

Prediction of aeroacoustics of deformable bodies with solid or porous surface through a boundary integral formulation

Beatrice De Rubeis^{1,a*}

¹Roma Tre University, Department of Civil, Computer Science and Aeronautical Technologies Engineering, Via della Vasca Navale 79, Rome, Italy, 00146

^abeatrice.derubeis@uniroma3.it

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Abstract. Novel boundary integral formulations suitable for the radiation of acoustic pressure from deformable, solid or porous, surfaces in arbitrary motion are theoretically/numerically developed. These will first be applied to a translating wing subject to unsteady bending and torsion. The influence of surface deformation on the evaluated perturbation fields will be assessed for different amplitude and frequency values of the bending and torsion modes. Subsequently, pressure will be radiated from a deformable porous sphere to validate the formulations for this type of surface.

Introduction

Nowadays a crucial issue for the aviation industries is the mitigation of the aircraft nuisance to comply with the increasingly demanding constraints in acoustic emissions.

Indeed, the growing interest in Urban Air Mobility (UAM) applications makes the design of more environmentally sustainable vehicles mandatory, representing a key point for the community acceptance of this new mobility concept. At the same time, the enhancement of conventional aircraft sustainability is also of primary importance from the perspective of eco-friendly next-generation aviation. To this aim, several international institutions and committees have established quantitative goals to achieve these common targets [1], in line with the Flightpath 2050 global vision [2].

For these reasons, many industries and research centers are making a great effort to develop new low-noise technologies to improve the sustainability of the current-adopted ones. Thus, to guarantee the sustainability of the aviation described in [2], the need to develop breakthrough concepts able to reduce both noise emissions and fuel consumption [3] has become unquestionable. In this framework, a lot of disruptive design solutions have been proposed like, for instance, Distributed-Propulsion layouts [4] based on the integration of the propulsive system with the aircraft airframe, with the possibility to exploit Boundary-Layer-Ingestion (BLI) technologies [5].

From all the above considerations, it is understood that an accurate description of the vehicle's acoustic emissions is undoubtedly of primary importance in the whole design process since its early stages. Because of this, it is essential to observe that the aeroacoustics of these novel concepts is affected by phenomena that might be negligible in standard configurations. Among these, for instance, the strong aerodynamic interactions occurring in multirotor propulsive systems, which may significantly modify performance and noise radiation of the aircraft more than in a standard configuration [6]. Furthermore, in highly-flexible innovative configurations, particular attention must be paid to the effects produced by the body deformation, which must be accurately taken into account both in aerodynamic and aeroacoustic simulations.

A significant amount of literature concerning the aeroacoustics of rigid bodies is available (see, for instance, the Farassat 1A formulation [7] commonly applied to evaluate noise emitted by rotating bodies (like helicopter rotor blades). However, since the distribution of perturbation fields produced by bodies moving in a fluid medium depends on the spatial orientation of the body

surface, it can be strongly affected by their deformation (particularly considering slender, flexible structures). Thus, the availability of aerodynamic and aeroacoustic formulations capable of considering such effects can be mandatory. Some acoustic formulations accounting for the body deformation are available in the literature, for instance, the approaches presented in [8] and [9], where the contribution of the blade deformation is included in the Farassat 1A formulation and in its compact-source version, respectively.

Specifically, starting from the most general form of wave equation governing the propagation of perturbations in a fluid medium, the Green function is introduced in order to derive the solution in terms of a boundary integral representation capable to take into account surface deformation effects. Two versions of the integral solution are determined for solid surfaces: one expressed in a reference frame that only observes the deformation of the surface (i.e. the one fixed with the rigid motion of the body) and one expressed in a reference frame fixed with the undisturbed medium. Given the lack of investigation on this topic in the literature, the proposed formulations are validated by cross-comparison of the predictions provided by the integral solutions expressed in the two reference frames. The numerical investigation concerns the effect of surface deformation on the noise emitted by a translating wing subjected to unstable bending and torsional deformations. The effects of the amplitude and frequency of surface deformations on the radiated noise will be analysed by comparison with the acoustic emission of the rigid body, in order to assess the importance of including body deformations in aircraft aeroacoustics. To validate that the written formulations apply to porous surfaces as well, the pressure radiated from a deformable porous sphere containing a source inside was compared with the direct radiation from the source.

