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Sensitivity analysis of analytically—corrected acoustic metamaterials into the spacetime domain

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Abstract. The present work is focused on the sensitivity analysis of analytically--corrected acoustic metamaterials with respect to the variation of relevant parameters for aeronautical applications. The performance decay of acoustic metamaterials designed in static conditions when operating in a moving flow can be mitigated using a design process involving coordinate transformations capable of recasting the convective wave equation in the form of the static D'Alembertian. To this aim, a spacetime reformulation of the problem is needed in order to ensure the formal invariance of equations under the action of spacetime coordinate transformations. The considered test case sees a circumferential domain occupied by a metafluid embedded by a conventional medium characterized by a uniform background flow and within which a monopole source is located. All the numerical simulations are done in the frequency domain using the commercial finite element method solver. A parametric study is conducted to analyze the influence of the acoustic source position with respect to the background flow and the variation of the intensity of the latter.

Introduction

The present work's topic is related to the aircraft mobility sustainability task sought by the research efforts of the last decades. Particularly, it arises from the necessity to examine possible strategies to reduce the community noise generated by aircraft mobility. Given the discrepancy between the emission reduction request and the technological advances, a breakthrough is needed. With the advent of the concept of metamaterials, thanks to the work of Veselago, in [1], Pendry and Smith in [2, 3], possibilities have opened up to develop new strategies that could be implemented with current technologies and that could allow achieving the goals set. Firstly though for electromagnetic applications, the metamaterial devices are conceived as the repetition of a sample geometric architecture within the space. The lattice obtained, combined with the materials' properties, showed atypical behaviours not reachable in nature and highly adaptable for satisfying the most diverse application requirements.

Only after the development of the transformation optics techniques [4-6] their enormous potentiality has been actually exploited by releasing the metamaterial properties from the specific design and linking them only to the specific application. As a consequence, their applicability has also been extended to other fields of physics and engineering, acoustics among others.
Following those steps, Norris developed the mathematical model that describes the atypical behavior of acoustic metamaterials obtainable through coordinate transformation addressing the definition of *metacontinuum* able to produce the so called *acoustic mirage* [7, 8]

Both transformation optics techniques and the metacontinuum model of Norris rely on the concept of formal invariance of the governing equation under coordinate transformations provided that unconventional material properties are identified.

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However, their application to aeronautical contexts has led to a performance decay due to the presence of a background flow that couples the fluid dynamic phenomenon with the acoustic one as seen by Iemma in [9] and Iemma and Palma in [10-12]. In fact, the convection effects result in a wave operator defined through mixed space and time derivatives that make the formal invariance fails. The solution proposed by the authors could be seen as a generalization of the coordinate transformation approach where the formal invariance of the governing equation is restored, describing the propagation phenomenon of the acoustic perturbation in the spacetime domain. In this new manifold, the transformation considered is based on Taylor's or Prandtl-Glauert's transformations and is applied directly to the acoustic metacontinuum model. Hence, analytically adapted acoustic metamaterial properties to convective applications are obtained.

The validation process of the proposed design strategy started in Colombo, Palma, and Iemma in [13] and here is further developed. Specifically, this work introduces a sensitivity analysis of the adapted metamaterial behaviour with respect to the variation of characteristic parameters of aeronautical problems, such as the relative position of the sound source with respect to the background flow and the Mach number intensity. Moreover, since the purpose is to highlight the corrections' behaviour, the acoustic metacontinuum properties are designed to mimic a domain filled with air. Firstly the mathematical model of the generic acoustic metacontinuum is shown, and subsequently, the methodology for its extension for convective applications is presented in the spacetime domain. Then the numerical set-up and the numerical results obtained with the parametric study are discussed.

