Tiltrotor whirl-flutter stability analysis using the maximum Lyapunov characteristic exponent estimated from time series

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Abstract. Stability analysis and assessment are fundamental in the analysis and design of dynamical systems. Particularly in rotorcraft dynamics, problems often exhibit time-periodic behavior, and modern designs consider nonlinearities to achieve a more accurate representation of the system dynamics. Nonlinearities in rotorcraft may arise from factors such as nonlinear damper constitutive laws or the influence of fluid-structure interaction, among others. Regardless of their origin, quantifying the stability of nonlinear systems typically relies on calculating their Jacobian matrix. However, accessing the Jacobian matrix of a system is often challenging or impractical, calling for the use of data-driven methods. This introduces additional complexity in capturing the characteristic dynamics of the system. Hence, a data-driven approach is proposed that utilizes the Largest Lyapunov Characteristic Exponent, obtained by analyzing the system's time series.

Introduction

When faced with a nonlinear problem in its general form,

$$\dot{x} = f(x, t)$$

stability is a local property of a specific solution, $x(t)$, resulting from a corresponding set of initial conditions, $x(t_0) = x_0$, i.e., of a Cauchy problem. One commonly encountered instance is the Linear, Time-Invariant (LTI) scenario. In this context, Lyapunov Characteristic Exponents (LCEs) quantify the growth or decay rate of disturbances from a typical solution in the nonlinear differential problem across distinct directions within the state space, providing insight into the stability of the reference solution in relation to these directions. Consider a solution $x(t)$ for $t \geq t_0$ (some call it the 'fiducial trajectory'), and a solution ${}_i x(t)$ of the problem.

$${}_i\dot{x} = f_{/x}|_{x(t),t} \; {}_i x, \qquad {}_i x(t_0) = {}_i x_0$$

for a perturbation ${}_i x_0$ of arbitrary magnitude and direction. LCEs are defined as

$$\lambda_i = \lim_{t\to\infty} \frac{1}{t} \| \, {}_i x(t) \|$$

When all the Lyapunov Characteristic Exponents (LCEs) are negative, the solution exhibits exponential stability. Conversely, if at least one LCE is positive, the reference solution is unstable or may lead to a chaotic attractor. When the largest LCE – or LCEs – are zero, a limit cycle oscillation (LCO) can be expected. This means that in the state space there exists one or multiple independent directions along which the solution neither expands nor contracts, converging instead to a self-sustained periodic motion. When multiple largest LCEs are equal to zero, a higher-order periodic or quasi-periodic attractor arises, such as a multi-dimensional torus. It is important to exercise caution when interpreting the LCEs as eigenvalues of Linear, Time-Invariant (LTI) problems, as demonstrated in [5], as they are not always equivalent.

Jacobian-Less Methods: Max LCE from Time Series

The MLCE is the LCE associated with the least damped principal direction of the problem, which represents the most critical stability indicator. Among the algorithms proposed in the literature, the one proposed by Rosenstein et al. [1] is used in this work. It is defined by the following steps. By utilizing the trajectory matrix, $\boldsymbol{X} \in \mathbb{R}^{M\times m}$, the full phase-space can be reconstructed using the time delay method, if needed, along with estimating the embedding dimension, m (estimated following Takens' theorem), and the reconstruction delay, J, where $M = N - (m-1)J$ and N is the length of the time series. In this context, each column of matrix $\boldsymbol{X}$ is a phase-space vector.

$$\boldsymbol{X} = [\boldsymbol{X}_1\ \boldsymbol{X}_2 \ldots \boldsymbol{X}_m]$$