Methodology

Let us consider a moving body producing perturbations within a fluid. In a frame of reference $\mathcal{R}(\mathbf{x})$, fixed to the undisturbed fluid (FFR), the potential aerodynamics and noise radiation problems can be described by models that are expressed as subcases of the following general wave equation problem for the generic field, $\Psi(\mathbf{x}, t)$

$$\nabla^2 \bar{\Psi} - \frac{1}{c^2} \frac{\partial^2 \bar{\Psi}}{\partial t^2} = \chi + \mathbf{z} \cdot \nabla H + \nabla \cdot (\mathbf{Z} \nabla H) + k_1 \frac{\partial H}{\partial t} + \frac{\partial}{\partial t} \left(k_2 \frac{\partial H}{\partial t} \right) + \frac{\partial}{\partial t} (\mathbf{w} \cdot \nabla H) \quad (1)$$

where $\bar{\Psi} = \Psi H(f)$ is the extension of the field Ψ to the whole domain \mathbb{R}^3 , with $H(f)$ denoting the Heaviside function with the argument $f(\mathbf{x}, t)$ representing the body surface. In addition, c denotes the speed of sound in the undisturbed fluid, while $k_1, k_2, \mathbf{z}, \mathbf{w}$ and \mathbf{Z} are scalar, vector and tensor forcing terms which depend on the specific aerodynamic or aeroacoustic application.

The application of the Green function method provides an integral solution of Eq. (1), which has been extensively used and validated in the past for potential aerodynamics and aeroacoustics of rigid bodies (see, for instance, [10], [11] and [12]). In the following, the extension of the boundary integral solution of Eq. (1) to deformable bodies is briefly described. It derives from the boundary integral formulation for the potential aerodynamics of deformable bodies introduced in [13] which, in turn, is developed as the extension of a boundary integral formulation for rigid bodies expressed in a frame of reference rigidly connected to the body (BFR, $\mathcal{R}(\mathbf{y})$).

Note that, in the case of a rigid body, the introduction of the BFR is particularly useful since, differently from the FFR description, the integration domain (namely, the locus of the emitting points at the instants of emission of the signals that reach the microphone at the observation time) coincides with the real surface of the body.

Following [13], for the development of the integral formulation for deformable bodies, let us introduce a curvilinear coordinate system (ξ^1, ξ^2, ξ^3) that follows the deformation of the body, such that: $\xi^3(\mathbf{y}, t) = \mathbf{0}$ identifies the motion of the body surface \mathcal{S} with respect to the BFR, and

each point $\mathbf{y} \in \mathcal{S}$ is associated to a point in the plane $(\xi^1, \xi^2) \in \Omega$. Thus, close to the surface of the body one has $\nabla_{\mathbf{y}} \xi^3 = \mathbf{n}$ (with \mathbf{n} denoting the outward unit normal vector of surface \mathcal{S}), and the Heaviside function on the body surface is expressed as $H[\xi^3(\mathbf{y}, t)]$.