Methodology

Relying on the definition of the acoustic metacontinuum model of Norris is possible to obtain atypical behaviours thanks to the proper characterization of the medium in terms of inertia and stiffness properties. In the most general case, those arise when anisotropic inertia and bulk modulus are defined. Following Norris' formulation, once the properties are identified, the governing equation that describes the propagation of the acoustic perturbation within such medium assumes the form

$$-\partial_{tt}p + K_{ref}\boldsymbol{S}:\nabla(\boldsymbol{\rho}^{-1}\boldsymbol{S}\nabla p) = 0\,, \tag{1}$$

where K_{ref}, ρ_{ref} and $c_{ref} = \sqrt{K_{ref}/\rho_{ref}}$ are the referenced bulk modulus, density and speed of sound respectively. In Eq. (1) two second order tensors are present: the $\boldsymbol{\rho}$ and the **S** tensors that enclose the anisotropic inertia and the anisotropic stiffness respectively. The actual shape of those quantities depends on the specific application and are generally obtained through coordinate transformation. Since in the tested case the metacontinuum is thought to mimic a domain filled with air, the identity transformation is used and its parameters reduce to the following simple form:

$$K_{ref} = K_0, \qquad \boldsymbol{\rho} = \rho_0\boldsymbol{I}, \qquad \boldsymbol{S} = \boldsymbol{I}, \tag{2}$$

where the subscript "0" refers to the characteristic quantities of a conventional fluid. It is underlined that Eq. (1) is known only in terms of pseudo-pressure and in a quiescent fluid.

Analytical Correction for Acoustic Metacontinua

Here the mathematical tools that allow extending the applicability of acoustic metamaterials to aeronautical context are explained but, firstly, the reformulation of the governing equation into the spacetime domain is needed. In fact, since the wave operator of aeroacoustic phenomenon is defined through mixed space and time derivatives, the acoustic mirage approach is no longer applicable since the formal invariance no longer holds. However, the substitution of a point in the 3D euclidean space with a point-event in a 3D+1 pseudo-Riemannian manifold allows the recovery of the formal invariance under general spacetime coordinate transformation. In this new manifold, any partial differential equation describing the propagation of a scalar field could be recast into the spacetime domain as

$$\partial_\mu(W^{\mu\nu}\partial_\nu\varphi) = 0\,, \tag{3}$$

where $\boldsymbol{\partial} = (1/c_{ref}\ \partial_t,\ \partial_{xi})$ is the differential operator in the new manifold and $W^{\mu\nu}$ are the contravariant components of a second order tensor that involves all the information about the phenomenon observed. The tensor $\mathbf{W}$ is linked to the metric tensor by $\boldsymbol{W} = \sqrt{-g}\boldsymbol{g}^{-1}$. Thus, if coordinate transformations are introduced, they will modify directly the component of such tensor. Specifically, the analytical correction method is based on Taylor's and Prandl-Glauert's coordinate transformations. Both allow the transformation of convective patterns into static ones under the assumption of potential flow. Thus, their definition is directly linked to the characteristics of the background flow. Considering a description of the phenomenon into the spacetime domain, they assume the following forms:

$$\xi_0' = \beta\xi_0 + M_\infty\frac{\xi_1}{\beta}, \quad \xi_1' = \frac{\xi_1}{\beta}, \quad \xi_2' = \xi_2, \quad \xi_3' = \xi_3, \tag{4}$$

for Prantl-Glauert's one, whereas for Taylor's transformation, the form assumed is

$$\xi_0' = \xi_0 + M_\infty\widehat{\boldsymbol{\Phi}}(\boldsymbol{\xi}) = \xi_0 + M_\infty(\xi_1 + \hat{\phi}), \quad \xi_i' = \xi_i, \tag{5}$$

with the aerodynamic potential of the background flow normalized as $\widehat{\boldsymbol{\Phi}} = \Phi/\|\mathbf{U}_\infty\|$.
The application of such coordinate transformations goes under the definition of the transformation matrix $\boldsymbol{\Lambda}$ which has to be inversely applied in order to analytically introduce convection onto static propagation as

$$\boldsymbol{W}_c = \boldsymbol{\Lambda}^{-1}\boldsymbol{W}\boldsymbol{\Lambda}^{-T}\,. \tag{6}$$

Therefore, the transformation in Eq. (6) is thought to be applied to the spacetime reformulation of the governing equation (1), and the adapted metamaterial properties are obtained.

