Immersed boundary-conformal coupling of cylindrical IGA patches

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Abstract. In this work an Immersed-Boundary-Conformal coupling method for coupling shells is presented. The linear elastic static analysis is carried out using the Kirchhoff-Love shell model. The variational statement is discretized with an Isogeometric Analysis approach. The method employs auxiliary shell patches conformal to the interfaces which are coupled to the main ones using an Interior Penalty formulation. Results showing the potential of such approach to study multi-component shell structures are provided.

Introduction

Cylindrical beams are extensively used in many engineering sectors due to their high stiffness-to-weight ratios, with typical examples that can be found in civil, marine, and automotive sectors. Truss structure relying on cylindrical beams are usually investigated using one-dimensional elements that allow to characterize the general mechanical response of the assembly but do not describe accurately the displacement and stress fields near the interfaces between elements. Here, a two or three-dimensional analysis is typically required.

The Isogeometric Analysis (IGA) method [1] employs spline basis for both the description of the geometry and the approximation of the solution, allowing for a straightforward incrementation of the polynomial order while at the same time limiting the increase in the number of degrees of freedom. However, describing the geometry obtained from the intersection of two cylinders with splines is not an easy task. A possible approach consists in using an Immersed-Boundary description for each cylindrical patch. In details, an additional curve is used to embed each of the two parametric domains to describe the boundary corresponding to the interface between the intersecting cylinders. This approach results in a subdivision of each parametric domain in an active and a non-active region, and in a trimming of the elements in between the two. However, such approach requires a weakly imposed continuity between the patches and an ad-hoc refinement strategies for the region near the interface if a higher resolution is required.

In this work, the proposed coupling strategy relies on an auxiliary patch for each cylinder. These are constructed conformal to the interface on one side. As such, while the coupling of the rotational degrees of freedom is still performed in a weak sense, the coupling of the displacements is obtained in a strong sense by merging corresponding degrees of freedom. The other side of each patch is used to define the boundary of the corresponding parametric domain, and to enforce the continuity using an Interior Penalty approach. The main advantage of such strategy is the arbitrary of the auxiliary patches, that allows for a straightforward local refinement in the region close to the intersection.

To demonstrate the potential of the proposed approach, preliminary numerical results are provided regarding the creation of a boundary conformal patch for a Kirchhoff plate and the coupling of cylindrical shell through the Interior Penalty approach.

Kirchhoff-Love shells

The starting point of the proposed method is the Kirchhoff-Love shell equation. The weak form of the equation for a single patch shell is stated as finding the displacement field $\boldsymbol{u}$ such as $a(\boldsymbol{v},\boldsymbol{u}) = f(\boldsymbol{v}) \quad \forall \boldsymbol{v}$. Where the definition of the spaces for $\boldsymbol{u}$ and $\boldsymbol{v}$ depends on the essential boundary condition for the problem at hand (see [3]). The bilinear and the linear form are defined as:

$$a(\boldsymbol{v},\boldsymbol{u}) = \int_\Omega \boldsymbol{\varepsilon}(\boldsymbol{v}) : \boldsymbol{N}(\boldsymbol{u}) \mathrm{d}\Omega + \int_\Omega \boldsymbol{\kappa}(\boldsymbol{v}) : \boldsymbol{M}(\boldsymbol{u}) \mathrm{d}\Omega \tag{1a}$$

$$f(\boldsymbol{v}) = \int_\Omega \boldsymbol{v} \cdot \widetilde{\boldsymbol{F}} \mathrm{d}\Omega + \int_{\Gamma_{N_1}} \boldsymbol{v} \cdot \widetilde{\boldsymbol{T}} \mathrm{d}\Gamma + \int_{\Gamma_{N_2}} \theta_n(\boldsymbol{v}) \widetilde{M}_{nn} \mathrm{d}\Gamma + \sum_{C \in \chi} \left(v_3 \tilde{S}\right)\big|_C \,, \tag{1b}$$

where $\boldsymbol{\varepsilon}$ and $\boldsymbol{\kappa}$ are the membrane and bending strain respectively, $\boldsymbol{N}$ and $\boldsymbol{M}$ are the generalized force and moment, $\widetilde{\boldsymbol{F}}$, $\widetilde{\boldsymbol{T}}$, $\widetilde{M}_{nn}$ and $\tilde{S}$ are the applied domain force, boundary force, bending moment and corner force, θ_n is the bending rotation, and Ω, Γ_{N_1}, Γ_{N_2}, and C are the surface, the boundary where force and moment boundary conditions are applied and the corners, respectively. The interested reader is referred to [2] for the details regarding the formulation.