After constructing the trajectory matrix, the algorithm locates the nearest neighbor, $\boldsymbol{X}_{\hat{j}}$, of each point on the trajectory, which is found by searching the point that minimizes the distance from each reference point, $\boldsymbol{X}_j$. The distance is expressed as

$$d_j(0) = \min_{\boldsymbol{X}_{\hat{j}}} \left\| \boldsymbol{X}_j - \boldsymbol{X}_{\hat{j}} \right\|$$

where $d_j(0)$ is the initial distance from the jth point to its nearest neighbor, and $\|\cdot\|$ denotes the Euclidean norm. Nearest neighbors must have a temporal separation greater than the mean period ($\bar{T}$, the reciprocal of the mean frequency of the power spectrum, although it can be expected that any comparable estimate, e.g., using the median frequency of the magnitude spectrum, yields equivalent results) of the time series, $|j - \hat{j}| > \bar{T}$. The largest Lyapunov exponent is then estimated as the mean rate of separation of the nearest neighbors. The jth pair of nearest neighbors diverge approximately at a rate given by the largest Lyapunov exponent:

$$d_j(l) \approx C_j e^{\lambda_1 (l\,\Delta t)}$$

where C_j is the initial separation. By taking the logarithm of both sides which represents a set of approximately parallel lines, for $j = 1, \ldots, M$, each with a slope roughly proportional to λ_1. The largest Lyapunov exponent is calculated using a least-squares fit to the “average” line defined by

$$y(l) = \frac{1}{\Delta t}\langle \log d_j(l) \rangle$$

where $\langle\cdot\rangle$ denotes the average over all values of j.

XV-15 Model Whirl Flutter

Building upon the previous research conducted in [2], the aeroelastic simulation of the XV-15 tiltrotor is now considered. For this analysis, an aeroservoelastic model is employed, encompassing all significant structural components. The airframe model comprises various elements, such as the flexible wing, rigid fuselage, empennages, control surfaces (elevator, rudder, flaps, and flaperons), and nacelle tilt mechanisms. Additionally, the model, originally developed in [4], incorporates crucial cockpit elements (a seat and control inceptors - collective and cyclic) to explore rotorcraft-pilot couplings. To develop this model, the MBDyn multibody solver (https://mbdyn.org/) is utilized, to represent the fundamental frequencies and mode shapes of the complete aircraft, with specific emphasis on the wing-nacelle section. The proprotor, featuring a three-bladed stiff-in-plane rotor with a gimballed hub, comprises the blades, flexible yoke, and pitch control chain. The flexibility of the wing, rotor blades, and yokes of the two rotors is modeled using an original

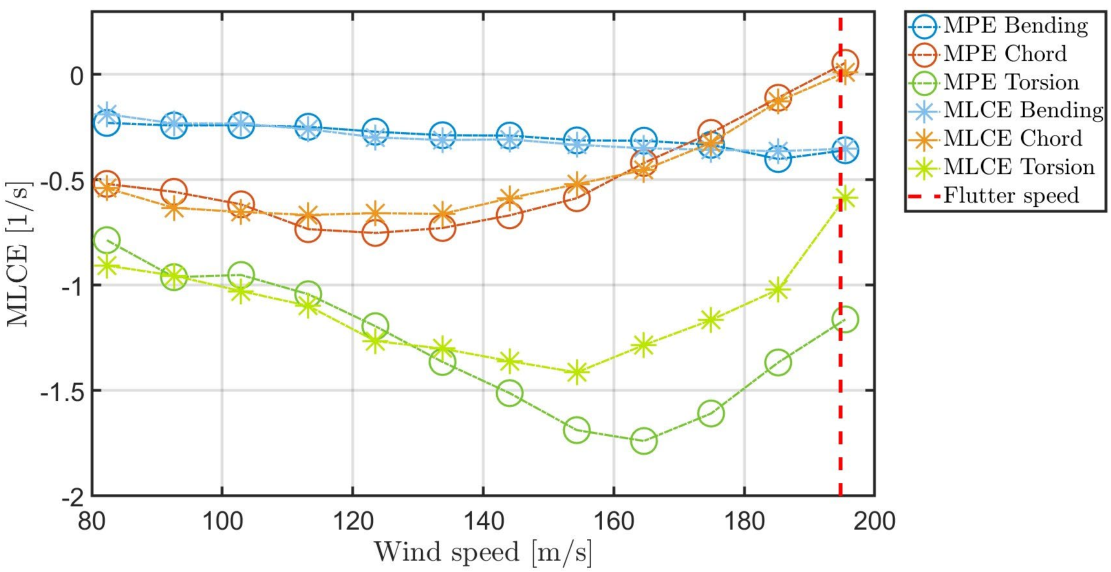

Figure 1: MLCE of the XV-15 in the standard operating condition for the airplane mode.

