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Nonlinear transient analyses of composite and sandwich structures via high-fidelity beam models

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Abstract. In this study, we employ low and high-fidelity finite beam elements to conduct geometrical nonlinear transient analyses of composite and sandwich structures. The equations of motion for various structural theories are derived in a total Lagrangian scenario using the Carrera Unified Formulation. The unified formalism's three-dimensional nature enables us to include all components of the Green-Lagrange strain tensor. To solve the equations, we utilize the Hilber-Hughes-Taylor (HHT)-α algorithm in conjunction with a Newton-Raphson procedure. We present the dynamic response of a sandwich stubby beam subjected to a step load, calculated using both equivalent-single layer and layer-wise approaches. Additionally, we discuss the effects of geometrical nonlinearity.

Introduction

In recent decades, the aerospace, automotive, and other engineering fields have faced new challenges that necessitate the adoption of sophisticated and lightweight components. These highly flexible structures are extensively utilized in various engineering applications as they can exhibit large displacements and rotations without undergoing plastic deformations. Furthermore, many of these components comprise sandwich structures made of composite materials to ensure a significant strength-to-weight ratio. As highlighted in numerous scientific papers [1,2], analyzing these structural configurations requires refined kinematic theories that can overcome the well-known limitations of the Euler-Bernoulli and the first-order-shear deformation theories.

This research aims to analyze the nonlinear dynamic behavior of composite and sandwich structures using variable-fidelity one-dimensional finite elements. The mathematical models are derived using the Carrera Unified Formulation (CUF). This hierarchical formalism enables the selection of the order of the structural model as an input of the analysis. Therefore, any theory can be obtained by arbitrarily expanding the generalized variables. Specifically, this work employs Lagrange (LE) and Taylor (TE) polynomials to develop kinematic expansions. According to the layer-wise concept, the LE models allow for the independent discretization of each lamina. In contrast, the Taylor-based models homogenize the cross-section properties with polynomials of arbitrary orders. Regardless of which theory is adopted, the governing equations and the related finite element arrays are formulated in terms of *Fundamental Nuclei* (FNs), the invariants of the methodology. The nonlinear equations are formulated using the total Lagrangian approach and solved using a suitable Newton-Raphson method. We utilize the Hilber-Hughes-Taylor (HHT)-α algorithm as the implicit time integration scheme to evaluate the nonlinear dynamic response. This algorithm proves particularly effective in stabilizing the time integration process under highly nonlinear effects.

To highlight the relevant discrepancies between low- and high-fidelity solutions in studying the response of laminated beams, we consider a stubby sandwich structure characterized by a significant degree of anisotropy between the core and external layers. Moreover, we have compared geometrical linear and nonlinear solutions for two different load magnitudes.

Unified formulation of geometrical nonlinear beam theory

The theoretical basis required for solving transient analyses in geometrically nonlinear regimes can be found in [3]. However, to ensure the self-contained nature of this work, we provide some basic equations here. Based on the one-dimensional finite element unified formulation, we express the displacement vector $\boldsymbol{u}^T(x,y,z,t) = (u_x, u_y, u_z)$ as a sum of products between cross-sectional (defined over the x-z plane) functions $F_\tau(x,z)$, finite element shape functions $N_i(y)$ (defined over the y-axis) and the nodal unknown vector $\boldsymbol{q}_{\tau i}(t)$

$$\boldsymbol{u}(x,y,z,t) = F_\tau(x,z)N_i(y)\boldsymbol{q}_{\tau i}(t) \qquad \tau = 1,\dots,M \text{ and } i = 1,\dots,nn_{el} \tag{1}$$

In Eq. 1, the subscripts indicate summation, while M and nn_{el} represent the number of functions included in the structural model and the number of nodes belonging to a single finite beam element, respectively. As previously mentioned, the geometrically nonlinear FE governing equations are obtained according to a total Lagrangian formulation by including all Green–Lagrangian strain tensor components. The strain–displacement relation and the constitutive law reported in Eq. 2 are obtained using the linear and nonlinear differential operators, $\boldsymbol{b}_l$ and $\boldsymbol{b}_{nl}$, respectively, and the stiffness matrix for linear elastic materials, $\boldsymbol{C}$.

$$\boldsymbol{\varepsilon} = (\boldsymbol{b}_l + \boldsymbol{b}_{nl})\boldsymbol{u} \qquad \boldsymbol{\sigma} = \boldsymbol{C}\,\boldsymbol{\varepsilon} \tag{2}$$