Numerical Results

The numerical results obtained concerned the parametric study of the behaviour of both the corrections proposed in (4) and (5). Numerical simulations have been done for a monochromatic acoustic perturbation exploiting the reformulation of the problem in the frequency domain. Specifically, the interest relies on the capability of the coordinate transformation approach to fictitiously induce flow in a domain where static propagation was originally defined.

To do so, a domain completely occupied by air is considered and virtually subdivided into two concentric regions, namely an inner circle of radius R_{in} and an annulus with outer radius $R_{out} = 30\ R_{in}$. The air in the inner circle is actually modelled by an acoustic metafluid, whose parameters mimic air following Eq. (2) . The simulations are performed using the commercial FEM solver Comsol Multiphysics [14] and the corrected metafluid equations (Eq. (3) for acoustic pressure applying the corrections of Eq. (6)) are implemented using the Partial Differential Equation (PDE) module. The acoustic propagation in the outer *hosting* fluid domain is modelled through the Linearized Euler Equations, with the dedicated module in the mentioned software. The continuity equation for the hosting fluid domain is forced by a monopole source with unitary intensity. Its location varies along an arc of radius $R_{source} = 7.5\ R_{in}$, centered in $\mathbf{x} = (0,0)$, from θ = -45° (upstream direction) to θ = 45° (downstream direction), with θ being the angle respect to the vertical axis. Moreover, the hosting fluid is characterized by a background flow; numerical simulations are carried out with a Mach number associated with the free stream ranging from 0.1 to 0.3. To numerically reproduce the free space propagation, the Asymptotic Far Field Radiation condition is imposed on the outer boundary. The Dirichlet and Neumann boundary conditions are imposed as described in [13] at the interface between the two domains for the correct exchange of information between the two modules and to guarantee the acoustic field continuity among the two domains in terms of pressure and normal velocity component.

The underlying hypothesis of the present work is that by applying the above mentioned analytical corrections to the tensor **W**, the convective effects induced by the presence of a flow are simulated and virtually introduced in the acoustic propagation within the metacontinuum. In this way, the scattering that would appear coupling the static air-metafluid with the flowing air in the hosting domain should be abated, obtaining the propagation pattern of a monopole source in a free field in the presence of aerodynamic convection. The scattering cross section σ is defined as the merit parameter to evaluate the residual scattering happening using the corrected metacontinuum to model the air in the inner circle:

$$\sigma = \int_C \left| L_{p_{metacontinuum}} - L_{p_{idealcase}} \right| dl \ , \tag{7}$$

where p = 20 [μPa] is the reference pressure for the evaluation of the L_p. Figure 1 shows the values for σ on a circle of radius $R_{SCS} = 1.5\ R_{in}$. The parameter σ is evaluated for each source position θ = ± [15°, 25°, 35°, 45°] and Mach number associated with the background flow M_∞ = [0.1, 0.2, 0.3].

