Low-energy earth-moon mission analysis using low-thrust optimal and feedback control

A. Almonte[1,a], I. Ziccardi[1,b], A. Adriani[1,c], A. Marchetti[1,d], M. Pontani[2,e]

[1]Thales Alenia Space Italia (TAS-I), Via Saccomuro 24, 00131 Rome, Italy

[2]Department of Astronautical, Electrical, and Energy Engineering, Sapienza University of Rome, via Salaria 851, 00138 Rome, Italy

[a]alessio.almonte@gmail.com, [b]irene.ziccardi@thalesaleniaspace.com, [c]andrea.adriani@thalesaleniaspace.com, [d]andrea.marchetti@thalesaleniaspace.com, [e]mauro.pontani@uniroma1.it

Keywords: Earth-Moon Transfers, Low Energy Transfers, Low-Thrust Spacecraft, Optimal Control, Feedback Guidance and Control, Particle Swarm Optimization

Abstract. This work is focused on designing a low-energy orbit transfer in the Earth-Moon system, aimed at reaching stable capture in a highly elliptical lunar orbit, with the use of low-thrust propulsion. The mission at hand includes three different phases: low-energy ballistic transfer starting from Earth, low-thrust minimum-fuel arc, and low-thrust lunar orbit insertion using variable-thrust nonlinear orbit control. First, a reference trajectory is generated in the framework of the Patched Planar Circular Restricted Three-Body Problem (PPCR3BP), leveraging invariant manifold dynamics. Trajectory propagation is performed using the Bicircular Restricted Four-Body Problem (BR4BP) model. Particle swarm optimization is applied for trajectory refinement and to detect the subsequent minimum-fuel low-thrust arc. Finally, the lunar orbit is entered thanks to the use of variable-thrust nonlinear orbit control.

Introduction

Low-energy Earth-Moon transfers have been studied extensively in the last decades. Some missions have already exploited the results from these studies, leading to considerable propellant savings and other advantages, such as flexibility in target orbit selection, extended launch windows, and more relaxed operational schedules. At the end of the 60s Conley [1] used elements of dynamical systems theory to identify temporary lunar capture conditions. Three decades ago, Belbruno and Miller [2] developed the Weak Stability Boundary (WSB) technique and applied it to lunar transfers, discovering a low-energy transfer through the equilibrium regions of the Sun-Earth-Moon system. Koon *et al* [3] obtained similar results by following the Conley methodology, taking advantage of invariant manifolds of planar Lyapunov orbits around the Earth-Moon L2 libration point. More recently Mingotti *et al* [4] designed low-energy low-thrust transfers, using PPCR3BP for the low-energy trajectory arc and optimal control with a direct method for the low-thrust lunar capture arc. An alternative approach to reach a stable capture orbit is represented by variable-thrust nonlinear orbit control, as described by Gurfil in [5] and Pontani *et al.* in [6]. In fact, using a feedback control law allows applying real-time control and compensate perturbations, with no need of a reference trajectory.

This work is focused on designing a low-energy Earth-Moon transfer starting from a GTO parking orbit and aimed at reaching a stable lunar capture orbit, with the use of low-thrust propulsion. Several design approaches are being employed: (a) use of the invariant manifold dynamics, to obtain a low-energy planar reference trajectory from Earth to Moon, (b) optimal control with the use of the particle swarm algorithm (PSO) to detect the subsequent minimum-fuel low-thrust arc, and (c) variable-thrust nonlinear orbit control to enter the desired lunar orbit. This work intends to show that the mission design techniques proposed in this study represent a convenient approach to preliminary Earth-Moon mission analysis.