Noise radiation in the body frame

From the general integral solution of Eq. (1), the integral formulation for sound radiation is derived, as well. To this purpose, for \mathbf{p}_0 and ρ_0 denoting pressure and density of the undisturbed fluid, let us introduce the acoustic pressure, $\mathbf{p}' = c^2(\rho - \rho_0)$ and the compressive tensor, $\mathbf{P} = (\mathbf{p} - \mathbf{p}_0)\mathbf{I}$. Then, for $\mathbf{v}_T = \mathbf{v} + \mathbf{v}_D$, Eq. (1) reduces to the Ffowcs Williams and Hawkings equation for $\bar{\Psi} = \bar{p}'$, $\chi = -\nabla_{\mathbf{y}} \cdot \nabla_{\mathbf{y}} \cdot (\mathbf{HT})$, $\mathbf{Z} = \mathbf{P}$ and $\mathbf{w} = -\rho_0 \mathbf{v}_T$, while $\mathbf{z} = \mathbf{0}$, $\mathbf{k}_1 = \mathbf{0}$ and $\mathbf{k}_2 = \mathbf{0}$. Thus, the integral solution of the general differential model in Eq. (1) provides the following BFR representation of the acoustic pressure radiated by deformable bodies [14]

$$\begin{aligned} \bar{p}'(\mathbf{y}_*, t_*) &= \int_0^\infty \iint_{\Omega} -\hat{G}_0 \nabla_{\mathbf{y}} \cdot \nabla_{\mathbf{y}} \cdot (\mathbf{HT}) \hat{J} \Big|_{g=0} d\xi^1 d\xi^2 d\xi^3 \\ &\quad - \iint_{\Omega} [(\mathbf{Pn}) \cdot \nabla_{\mathbf{y}} \hat{G}_0 \hat{J}] \Big|_{\xi^3=0, g=0} d\xi^1 d\xi^2 \\ &\quad + \iint_{\Omega} \left[\frac{1}{(1 + \mathbf{v}_D \cdot \nabla_{\mathbf{y}} \hat{\theta})} \frac{\partial}{\partial t} \Big|_{\xi} [(\mathbf{Pn}) \cdot \nabla_{\mathbf{y}} \hat{\theta} \hat{G}_0 \hat{J}] \right] \Big|_{\xi^3=0, g=0} d\xi^1 d\xi^2 \\ &\quad - \iint_{\Omega} [\rho_0 \mathbf{v}_T \cdot \mathbf{n} \mathbf{v} \cdot \nabla_{\mathbf{y}} \hat{G}_0 \hat{J}] \Big|_{\xi^3=0, g=0} d\xi^1 d\xi^2 \\ &\quad - \iint_{\Omega} \left[\frac{1}{(1 + \mathbf{v}_D \cdot \nabla_{\mathbf{y}} \hat{\theta})} \frac{\partial}{\partial t} \Big|_{\xi} [\rho_0 \mathbf{v}_T \cdot \mathbf{n} (1 - \mathbf{v} \cdot \nabla_{\mathbf{y}} \hat{\theta}) \hat{G}_0 \hat{J}] \right] \Big|_{\xi^3=0, g=0} d\xi^1 d\xi^2 \end{aligned} \tag{2}$$

Where $\hat{G}_0 = -\mathbf{1}/[4\pi r(\mathbf{1} + \mathbf{M}_r)]|_{\hat{\theta}}$, with $(\dots)|_{\hat{\theta}}$ indicating evaluation at time $t_* - \hat{\theta}$ (the symbol $\hat{\theta}$ denotes the time taken by the signal to propagate from a point in the BFR to the observer). In addition, $\mathbf{M}_r = \mathbf{M} \cdot \mathbf{e}_r$, $\mathbf{M} = \mathbf{v}/c$, with $\mathbf{e}_r = \mathbf{r}/r$, while \mathbf{v} and \mathbf{v}_D are rigid and deformation velocity of the body, respectively, and $\hat{J} = |\mathbf{a}_1 \times \mathbf{a}_2 \cdot \mathbf{a}_3|/|\mathbf{1} + \mathbf{v}_D \cdot \nabla_{\mathbf{y}} \hat{\theta}|$ is the Jacobian of the curvilinear co-ordinates transformation, with \mathbf{a}_β denoting the covariant basis vectors of the curvilinear co-ordinate system. Furthermore, the symbol $(\dots)|_{\xi^3=0}$ indicates evaluation over the body surface, whereas the symbol $(\dots)|_{g=0}$ represents evaluation at the instant of emission from the point of the deformed surface of the signal that at the time t_* reaches the observer.

