Reduced-attitude stabilization for spacecraft boresight pointing using magnetorquers

Fabio Celani

School of Aerospace Engineering, Sapienza University of Rome, Via Salaria 851 00138 Roma Italy

fabio.celani@uniroma1.it mail

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Abstract. This paper presents a method for achieving a desired boresight pointing of an instrument on a spacecraft using only magnetorquers as torque actuators. The desired pointing direction is inertially fixed. The proposed method is of proportional-derivate type and stabilizes the boresight pointing. Numerical simulations illustrate the effectiveness of the method and show that convergence to the desired pointing direction occurs faster than employing a three-axis stabilization approach.

Introduction

Pointing of the boresight of an instrument is a common task of many spacecraft during their operational life. For example, it might be required to point a telescope at a star, to point a transmitting or receiving antenna at a ground station, or to point the Sun with solar panels. A common approach for boresight control is performing three-axis attitude control which requires knowledge of the full attitude of the spacecraft. However, in boresight pointing applications, only the pointing direction is relevant and the rotation about the boresight is not considered. In this sense, full attitude knowledge is not required for boresight control, and it suffices to measure the reduced-attitude vector defined on the two-dimensional sphere. Motivated by this practical consideration this research presents a method for reduced-attitude stabilization for a spacecraft that uses only magnetorquers as torque actuators.

Magnetorquers are planar current-driven coils rigidly placed on the spacecraft typically along three orthogonal axes. The interaction between the magnetic dipole moment generated by those coils and the Earth magnetic field creates a torque that attempts to align the magnetic dipole moment in the direction of the field. Magnetorquers present the following benefits: (i) they are simple, reliable, and low cost; (ii) they need only renewable electrical power to be operated; (iii) they save weight with respect to any other class of torque actuators. On the other hand, magnetorquers have the following important limitations: (i) the control torque generated by magnetorquers is constrained to belong to the plane orthogonal to the Earth magnetic field; (ii) the maximum torque they can generate is substantially smaller than for other types of torque actuators. Due to these limitations, using only magnetorquers for attitude stabilization leads to smaller pointing accuracy and slower convergence compared to other torque actuators. Thus, it is considered a feasible option especially for low-cost micro and nano satellites, and for satellites with a failure in the main torque actuators.

This work proposes a boresight stabilization law for an inertially pointing spacecraft equipped only with magnetorquers as torque actuators. Numerical simulations for a case study show the effectiveness of the proposed law.

Spacecraft Boresight Pointing

Consider the problem of aligning a body-fixed boresight axis of an instrument on a spacecraft with an inertially-fixed reference axis. In addition, once the alignment condition is achieved the spacecraft must not rotate about the reference axis to avoid blurred measurements from the instrument. The body-fixed boresight axis is expressed by the constant column matrix of its body coordinates $\boldsymbol{a} \in \mathbb{S}^2 = \{\boldsymbol{v} \in \mathbb{R}^{\boldsymbol{3}}: \boldsymbol{v}^T\boldsymbol{v} = \boldsymbol{1}\}$. The inertially-fixed reference axis is expressed by the constant column matrix $\boldsymbol{a}_r^i \in \mathbb{S}^2$ of its coordinates along the standard Earth-Centered Inertial (ECI) frame [1]. Let $\boldsymbol{R}$ be the rotation matrix that transforms a column matrix of body coordinates into the corresponding column matrix of ECI coordinates. Thus, the reference axis expressed in body-coordinates is given by $\boldsymbol{a}_r^b = \boldsymbol{R}^T\boldsymbol{a}_r^i$. Column matrix $\boldsymbol{a}_r^b$ describes a reduced attitude and obeys the following kinematics [2]

$$\dot{\boldsymbol{a}}_r^b = \boldsymbol{a}_r^b \times \boldsymbol{\omega} \tag{1}$$

where $\boldsymbol{\omega}$ is the column matrix of body coordinates of the angular velocity of the body frame with respect to the ECI frame. The spacecraft is modeled as a rigid body and its attitude dynamics are given by [2]

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\boldsymbol{J}\boldsymbol{\omega}) = \boldsymbol{T}_c + \boldsymbol{T}_d \tag{2}$$

where $\boldsymbol{J}$ is the spacecraft inertia matrix, $\boldsymbol{T}_c$ is the column matrix of body coordinates of the control torque, and $\boldsymbol{T}_d$ is column matrix of body coordinates of the sum of all disturbance torques acting on the spacecraft. The control torque is generated by three magnetorquers aligned with the body axes. Thus, it can be expressed as follows

$$\boldsymbol{T}_c = \boldsymbol{m}_c \times \boldsymbol{b}^b \tag{3}$$

where $\boldsymbol{m}_c$ is the column matrix of the control magnetic dipole moments generated by the three magnetorquers, and $\boldsymbol{b}^b$ is the column matrix of body coordinates of the geomagnetic induction.