MISSION Analysis and results

The mission is composed of two phases, analyzed using different reference frames, to simplify the design. Phase 1 covers the low-energy low-thrust Earth-Moon transfer. A planar low-energy exterior transfer is found using invariant manifold dynamics in the PPCR3BP framework. First, two convenient Jacobi constants $C_{SE} = 3.0075$ and $C_{EM} = 3.15$ are selected for the Sun-Earth and Earth-Moon three body system. Then, Sun-Earth L2 and Earth-Moon L2 planar Lyapunov orbits associated with those constants are found by exploiting their symmetry properties with the use of PSO. Invariant manifolds are propagated from Lyapunov orbits. Invariant manifolds in PCR3BP are a subset of the phase space, and separate bouncing trajectories from transit orbits. Taking advantage of this property, invariant manifolds are cut with Poincaré sections, reducing the phase space dimension to 2. Manifold cuts and Poincaré sections are shown in Figure 1, where section S_A cuts stable manifold $W_S^+(\gamma_2)$ and section S_B cuts unstable manifold $W_U^-(\gamma_2)$ of Sun-Earth L2 Lyapunov orbit γ_2; instead, section S_C cuts stable manifold $W_S^+(\delta_2)$ of Earth-Moon L2 Lyapunov orbit δ_2.

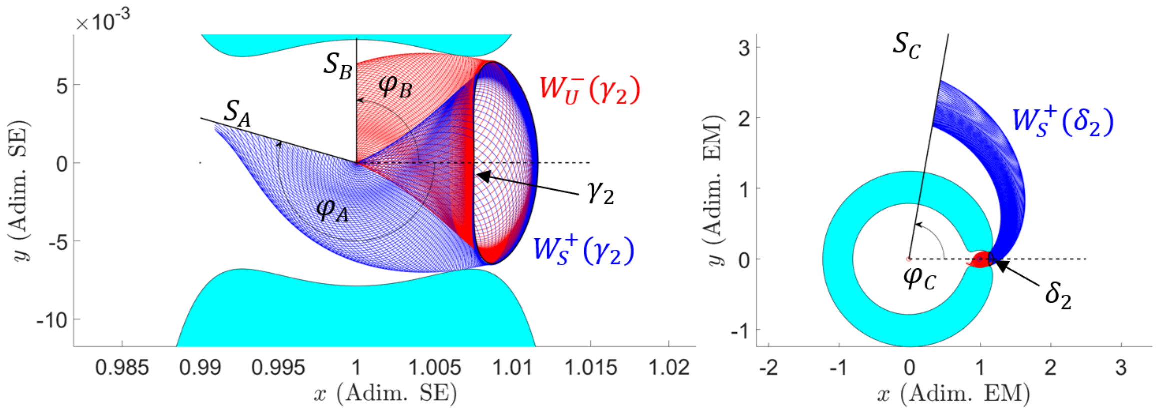

Figure 1: Manifolds cut and Poincaré sections

Intersections are evaluated in the $r_2 - \dot{r_2}$ plane, shown in Figure 2, where r_2 is the distance and $\dot{r_2}$ the radial velocity with respect to Earth, secondary body in the Sun-Earth three-body system.