Noise radiation in the air frame

An equivalent integral formulation for the acoustic pressure fields related to moving deformable bodies can be derived in the FFR.

This is readily derived from Eq. (2) by observing that $\mathbf{v} = \mathbf{0}$ (the velocity of the fluid reference frame is null), replacing \mathbf{v}_D with \mathbf{v}_T (in the air frame of reference the velocity of the body is given by the superposition of that related to the rigid body motion with that due to the deformation), replacing \hat{G}_0 with $G_0 = -\mathbf{1}/(4\pi r)$ (Green's function in the air reference), and replacing $\hat{\theta}$ with $\theta = r/c$ (time taken by a signal to propagate from a fixed point in the FFR to the observer). Thus, the following representation of the acoustic pressure radiated by deformable bodies in the fluid frame of reference is obtained [14]

$$\begin{aligned}
 \bar{p}'(\mathbf{x}_*, t_*) &= \int_0^\infty \iint_{\Omega} -G_0 \nabla \cdot \nabla \cdot (HT) J \Big|_{g=0} d\xi^1 d\xi^2 d\xi^3 \\
 &\quad - \iint_{\Omega} [(\mathbf{Pn}) \cdot \nabla G_0 J] \Big|_{\xi^3=0, g=0} d\xi^1 d\xi^2 \\
 &\quad + \iint_{\Omega} \left[\frac{1}{(1 + \mathbf{v}_T \cdot \nabla \theta)} \frac{\partial}{\partial t} \Big|_{\xi} [(\mathbf{Pn}) \cdot \nabla \theta G_0 J] \right] \Big|_{\xi^3=0, g=0} d\xi^1 d\xi^2 \\
 &\quad - \iint_{\Omega} \left[\frac{1}{(1 + \mathbf{v}_T \cdot \nabla \theta)} \frac{\partial}{\partial t} \Big|_{\xi} [\rho_0 \mathbf{v}_T \cdot \mathbf{n} G_0 J] \right] \Big|_{\xi^3=0, g=0} d\xi^1 d\xi^2
 \end{aligned} \tag{3}$$

where $J = |\mathbf{a}_1 \times \mathbf{a}_2 \cdot \mathbf{a}_3| / |\mathbf{1} + \mathbf{v}_T \cdot \nabla \theta|$ is the Jacobian of the curvilinear co-ordinates transformation in the FFR.

For rigid body applications, the integral formulation in the BFR is convenient because the integration domain coincides with the real surface of the body. This does not occur in the case of deformable bodies. However, as already stated, the BFR formulation is introduced in order to provide analytical/numerical comparison with the FFR integral formulation for cross-validation purposes.

Noise radiation from a porous surface

From the general integral solution of Eq. (1) we also derive the integral formulation for sound radiation from porous surfaces. Making the same assumptions as for the formulations for solid surfaces and defining \mathbf{u} as the fluid velocity, Eq. (1) reduces to the Ffowcs Williams and Hawkings equation for porous surfaces for $\bar{\Psi} = \bar{p}'$, $\chi = -\nabla \cdot \nabla \cdot (HT)$, $\mathbf{Z} = \mathbf{P} + \rho \mathbf{u} \otimes (\mathbf{u} - \mathbf{v}_T)$ and $\mathbf{w} = -\rho_0 \mathbf{v}_T - \rho(\mathbf{u} - \mathbf{v}_T)$, while $\mathbf{z} = \mathbf{0}$, $\mathbf{k}_1 = \mathbf{0}$ and $\mathbf{k}_2 = \mathbf{0}$. Therefore, the integral solution of the general differential model in Eq. (1) gives the following FFR representation of the acoustic pressure radiated by deformable porous surfaces:

