

Comparative study of shock response synthesis techniques for aerospace applications

Ada Ranieri^{1,a*}, Simone De Carolis^{1,b}, Giuseppe Carbone^{1,c}, Michele Dassisti^{1,d},
Andrea De Cesaris^{2,e}, Leonardo Soria^{1,f}

¹ Polytechnic University of Bari, Bari, Italy

²Sitael, Mola di Bari (BA), Italy

^aada.ranieri@poliba.it, ^bsimone.decarolis@poliba.it, ^cgiuseppe.carbone@poliba.it,
^dmichele.dassisti@poliba.it, ^eandrea.decesaris@sitael.com, ^fleonardo.soria@poliba.it

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Abstract. The Shock Response Spectrum (SRS) is a widely used tool for analyzing and characterizing the response of mechanical systems to shock and transient events. In the aerospace industry, the SRS is used to compute the severity of the shock event on the electrical and optical equipment of a spacecraft. However, the SRS only provides magnitude information and does not retain temporal or phase information. Moving to the time domain is not a straightforward process because a time history has a unique SRS, but the converse is not true. Therefore, it is challenging to find the right time history that reproduces an SRS when simulating a given input profile using pyrotechnic devices or when computing the response to a shock input profile in the time-domain. For a given SRS an infinite combination of time pulses is possible. Synthesizing an SRS involves recovering a time-domain pulse that can accurately replicate the given SRS. There are many methods which are already widely utilized in the aerospace industry, including the use of damped sinusoids, enveloped sinusoids and wavelets. In this paper we compare different techniques, with the objective of identifying the most suitable method based on the considered frequency range and type of impulse. The case study under consideration is an SRS input profile corresponding to a real industrial case. Three artificial SRS accelerations have been generated to replicate the input, and the percentage errors of each method in comparison to the reference signal have been assessed. Further development will involve the use of optimization algorithms to generate the SRS profile with the smallest possible error.

Introduction

One of the most significant challenges in the space industry is the design and testing of aerospace structures and systems for reliable and safe operation in harsh environments, including the sudden and impulsive loads occurring during the launch phase. The most intense events are commonly caused by pyrotechnic devices actuating at the base of the spacecraft. The firing of these devices results in impulsive loads characterized by high peak acceleration, high-frequency content, and short duration. This poses a significant threat to the reliability and safety of electrical and optical components of the spacecraft, which are sensitive to high frequency loads. To demonstrate its compliance to shock requirements, the structure has to be tested by applying the shock load on the base interface. The accepted standard for implicit description of the pyroshock environment is the Shock Response Spectrum (SRS), which is a useful tool for estimating the damage potential of the shock pulse and for test level specification. The SRS finds its first applications in the 50's by the seismic and aerospace community. An SRS is generated by plotting in the frequency domain the peak response of a series of Single Degree of Freedom (SDoF) oscillating systems subjected to the same transient base acceleration input. The damping is usually assumed to be 5% ($Q=10$), while

the natural frequency of each SDoF system is chosen to be different. The primary limitation of the SRS is its inability to provide temporal or phase information, as it only gives magnitude information. As a result, when subjecting a structure to electro-dynamic shaker testing for shock qualification, the SRS cannot be directly utilized [1]. Instead, it becomes necessary to synthesize an SRS-compatible acceleration time history. A similar challenge arises when analyzing nonlinear structures, where a modal approach is not feasible, and a modal transient analysis must be conducted to account for the phase among the peak responses of individual modes.

The aforementioned waveform can be obtained using a series of sinusoids [1,2] or wavelets [3], tailored to resemble an actual pyrotechnic shock pulse.

Shock Response Spectrum Synthesis

While a unique impulse in the time domain corresponds to a specific SRS, the opposite is not true. In fact, an SRS corresponds to an infinite number of possible pulses. As a result, there are several techniques available to obtain SRS-compatible acceleration time history. In this work we will investigate the accuracy of SRS synthesis throughout the summation of damped sines, enveloped sines and wavelets.