By substituting Eq. 1 and Eq. 2 into the principle of virtual work, it becomes possible to express the virtual variations of both strain energy (δL_{int}) and the work done by inertial forces (δL_{ine}) and external loads (δL_{ext}) in the CUF formalism.

$$\begin{aligned} \delta L_{int} &= \delta \boldsymbol{q}_{sj}^T \boldsymbol{K}_s^{ij\tau s} \boldsymbol{q}_{\tau i} \\ \delta L_{ine} &= \delta \boldsymbol{q}_{sj}^T \boldsymbol{M}^{ij\tau s} \ddot{\boldsymbol{q}}_{\tau i} \\ \delta L_{ext} &= \delta \boldsymbol{q}_{sj}^T \boldsymbol{F}^{sj} \end{aligned} \tag{3}$$

The FNs of the secant stiffness and mass matrix are denoted as $\boldsymbol{K}_s^{ij\tau s}$ and $\boldsymbol{M}^{ij\tau s}$, respectively, while $\boldsymbol{F}^{sj}$ represents the fundamental nucleus of the loading vector. Here, the indexes s and j are adopted for the virtual variations of the displacements and have the same bounds as τ and i. The assembled matrices and vectors associated with any arbitrary structural model are constructed by permuting these four indexes. Based on this notation, the equations of motion are

$$\boldsymbol{M}\ddot{\boldsymbol{q}}(t) + \boldsymbol{K}_s(\boldsymbol{q})\boldsymbol{q}(\boldsymbol{t}) = \boldsymbol{F}(t) \tag{4}$$

Equation 4 is solved by using the Newton-Raphson method and the HHT-α implicit time integration scheme. For a dynamic conservative problem, the linearization of the residual nodal forces leads to

$$\begin{aligned} \delta(\delta L_{int} + \delta L_{ine} - \delta L_{ext}) &= \delta \boldsymbol{q}_{sj}^T \left(\boldsymbol{K}_0^{ij\tau s} + \boldsymbol{K}_{T1}^{ij\tau s} + \boldsymbol{K}_\sigma^{ij\tau s}\right)\delta \boldsymbol{q}_{\tau i} + \delta \boldsymbol{q}_{sj}^T \boldsymbol{M}^{ij\tau s} \delta\ddot{\boldsymbol{q}}_{\tau i} = \\ &= \delta \boldsymbol{q}_{sj}^T \boldsymbol{K}_T^{ij\tau s} \delta \boldsymbol{q}_{\tau i} + \delta \boldsymbol{q}_{sj}^T \boldsymbol{M}^{ij\tau s} \delta\ddot{\boldsymbol{q}}_{\tau i} \end{aligned} \tag{5}$$

In Eq. 5, $\boldsymbol{K}_T^{ij\tau s}$ represents the FN of the tangent stiffness matrix (see [4]).

Results

Figure 1 and Table 1 present the dimensions, material properties, boundary conditions, and loading conditions of the sandwich beam. The structure was subjected to a pressure load that was constant in time but variable in magnitude.

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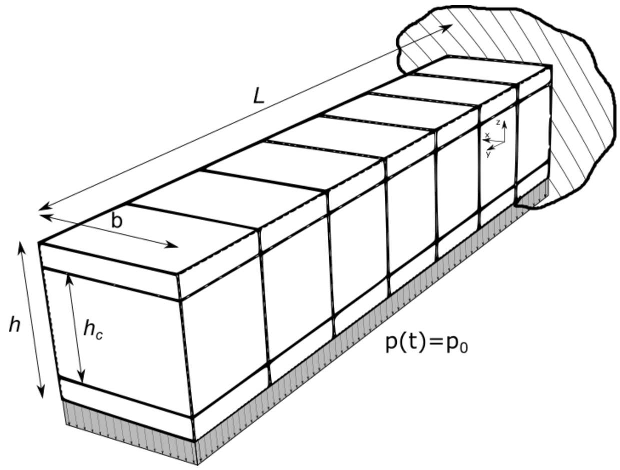

Figure 1: the cantilever sandwich beam.

Table 1: dimensions and material data of the sandwich beam.