The proposed boresight stabilization law is given by

$$\boldsymbol{m}_c = \boldsymbol{b}^b \times [k_p(\boldsymbol{a} \times \boldsymbol{a}_r^b) - k_d\,\boldsymbol{\omega}] \tag{4}$$

where k_p and k_d are positive scalar gains**.** The stabilization law is obtained by modifying the proportional-derivative like law for fully actuated spacecraft presented in [2]. Specifically, the cross product with $\boldsymbol{b}^b$ is introduced to enforce that $\boldsymbol{m}_c$ is perpendicular to $\boldsymbol{b}^b$**.** The latter property allows to save energy since Eq. 3 shows that a term in $\boldsymbol{m}_c$ parallel to $\boldsymbol{b}_b$ does not give any contribution to the control torque $\boldsymbol{T}_c$**.**

Stability Analysis

In this section a stability result of the proposed boresight stabilization law is presented. The stability analysis is carried out by adopting the inclined dipole model for the geomagnetic field [1]. Consider a circular orbit for the spacecraft and let $\boldsymbol{b}^i(t)$ denote the column matrix of ECI coordinates of the geomagnetic induction along the orbit. The adoption of the inclined dipole model leads to an almost periodic time behavior for $\boldsymbol{b}^i(t)$. Consider matrix

$$\boldsymbol{\Gamma}^i(t) = \boldsymbol{b}^i(t)^T\boldsymbol{b}^i(t)\boldsymbol{I}_{3\times3} - \boldsymbol{b}^i(t)\boldsymbol{b}^i(t)^T \tag{5}$$

where $\boldsymbol{I}_{3\times3}$ is the identity matrix. Note that $\boldsymbol{\Gamma}^{\boldsymbol{i}}(\boldsymbol{t})$ is also almost periodic, and consider the following average value

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$$\mathbf{\Gamma}_{av}^{i} = \lim_{T\to\infty} \frac{\mathbf{1}}{T} \int_{\mathbf{0}}^{T} \mathbf{\Gamma}^{i}(\tau)\, d\tau \tag{6}$$

It can be shown that the following stability result holds true.

Proposition. If $\mathbf{det}(\mathbf{\Gamma}_{av}^{i}) \neq \mathbf{0}$ then it is possible to determine positive values for k_p and k_d so that the equilibrium $(\boldsymbol{a}_r^b, \boldsymbol{\omega}) = (\boldsymbol{a}, \mathbf{0})$ of the closed-loop in Eqs. 1-4 with $\boldsymbol{T}_d = \mathbf{0}$ is asymptotically stable.

It can be obtained that condition $\mathbf{det}(\mathbf{\Gamma}_{av}^{i}) \neq \mathbf{0}$ is fulfilled if the orbit inclination is not too low.

Numerical Simulations

The goal of this section is to validate by numerical simulations the boresight stabilization law in Eq. (4). Consider a spacecraft with inertia matrix equal to $\boldsymbol{J} = \text{diag}[27\ 25\ 17]\ \text{kg m}^2$ which follows a circular orbit with 450 km altitude, 87 deg inclination, and zero RAAN. The orbital period is about 5600 sec. The maximum value for each element of $\boldsymbol{m}_c$ is set to 10 A m^2. The body-fixed boresight axis and the inertially-fixed reference axes are set equal to $\boldsymbol{a} = \boldsymbol{a}_r^i = [0\ 0\ 1]^T$. Disturbance torques included in the simulations are gravity-gradient torque, residual magnetization torque, and aerodynamic drag torque modeled as in [3]. The gains of the stabilization law are set to $k_p = 4.9\ 10^4$ $k_d = 1.2\ 10^9$. Since $\boldsymbol{a} = \boldsymbol{a}_r^i$ the required boresight pointing can also be achieved through a three-axis attitude stabilization action that aligns the body frame with the ECI frame. Thus, the performances of the proposed boresight stabilization law are compared with those of the following proportional-derivative-like three-axis attitude stabilization law [2]

$$\boldsymbol{m}_c = \boldsymbol{b}^b \times \left[-k_p \sum_{i=\mathbf{1}}^{\mathbf{3}} (\boldsymbol{e}_i \times \boldsymbol{R}\boldsymbol{e}_i) - k_d\, \boldsymbol{\omega}\right] \tag{7}$$

in which $\boldsymbol{e}_i$ is the i-th column of the identity matrix $\boldsymbol{I}_{\mathbf{3}\times\mathbf{3}}$. The numerical values of k_p and k_d are the same as for the boresight stabilization law.

A Monte Carlo campaign of 40 simulations is run by considering random initial attitude and random initial angular velocity with maximum amplitude of 3 deg/sec. The time behaviors of the pointing error and of the control magnetic dipole moment are reported in Fig. 1. In each simulation the pointing accuracy is evaluated through the magnitude $\Theta_{max\ ss}$ which denotes the maximum pointing error in steady-state. Table 1 reports the mean values $\overline{\Theta_{max\ ss}}$ and the standard deviations $\sigma(\Theta_{max\ ss})$ obtained with the two types of stabilization.