Wavelets. A wavelet is a discrete waveform of limited duration that is suited for approximating data with sharp discontinuities [4]. The original signal can be reconstructed as a summation of a set of wavelets with specified parameters. The equation of a single wavelet $W_m(t)$ is:

$$W_m(t) = \begin{cases} 0, & \text{for } t < t_{dm} \\ A_m \sin\left[\frac{2\pi f_m}{N_m}(t - t_{dm})\right] \sin[2\pi f_m(t - t_{dm})], & \text{for } t_{dm} \leq t \leq \left[t_{dm} + \frac{N_m}{2f_m}\right] \\ 0, & \text{for } t > \left[t_{dm} + \frac{N_m}{2f_m}\right] \end{cases} \quad (1)$$

A discrete wavelet has a sinusoidal motion with a finite and odd number of half sine oscillations N_m with unique parameters for frequency f_m , amplitude A_m and time delay t_{dm} .

Damped sinusoids. The sinusoid approach shows a difference in the way the rise, peak and decay of the waveform is controlled, compared to the previously presented method. In this case the parameters to control are slightly different:

$$W_m(t) = \begin{cases} 0, & \text{for } t < t_{dm} \\ A_m e^{-\xi_m 2\pi f_m(t-t_{dm})} \sin\left[\frac{2\pi f_m}{N_m}(t - t_{dm})\right] \sin[2\pi f_m(t - t_{dm})], & \text{for } t \geq t_{dm} \end{cases} \quad (2)$$

It can be noted an extra term ξ_m , that is the damped sinusoid damping ratio.

Enveloped sinusoids. The enveloped sinusoids with random phase angles approach is similar to the one of damped sinusoids. The equation for enveloped sinusoids is given by:

$$W_m(t) = E(t)A_m \sin(2\pi f_m t + \varphi_m). \quad (3)$$

Where φ_m are random phase angles for each frequency n. The rise, plateau and decay of $W_m(t)$ is controlled by an envelope function $E(t)$ rather than damping.

For all the three methods, iterations for the parameters of a set of m waveforms a time t yield a synthesized acceleration that is expressed as:

$$\ddot{x}(t) = \sum_{m=1}^{N_m} W_m(t). \tag{4}$$

An example of a synthesized time history from the SRS input in Table 1 with a duration of $T=0.06\text{ s}$ can be seen in Fig.1.