Interior penalty coupling

When dealing with multiple patch shell structures, a coupling strategy is necessary. In some cases, when the patches are conformal to the interface, the coupling condition can be imposed strongly by appropriately selecting the spaces for $\boldsymbol{u}$ and $\boldsymbol{v}$. However, in general, the coupling condition needs to be imposed weakly through integrals along the interface. This is particularly true when the patches are trimmed in an immersed boundary approach. In such cases, the problem becomes finding $\boldsymbol{u}_h$ such as

$$\sum_{p=1}^{N_p} a_p(\boldsymbol{v}_h, \boldsymbol{u}_h) + \sum_{i=1}^{N_i} b_i(\boldsymbol{v}_h, \boldsymbol{u}_h) = \sum_{p=1}^{N_p} f_p(\boldsymbol{v}_h) \quad \forall \boldsymbol{v}_h \tag{2}$$

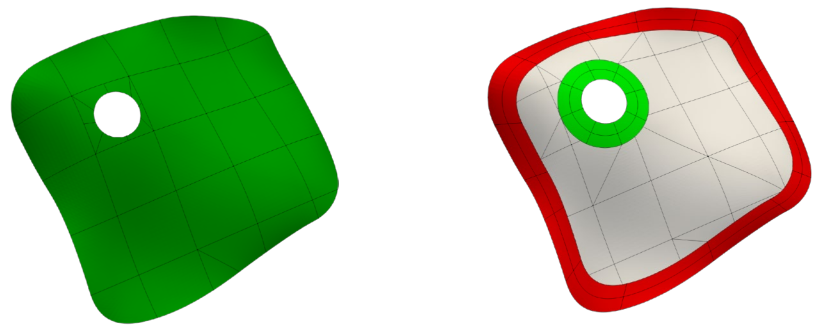
where $\boldsymbol{u}_h$ and $\boldsymbol{v}_h$ are the discretized displacements and test function, respectively. $a_p(\boldsymbol{v}_h, \boldsymbol{u}_h)$ and $f_p(\boldsymbol{v}_h)$ represent the discretized version of Eq. (1) for the p-th of the N_p patches. $b_i(\boldsymbol{v}_h, \boldsymbol{u}_h)$ is the bilinear form associated to the i-th of the N_i interfaces and is defined as

$$\begin{aligned} b_i(\boldsymbol{v}_h, \boldsymbol{u}_h) = \int_{\Gamma_h^I} (\{\boldsymbol{T}(\boldsymbol{v}_h)\} \cdot [\![\boldsymbol{u}_h]\!] + [\![\boldsymbol{v}_h]\!] \cdot \{\boldsymbol{T}(\boldsymbol{h}_h)\}) d\Gamma + \\ + \int_{\Gamma_h^I} (\{M_{nn}(\boldsymbol{v}_h)\} [\![\theta_n(\boldsymbol{u}_h)]\!] + [\![\theta_n(\boldsymbol{v}_h)]\!] \{M_{nn}(\boldsymbol{v}_h)\}) d\Gamma + \\ + \mu_D \int_{\Gamma_h^I} [\![\boldsymbol{v}_h]\!] \cdot [\![\boldsymbol{u}_h]\!] d\Gamma + \mu_R \int_{\Gamma_h^I} [\![\theta_n(\boldsymbol{v}_h)]\!] [\![\theta_n(\boldsymbol{u}_h)]\!] d\Gamma \end{aligned} \tag{4}$$

where Γ_h^I is the i-th interface, $\boldsymbol{T}$ and M_{nn} are the fluxes associated with the Ersatz force and bending moment (see [3]), and $[\![a]\!]$ and $\{a\}$ are the standard jump and average operator. μ_D and μ_R are the penalty parameters associated with the displacements and the rotation, that are chosen here as $\mu_d = \beta \frac{Et}{h}$ and $\mu_r = \beta \frac{Et^3}{h}$, where E, t, and h are the maximum young modulus of the material, the shell thickness, and mesh size, and β is a problem independent constant chosen here as 10^3.

The Immersed Boundary-Conformal approach

In the immersed boundary paradigm, the shell surface is embedded within a trimming curve, resulting in the presence of entire, partial, and empty elements. On the partial elements boundary and coupling condition can only be imposed in a weak sense. In the proposed method, an auxiliary boundary conforming patch is generated along the trimming curve. Although coupling between the auxiliary patch and the internal domain becomes necessary, it occurs in a more internal region of the domain where local phenomena are less likely to occur. Furthermore, these auxiliary patches can be refined as needed to capture these local phenomena more accurately. One of the difficulties associated with the proposed method lies in the integration in the trimmed elements. Here, the algorithm described in [4] is adopted, which is based on a high-order reparameterization of the trimmed elements.