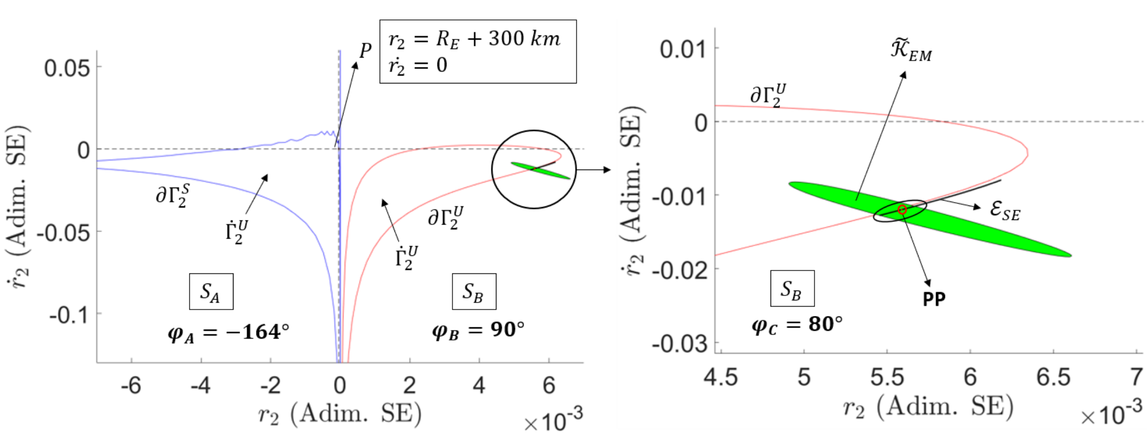

Figure 2: Manifolds intersection in $r_2 - \dot{r}_2$ plane

From figure 2 it is possible to see that the angle φ_A is tuned to move the $W_S^+(\gamma_2)$ stable manifold cut $\partial\Gamma_2^S$ close to the starting point P corresponding to GTO pericenter conditions. Trajectories starting from P in S_A end in the $\mathcal{E}_{SE}$ set on section S_B, at external points close to the $W_U^-(\gamma_2)$ unstable manifold cut $\partial\Gamma_2^U$. Tuning angle φ_C allows moving the set $\widetilde{\mathcal{K}}_{EM}$, until intersection with $\mathcal{E}_{SE}$ occurs. The set $\mathcal{E}_{SE} \cap \widetilde{\mathcal{K}}_{EM}$ represents temporary ballistic capture trajectories and here the Patch Point (PP) between Sun-Earth and Earth-Moon trajectory arcs is chosen. The PP conditions are then integrated backward using the Bicircular Restricted Four-Body Problem (BR4BP), to reach (through backward integration) the GTO orbit, with parameters $a = 24363.57$ km, $e = 0.7036$, $i = 23.45°$, $\Omega = 0°$, $\omega = 163.72°$, $\theta_* = 0°$. The velocity change to perform Translunar Injection (TLI) is $\Delta v^{(TLI)} = 671.8$ m/s and the time of flight is $\Delta t_1 = 142.47$ days. The small maneuver necessary at patch point to link the trajectories is substituted with a low-thrust arc obtained with minimum-fuel optimal control. The low thrust propulsion system is identified by $u_T^{(\max)} = \frac{T_{max}}{m_0} = 2 \cdot 10^{-4}$ m/s and $c = g_0 I_{sp} = 18.142$ km/s. The state vector is defined as $\boldsymbol{X} = \left[x, y, v_x, v_y, \frac{m}{m_0}\right]^T = [x_1, x_2, x_3, x_4, x_5]^T$ and the control vector is $\boldsymbol{u} = [u_T, \alpha]$, where $u_T = T/m_0$ with $0 \leq u_T \leq u_T^{(\max)}$ and α is the angle between the thrust direction and the line from the Sun and the Earth-Moon barycenter. The objective of minimum-fuel optimal control is to find the control $\boldsymbol{u_T}$ and the constant parameters vector $\boldsymbol{p}$ such that the cost function $J = -m_f$ is minimized, while satisfying the state equations $\dot{\boldsymbol{X}} = \boldsymbol{f}(\boldsymbol{X}, \boldsymbol{u}, t, \boldsymbol{p})$ and the boundary conditions $\boldsymbol{\Psi}(\boldsymbol{X_0}, \boldsymbol{X_f}, t_0, t_f, \boldsymbol{p}) = \boldsymbol{0}$. Additional constraints are added to the Lunar Orbit Insertion (LOI) condition, limited to orbits with $e \in (0.6, 0.7)$, to arrive at a capture orbit, and $r_P = R_M + 100$ km to avoid Moon impact. These constraints are written in terms of equality constraints using the parameter vector $\boldsymbol{p}$. Exploiting the necessary optimality conditions and the Pontryagin minimum principle it is possible to obtain the control law, depending on co-state vector $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5]^T$. The minimum set of unknown parameters is $\boldsymbol{\chi} = \{\boldsymbol{\lambda_0}, t_f, \boldsymbol{p}\}$ and is found using PSO. The minimum time of flight is $t_f = 13.30$ days, with $\frac{m_f}{m_0} = 0.994$. The time histories of the thrust angle are shown in Figure 3.