Table 1. Shock load input

Frequency [Hz]	Amplitude [g]
100	56
1000	2820
10000	2820

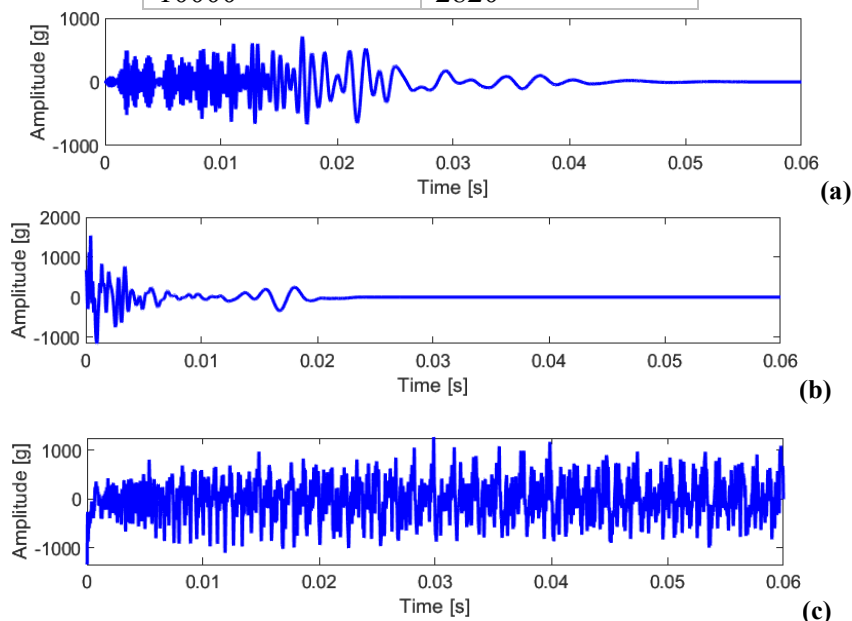
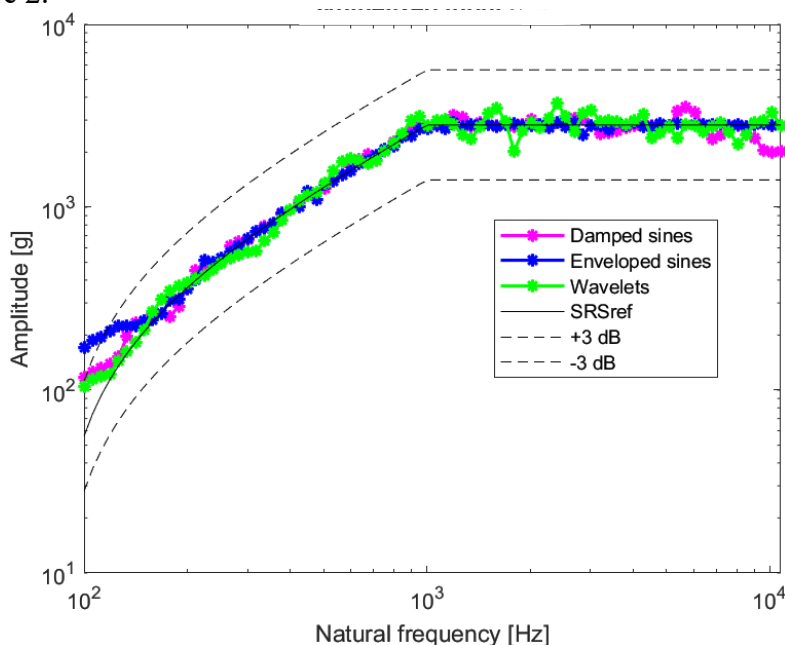


Figure 1. Reconstructed time history of a shock input with (a) wavelets, (b) damped sines, (c) enveloped sines

The synthesized accelerations have been converted to SRS and compared to the reference input as shown in Figure 2.



2. Synthesized SRS comparison

Table 2. Synthesis correlation coefficient

	Damped sines	Enveloped sines	Wavelets
100-200 Hz	0.98186	0.99566	0.99773
200-1000 Hz	0.97651	0.98699	0.9659
1000-10K Hz	0.98934	0.99421	0.98707

Furthermore, the Synthesis Correlation Coefficient (*COR*) [5] in Table 2 has been computed in low, middle and high frequency range to compare the efficiency.

$$COR = \frac{|\sum_{f_1}^{f_2} SRS_r(f_n)SRS_s(f_n)|^2}{\sum_{f_1}^{f_2} SRS_r(f_n)^2 \sum_{f_1}^{f_2} SRS_s(f_n)^2} \tag{5}$$

Where *SRS_r* and *SRS_s* are respectively the reference and synthesized SRS. Globally, a good level of accuracy (near the unity) has been achieved. In particular, the enveloped sines method seems to be the most effective. It can be observed that the methods are less accurate in the middle frequency range (200-1000 Hz).

Conclusions and future developments

In conclusion, the investigated techniques, namely the summation of damped sines, enveloped sines, and wavelets, have shown good levels of accuracy in reproducing the desired SRS input. Further studies should be conducted by exploring different parameter settings and types of input profiles to enhance the understanding of these techniques. Additionally, the development of an optimization algorithm, such as the least square fitting method or genetic algorithm, should be pursued to combine the methods and synthesize a single SRS that minimizes the error and achieves a higher level of accuracy.

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