(a) (b)

Figure 1 Same shell structure discretized using a trimmed single patch (a) and a combination of a main patch and two auxiliary boundary conformal ones (b).

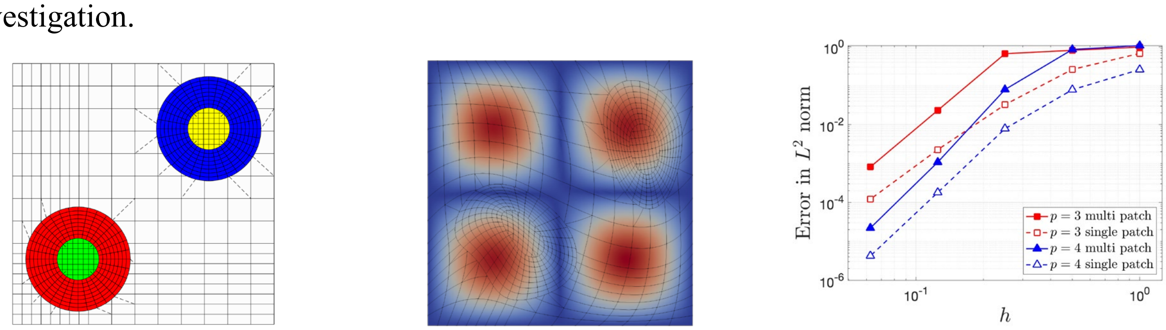
In Fig. (1), it is shown an example of a shell structure with boundary conformal patches aligned with the trimmed domain. In a similar fashion, we believe that coupling two intersecting surfaces creating an interface conformal patch for each surface would be beneficial to the accuracy of the analysis and is currently under investigation.

Numerical Results

In this session some preliminary results leading to the immersed boundary conformal coupling are presented.

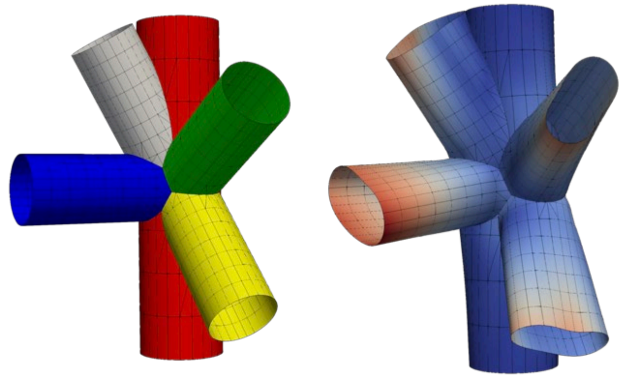
Kirchhoff plate with pseudo cut-outs: The plate shown in Fig. (2), characterized by edge size $L = 10$ m, Poisson ratio $\nu = 0.3$, thickness $t = 0.1$ m, Young modulus $E = 12(1-\nu^2)/t^3$Pa, and having simply supported boundary condition, is subjected to a constructed force that produces the displacement field described by $u_3 = \sin(2\pi x/L)\sin(2\pi y/L)$. In Fig. (2a) it is shown the parametric space with two pseudo cut-outs surrounded by boundary conformal patches. In this test the holes have been filled to easily retrieve the analytical solution. Fig(2b) shows the contour of the solution with superimposed mesh of the main and the auxiliary patches, it is pointed out the smoothness of the solution. Fig. (2c) shows the L_2 convergence for this test.

Intersecting cylindrical patches: Fig.(3a) shows a structure constituted of five intersecting cylindrical shells characterized by Young modulus $\nu = 0.3$, Poisson ratio $\nu = 0.3$, and thickness 10 mm. On the vertical cylinder, simply supported boundary conditions are applied at the opposite ends. A uniform domain force $f = [1,1,1]$ is applied on each patch. In Fig. (3b) it can be noted how the interior penalty method guarantees coupling conditions of the complex structure under investigation.



(a) (b) (c)

Figure 2 Kirchhoff plate with two pseudo cut-outs. (a) Parametric domain (b) physical domain with superimposed displacements contour (c) $\mathbf{L^2}$ *convergence of the displacements field.*



(a) (b)

Figure 3 Shell structure with multiple cylindrical patch (a). Contour of the magnitude of the displacement (b).

Conclusions

This contribute introduces the boundary conformal method for the coupling of shell patches and provides initial results towards the implementation of a complete approach. The proposed formulation is based on the interior penalty method for coupling shell patches, the utilization of auxiliary interface conformal patches, and a high-order integration scheme for trimmed elements. The presented results suggest the potential of the method, which is currently being developed.

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