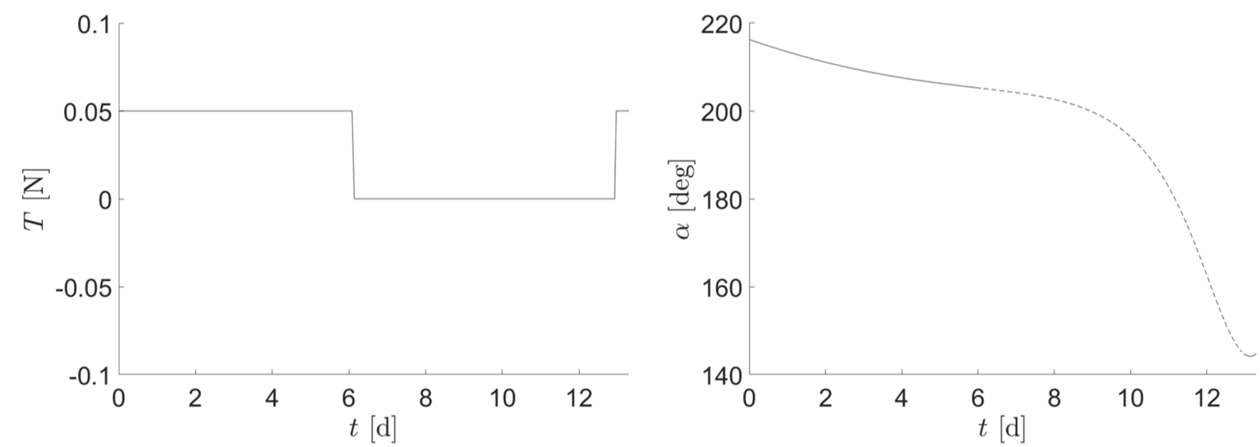

Figure 3: Time histories of thrust T and thrust direction α

In Phase 2 of the mission nonlinear control was employed to enter a stable lunar orbit. This control enjoys quasi-global stability properties and allows compensating perturbations [6]. The main objective of this phase is convergence to the target orbit, while compensating perturbations due to Earth and Sun. In this framework the state vector is given by the Modified Equinoctial

Elements (MEE) and the mass ratio, i.e. $\boldsymbol{X} = \left[p, l, m, n, s, q, \frac{m}{m_0}\right]^T = [x_1, x_2, x_3, x_4, x_5, x_6, x_7]^T = [\boldsymbol{z}, x_6, x_7]^T$, whereas the control vector is $\boldsymbol{u_T} = \boldsymbol{T}/m_0$ with $0 \le u_T \le u_T^{(\max)}$. The target set is defined in terms of MEE as

$$\boldsymbol{\Psi} = \left[x_1 - a_d(1-e_d^2),\ x_2 - e_d cos(\Omega_d + \omega_d),\ x_3 - e_d sin(\Omega_d + \omega_d),\ x_4 - \tan\left(\frac{i_d}{2}\right)\cos(\Omega_d),\ x_5 - \tan\left(\frac{i_d}{2}\right)\sin(\Omega_d)\right]^T.$$

The dynamics is governed by the Lagrange Equations with MEE [6]. The control law is derived in [6] and yields $\boldsymbol{a_T}$**,** i.e. the thrust acceleration as a function of the state, the boundary condition violation, and the perturbing acceleration. The latter includes the effect of Earth and Sun as third bodies. Matrix $\boldsymbol{K}$ is a positive definite diagonal matrix of gains, selected after trial-and-attempt tuning. The target orbit is reached in a time of flight $t_f = 77.52$ days, with $\frac{m_f}{m_0} = 0.920$. After 100 days from Lunar Orbit Injection (LOI) the mass ratio reduces to $\frac{m_f}{m_0} = 0.909$, because propellant is used to compensate the perturbations. The orbit elements of the planar capture orbit reached at the end of Phase 1, together with target parameters and parameters after 100 days of propagation, are shown in Table 1. The trajectory in Phase 2 and the full trajectory are shown in Figure 4.

	a [km]	e	i [deg]	Ω [deg]	ω [deg]
LOI	108058	0.7000	0	-23.78	48.14
Target orbit	9751	0.6870	55.70	120.00	90.00
Final	9772	0.6871	55.71	120.01	89.99

