiffness matrix $\boldsymbol{K}_E$ for a general virtual element E is given by the sum of two terms. The first term, named the consistency term, is given by

$$\boldsymbol{K}_E^c = A_E \, \boldsymbol{\Pi}_E^T \, \boldsymbol{C} \, \boldsymbol{\Pi}_E \tag{2}$$

where $\boldsymbol{C}$ is the material tangent stiffness tensor in Voigt notation. $\boldsymbol{K}_E^s$ is a stabilization term whose presence is motivated by the need to avoid zero-energy modes not associated with rigid body motions. The loss of material integrity is governed by the internal damage variable ω, $0 \leq \omega \leq 1$. The constitutive equations for an isotropic damage model is

$$\boldsymbol{\sigma} = (1 - \omega) \, \boldsymbol{C}^0 \, \boldsymbol{\varepsilon}_\Pi \tag{3}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}_\Pi$ collect the Voigt components of the stress and strain respectively, and $\boldsymbol{C}^0$ is the elasticity matrix for the pristine elastic material. The evolution of damage is governed by the linear softening law

$$\omega(\kappa) = \frac{\kappa_f}{\kappa_f - \kappa_0}\left(-\frac{\kappa_0}{\kappa}\right) \tag{4}$$

and loading-unloading conditions

$$f(\boldsymbol{\varepsilon}, \kappa) = \varepsilon_{eq}(\boldsymbol{\varepsilon}) - \kappa \leq 0, \qquad \dot{\kappa} \geq 0, \qquad f(\varepsilon, \kappa)\,\dot{\kappa} = 0 \tag{5}$$

in which f is the damage loading function, ε_{eq} is the modified Von Mises equivalent strain [8], and κ is an internal variable that corresponds to the maximum level of equivalent strain ever reached in the previous history of the material. The stress at a generic point $\boldsymbol{x}$ and at a generic loading increment λ is given by $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\lambda, \boldsymbol{x}, \boldsymbol{\varepsilon}_\Pi, \mathcal{H})$, where $\mathcal{H}$ contains the history variables of the damage model. The tangent material stiffness matrix $\boldsymbol{C}$ at a certain time t is consistently computed from the constitutive law as

$$\boldsymbol{C}(t, \boldsymbol{x}, \boldsymbol{\varepsilon}_\Pi, \mathcal{H}) = \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}_\Pi} \tag{6}$$

To avoid damage localisation and mesh dependency of the solution, an integral-type nonlocal damage model has been employed. The adopted weight function is the truncated quadratic polynomial function [4].

Numerical Tests

The specimen design used for the following numerical tests is based on the Extended Compact Tension, EC(T), specimen, shown in Fig. 1(a). The EC(T) has been developed for fatigue and fracture testing of solid (dense) materials but has been adapted and used for characterizing lattice structures. The rectangular specimen consists of repeated unit cells of side length l. Each specimen is eleven unit cells wide (W), forty unit cells high (H, and two unit cells thick (t). Only half of the EC(T) specimens were modelled by applying appropriate symmetry boundary conditions as shown in Fig. 1(b). The unit cell of the lattice structure studied in this work is shown in Fig. 2(a) and is based on a two-dimensional representation of a body-centred cubic (GBCC) unit cell (Fig. 1(b)). Each unit cell has external dimensions of $l \times l$, with $l = 3.5$ mm. Each lattice unit cell is discretised with 176 polygonal virtual elements (Fig. 2(c), and the numerical model has 157882 degrees of freedom. Simulations are carried out under displacement control and plane-strain

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assumption. The material selected as the constituent material is an additive manufactured Ti-6Al-4V alloy whose mechanical and damage properties are [7]: Young's Modulus $E = 123$ GPa, Poisson's ratio $\nu = 0.3$, yield strength $\sigma_y = 932$ MPa and fracture strain $\kappa_f = 0.1105$. The interaction radius has been set to $R = 2$ mm. Two different unit cell configurations have been analysed, with truss diameter $d = 0.5$ mm and $d = 0.75$ mm. For each configuration, numerical tests have been performed with crack length $a = 2l$ and $a = 3l$.