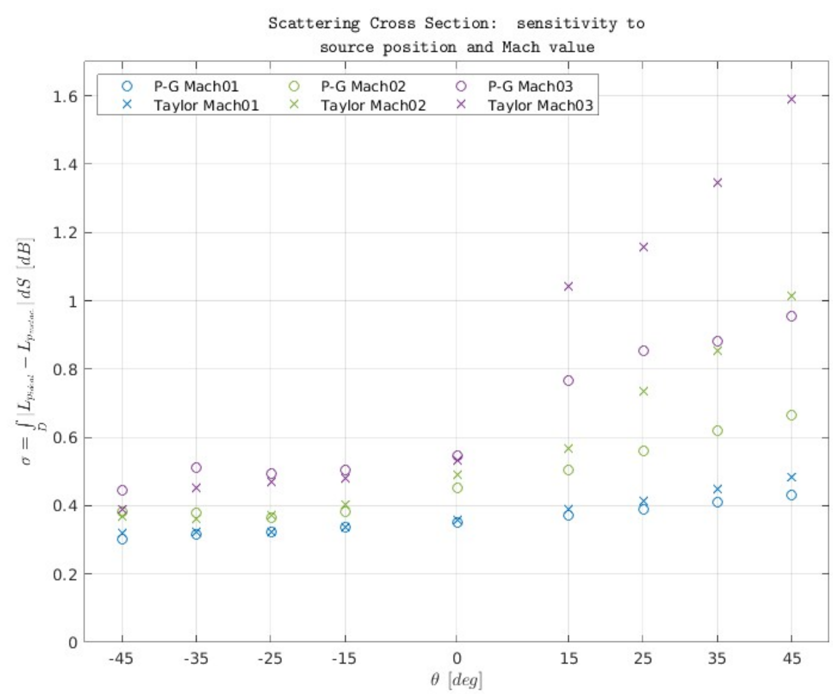
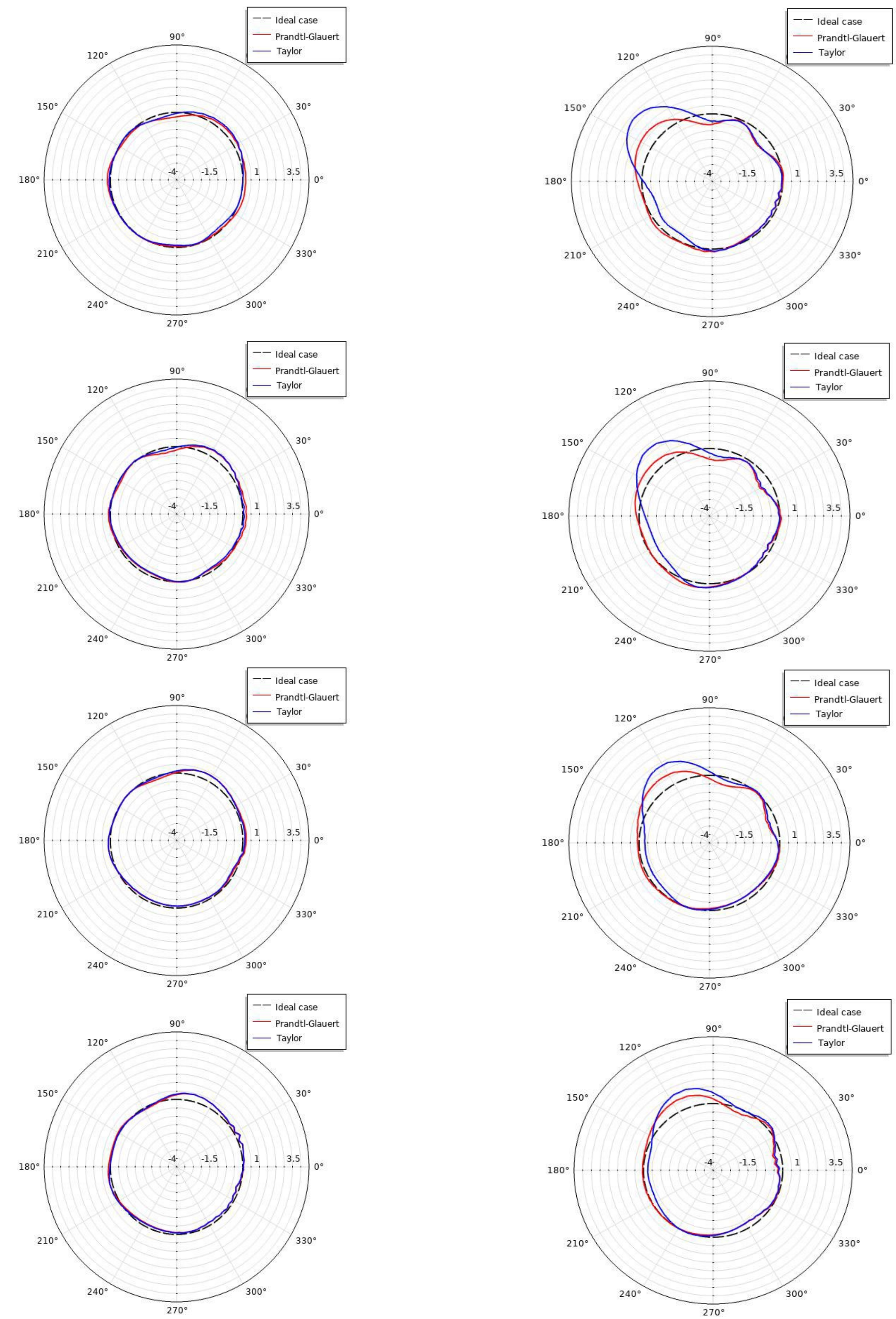