geometrically exact composite-ready beam finite element model known as “finite volume” [3], which is well-suited for multibody dynamics. The Whirl-Flutter phenomenon was observed through a two-phase approach involving excitation with a sinusoidal input through the swashplate, after reaching the desired trim configuration, followed by a free response phase. By linear interpolation, the instability region under the standard operating condition for the airplane mode was estimated to be U_∞ = 195.5 m/s (indicated by the dotted red line in Fig. 1). Notably, the proposed method successfully recovered the accurate estimation of the Whirl-Flutter instability previously documented in [4], validating its reliability. However, it is important to note that time series analysis methods are sensitive to changes in the observation window. Due to the system's rapid convergence compared to other directions, evaluating it accurately poses challenges, particularly in the case of torsion. To verify the obtained results, a comparison was made with those obtained using the Matrix Pencil Estimation method (MPE). In the chord direction (with a configuration in airplane mode and idle engines), a time series depicting a slow limit cycle was obtained at U_∞ = 185.2 m/s (Fig. 2). Interestingly, when solely considering the linear component, the system appears to exhibit growth. However, the system converges to a limit cycle. The amplitude, A, in Fig. 2 used to compare the solution obtained by the MPE method is determined by selecting the maximum value present in the time series.

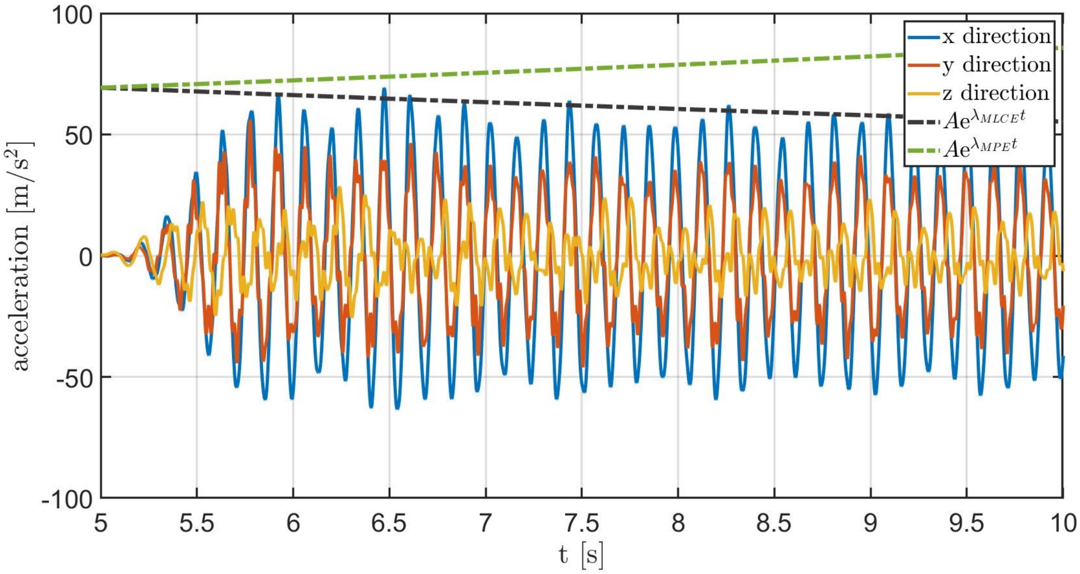

Figure 2: Time history with an input perturbation in the chord, at the hub location.

Conclusion

The presented approach is employed for investigating the whirl flutter instability of a tiltrotor. By extensively exploring its nonlinear dynamics, it becomes feasible to expand the understanding of various instabilities, particularly in cases where linear methods fall short in capturing the entirety of the response domain. Future advancements will involve coupling the method with the vortex particle solver DUST (https://www.dust-project.org/) to have a more detailed account for the aerodynamic interactions.

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