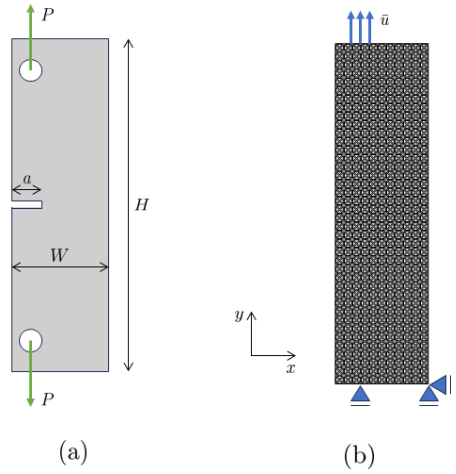

Figure 1: (a) Extended Compact Tension, EC(T), specimen; (b) computational model.

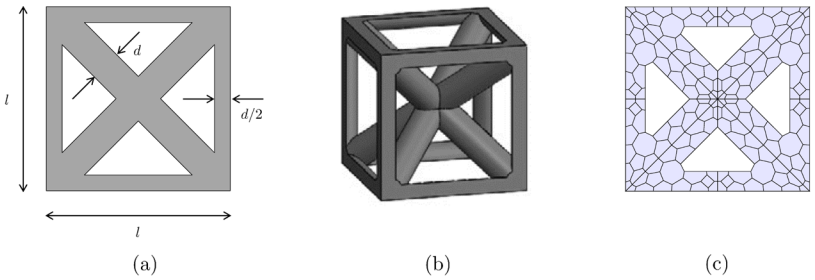

Figure 2: (a) 2D representation of the GBCC unit cell geometry; (b) actual 3D geometry.

Conclusions

A nonlinear VEM formulation combined with a continuum damage model has been employed to model the fracture behaviour of two-dimensional metallic lattice material under static loading. VEM's effectiveness in modelling complex morphologies such as lattice structures has been verified. The load as a function of the load-line displacement remains relatively linear for most specimens until close to reaching the critical load and a reduction of the critical load with initial crack extension can be observed for both unit cell configuration.

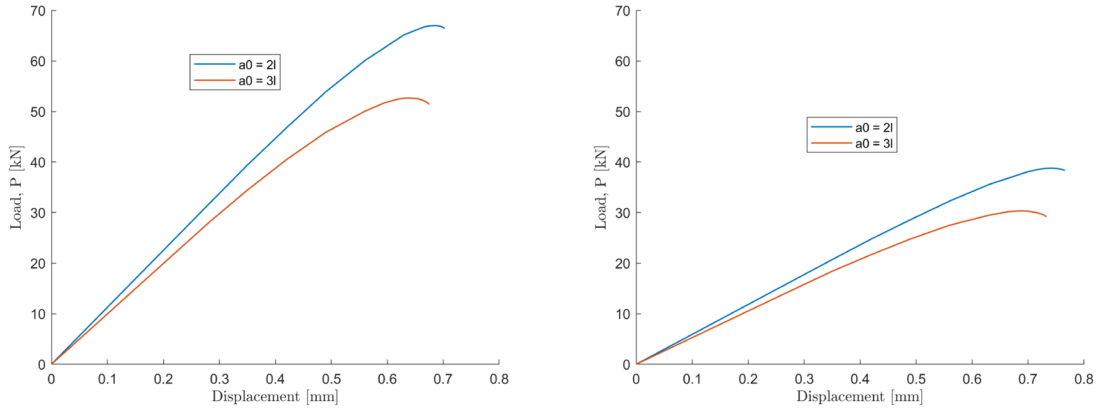

Figure 3: Load-displacement responses of the (a) 0.75 mm and (b) 0.50 mm uniform diameter EC(T) specimens showing the applied load as a function of load-line displacement for different initial crack lengths.

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