Figure 1. *Point plot of the σ evaluated for M_∞ = [0.1, 0.2, 0.3] and for every source location θ considered*
- Prandtl-Glauert's (o-marks) and Taylor's (x-marks) correction sensitivity comparison



(a) (b)

Figure 2. *Polar [dB] diagram of the scattered field for $M_\infty = 0.3$ at $r = 1.5\ R_{in}$ - Prandtl-Glauert's (red) and Taylor's (blue) correction sensitivity to the variation of the source location - left column 2a relative to $\theta = -[45°, 35°, 25°, 15°]$ values; right column 2b relative to $\theta = [45°, 35°, 25°, 15°]$ values.*

A trend in the σ appears from Fig. 1: the more the source location moves downstream, the higher the values of the SCS. The effect of Mach is more evident for the downstream θ positions, increasing the values of σ. This is also in agreement with previous findings in Iemma and Palma [11]. At least for Taylor's coordinate transformation, the authors addressed the effect of the neglected terms as a pattern field connected to a dipole propagation in the case of an acoustic disturbance impinging an impermeable body. The study highlights the presence of sectors of high intensity where the correction loses part of its efficiency. In the case considered, it seems that the sector where the misbehaviour of the corrections arises coincides with the downstream location of the acoustic source. Thus, it is plausible that the same considerations done by the authors for their application of Taylor's transformation could also be extended here. Even with limited amplitude compared to the Taylor one, the performances of the analytic corrections seem to depend on the source position also for the PG transformation. The causes are not completely clear to the authors. In order to verify these effects, the insertion loss (IL) is evaluated on R_{SCS} and shown in a polar plot. In Fig. 2 the results obtained with the application of each analytical correction for every θ angle and with the Mach number set to its higher value are shown. The scattering directivity is increased when the source is moved towards downstream positions. Moreover, some small ripples appear in the IL, suggesting their origin to be sought in possible numerical issues.

Conclusions

The parametric study conducted has brought more insights into the coordinate transformations' behaviour. The qualitative analysis of their sensitivity to the variation of the characteristic aeronautical parameters considered has led to the identification of an influence zone where the corrections are losing part of their efficacy. The fictitious introduction of flow in the metafluid domain seems more efficient whenever the source location is in the upstream zone. This condition guarantees a low value of the merit parameter σ independently from the Mach number value for both coordinate transformations. The present analysis confirms previous works' findings and represents a step forward in defining more suitable strategies for using adapted meta-devices in aeronautical contexts. However, a higher comprehension of the coordinate transformations behaviour requires the analytical approximation's effect quantification and its influence on the performances of convective-designed metamaterials.

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