Geometrical data	Material data	Face	Core
L = 0.1 [m]	Young's modulus, E [GPa]	200	0.66
h = b = 0.02 [m]	Poisson's ratio	0.27	0.3
h_c = 0.014 [m]	Density [kg/m^3]	7800	60

The finite element models utilized in this study consisted of seven 4-node beam elements placed along the longitudinal axis. Transient analyses were performed using the Taylor-based expansions of second (TE2) and third order (TE3) - as well as a layer-wise model consisting of three (one per layer) bi-cubic Lagrange elements (3-LE16) placed over the cross-section. The degrees of freedom (d.o.f.) corresponding to the TE2, TE3 and 3LE16 solutions were 396, 660, and 2640, respectively. In Figure 2, the transverse deflection of the sandwich beam is depicted for two different values of the applied pressure. As expected, the equivalent-single-layer solutions, although improved compared to classical beam models, significantly underestimate the deformation of the structure when compared to the layer-wise prediction. Additionally, it is noteworthy that both geometrically linear and nonlinear approaches provide almost indistinguishable results for relatively small values of p_0, irrespective of the structural theory employed. However, it can be observed that with the increase of the load value, the linear and nonlinear curves obtained using the 3-LE16 model differ significantly. This effect can be attributed to the superior ability of the layer-wise model to accurately describe the deformation and stress fields compared to the equivalent-single-layer kinematics.

Summary

This work presented preliminary results concerning composite and sandwich structures' geometrical nonlinear transient responses calculated with one-dimensional finite element models based on various kinematic assumptions. The comparisons between low- and high-fidelity solutions demonstrated the importance of accurately describing the cross-section deformations, especially in the nonlinear regime. Further results will be provided on composite laminated structures subjected to different loads.

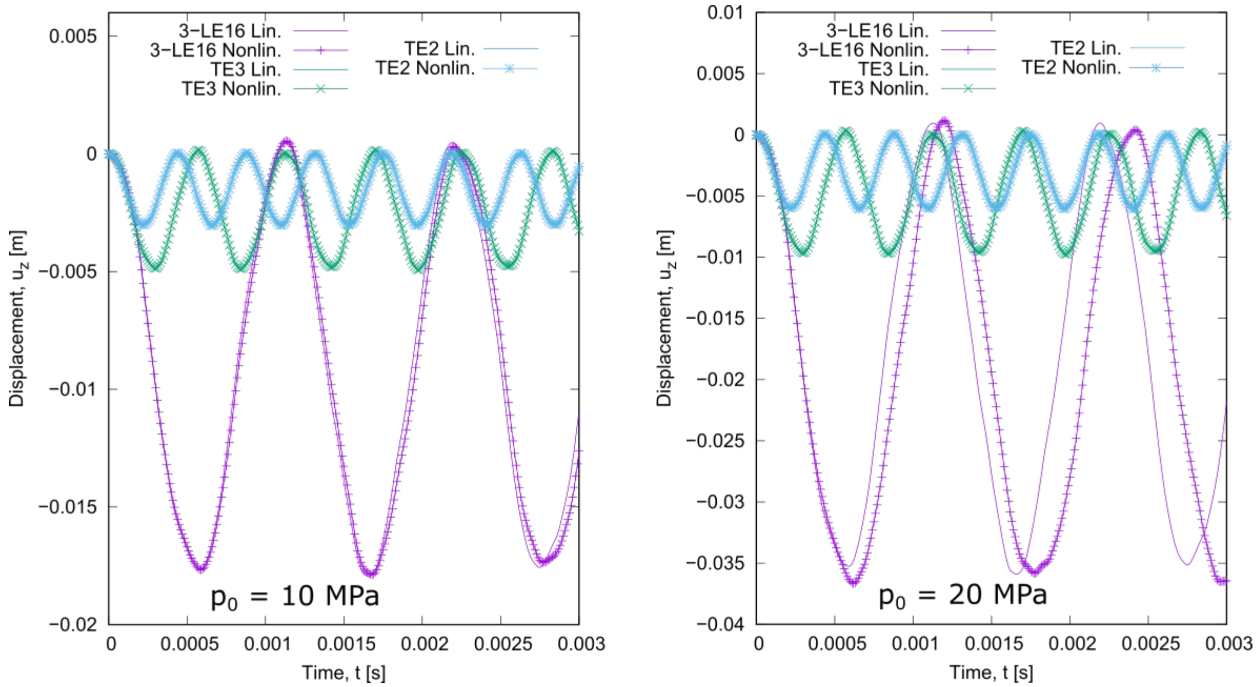

Figure 2: transient responses of the sandwich beam for two pressure values.

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