Table 1. Statistics of the Monte Carlo campaign

type of stabilization	$\overline{\Theta_{max\ ss}}$ [deg]	$\sigma(\Theta_{max\ ss})$ [deg]
boresight	17.68	3.22
three-axis	18.25	0.60

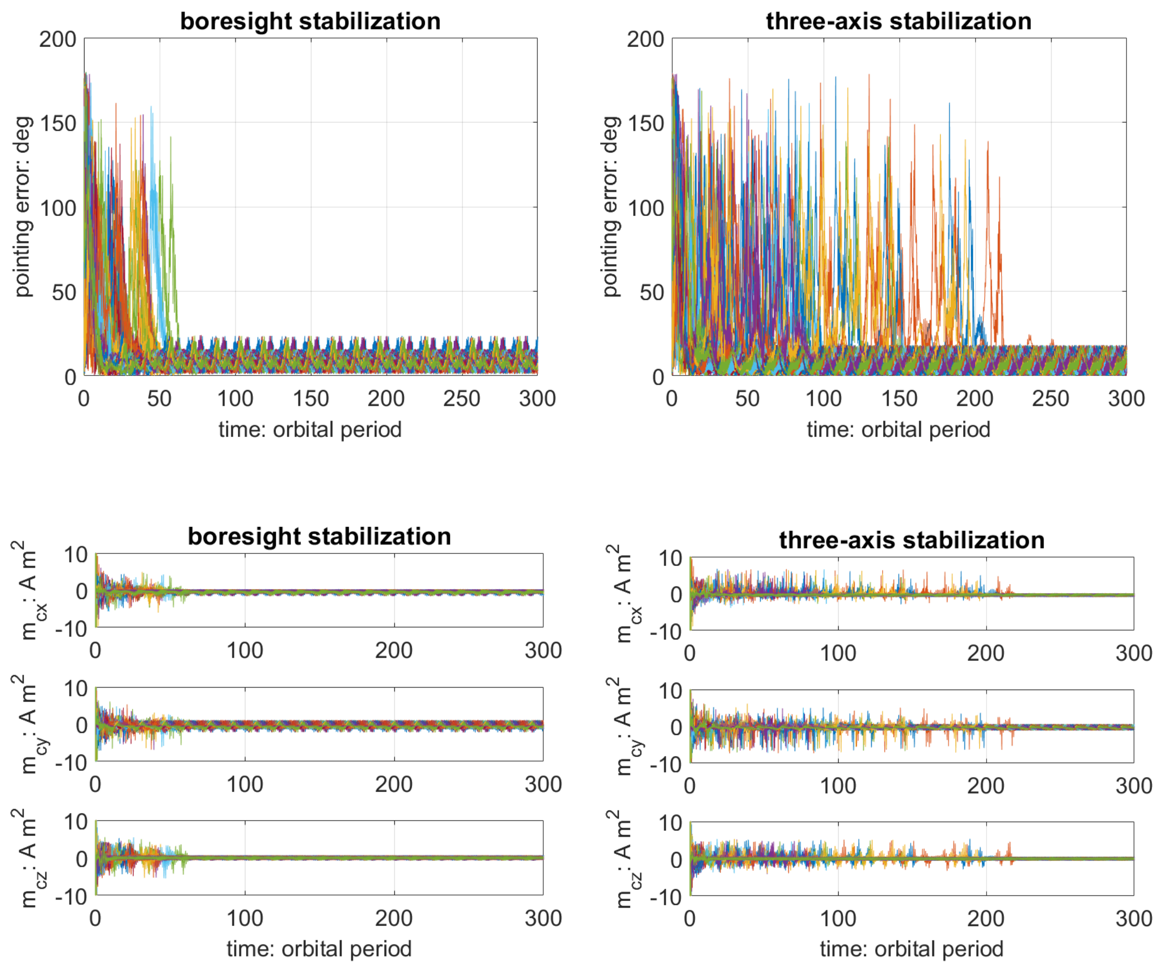

Figure 1. Time histories of the pointing error and of the control magnetic dipole moment.

The statistics show that the pointing accuracies of the two types of stabilization are similar. However, through visual inspection of the time behaviors of the pointing error obtain that boresight stabilization achieves a faster convergence to the steady-state behavior. Specifically, by employing boresight stabilization converge is obtained after approximately 60 orbital periods in the worst simulation run whereas with three-axis stabilization it is obtained only after about 250 orbital periods in the worst case. Faster convergence is probably due to the fact that the objective of boresight stabilization is achieving the desired alignment only for the boresight axis rather than for of all three body axis.

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