Table 1: orbit elements at LOI and along the target orbit

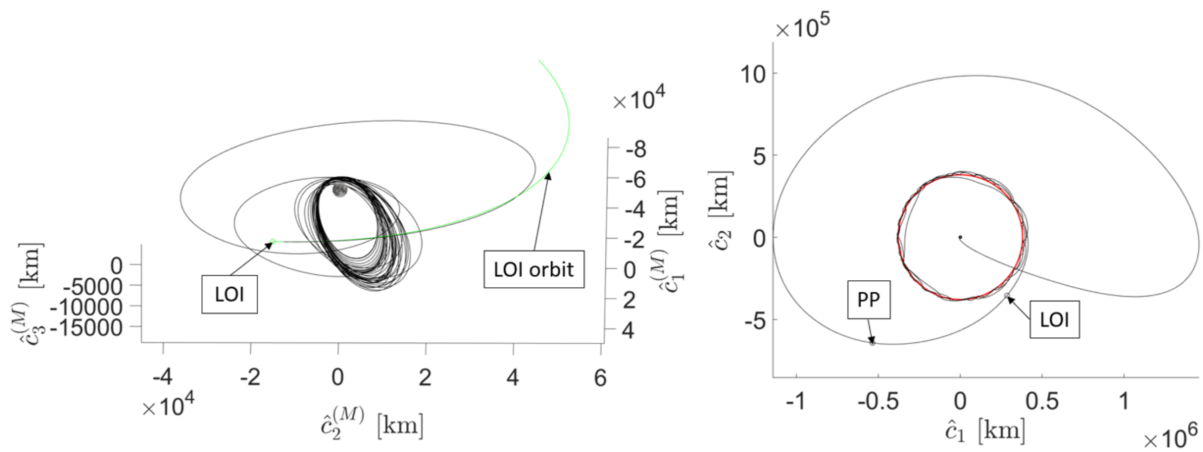

Figure 4: a) Trajectory in Phase 2, in MCI reference frame b) Full transfer in ECI reference frame

Concluding Remarks

This paper proposes a preliminary mission analysis for a low-energy low-thrust Earth-Moon transfer, starting from a GTO orbit and aimed at reaching a lunar highly elliptical orbit. In Phase 1 of the mission, regarding the Earth-Moon transfer, invariant manifold dynamics is used to obtain a low-energy planar reference trajectory, thus reducing the trajectory design to the research of a point in the phase space. The PSO algorithm allows further refininement of the trajectory in the framework of the BR4BP. Then, the same algorithm is employed to find the subsequent minimum-fuel low-thrust arc. In Phase 2 the final highly elliptic lunar orbit is reached using variable thrust nonlinear orbit control, with perturbations compensation and no need of a reference trajectory.

Assuming that the departure from GTO is demanded to the launch vehicle, as a part of its operations, the overall propellant consumption for the spacecraft equals 8% of its initial mass. In the end, the combination of the techniques described in this study allows defining an Earth-Moon mission profile with modest propellant consumption. In principle, the methodology at hand is also appliable to a variety of departure and target orbits in the Earth-Moon system.

References

[1] Conley, C., On the Ultimate Behavior of Orbits with Respect to an Unstable Critical Point I. Oscillating, Asymptotic, and Capture Orbits, Journal of Differential Equations, No. 5, 1969, pp 136–158. https://doi.org/10.1016/0022-0396(69)90108-9

[2] Belbruno, E. A., Miller, J. K., Sun-Perturbed Earth-to-Moon Transfers with Ballistic Capture, Journal of Guidance, Control and Dynamics, Vol. 16, No.4, 1993, pp 770-775. https://doi.org/10.2514/3.21079

[3] Koon, W. S., Lo, M. W., Marsden, J. E., Ross, S. D., Low Energy Transfer to the Moon, Celestial Mechanics and Dynamical Astronomy, Vol. 81, 2001, pp 63-73. https://doi.org/10.1007/978-94-017-1327-6_8

[4] Mingotti, G., Topputo, F., Bernelli-Zazzera, F., Efficient Invariant-Manifold, Low-Thrust Planar Trajectories to the Moon, Communications in Nonlinear Science and Numerical Simulation, Vol. 17, Issue 2, 2012, pp 817-831. https://doi.org/10.1016/j.cnsns.2011.06.033

[5] Gurfil, P., Nonlinear feedback control of low-thrust orbital transfer in a central gravitational field, Acta Astronautica, Vol. 60, 2007, pp 631-648. https://doi.org/10.1016/j.actaastro.2006.10.001

[6] Pontani, M., Pustorino, M., Nonlinear Earth orbit control using low-thrust propulsion, Acta Astronautica, Vol. 179, 2021, pp 296–310. https://doi.org/10.1016/j.actaastro.2020.10.037