$$\begin{aligned}
 \bar{p}'(\mathbf{x}_*, t_*) &= \int_0^\infty \iint_{\Omega} -G_0 \nabla \cdot \nabla \cdot (HT) J \Big|_{g=0} d\xi^1 d\xi^2 d\xi^3 \\
 &\quad - \iint_{\Omega} [(\mathbf{Pn} + \rho [\mathbf{u} \otimes (\mathbf{u} - \mathbf{v}_T)] \mathbf{n}) \cdot \nabla G_0 J] \Big|_{\xi^3=0, g=0} d\xi^1 d\xi^2 \\
 &\quad + \iint_{\Omega} \left[\frac{1}{(1 + \mathbf{v}_T \cdot \nabla \theta)} \frac{\partial}{\partial t} \Big|_{\xi} [(\mathbf{Pn} + \rho [\mathbf{u} \otimes (\mathbf{u} - \mathbf{v}_T)] \mathbf{n}) \cdot \nabla \theta G_0 J] \right] \Big|_{\xi^3=0, g=0} d\xi^1 d\xi^2 \\
 &\quad - \iint_{\Omega} \left[\frac{1}{(1 + \mathbf{v}_T \cdot \nabla \theta)} \frac{\partial}{\partial t} \Big|_{\xi} [(\rho_0 \mathbf{v}_T + \rho(\mathbf{u} - \mathbf{v}_T)) \cdot \mathbf{n} G_0 J] \right] \Big|_{\xi^3=0, g=0} d\xi^1 d\xi^2
 \end{aligned} \tag{4}$$

where G_0, J and θ are exactly the same as defined for the formulation for solid surfaces in the FFR. It is also useful to note that for solid surfaces, the $\mathbf{u} - \mathbf{v}_T$ term is null, and Eq. (4) reduces to Eq. (3).

Numerical Results

A preliminary numerical investigation on the body deformation effects on the aerodynamic and acoustic fields radiation is here addressed. To this purpose, an upswept, untapered wing in uniform translation, with NACA 0012 airfoil sections, semi-span equal to **1.5 m**, and chord equal to **1 m** is considered. In a right-handed BFR with origin at the mid-wing section leading edge, \mathbf{y}_1 -axis

coincident with the advancing direction and pointing forward, and y_2 -axis directed starboard, the observer is placed at $[0, -2 \text{ m}, -0.5 \text{ m}]$. A lifting case is examined by setting the wing angle of attack equal to 4° . The aerodynamic solution over the body surface is evaluated by an aerodynamic integral solver for the rigid body motion, with the deformation only taken into account in the definition of the boundary conditions. The body deformation is expressed as $\mathbf{x}_D(\mathbf{y}, t) = \Psi_b(\mathbf{y})\mathbf{q}_b(t) + \Psi_t(\mathbf{y})\mathbf{q}_t(t)$, with the two bending and torsion shape functions given by

$$\psi_b(\mathbf{y}) = \begin{Bmatrix} 0 \\ 0 \\ y_2^2 \end{Bmatrix}; \quad \psi_t(\mathbf{y}) = \begin{Bmatrix} y_3 \sin(\Omega_t y_2) \\ 0 \\ -y_1 \sin(\Omega_t y_2) \end{Bmatrix} \quad (5)$$

Where $\Omega_t = \pi/b$, with b denoting the semi-span, whereas the lagrangean coordinates are expressed as $q_b(t) = A \cos(\omega_b t)$ and $q_t(t) = B \cos(\omega_t t)$ with ω_b and ω_t representing the pulsations of the bending and torsion deformations, respectively. First, for several Mach numbers, Fig. 1 shows the comparisons between the time signatures at the observer position evaluated through the integral formulations written in the FFR and in the BFR, respectively for the acoustic pressure. In this case the wing deformation is defined with $A = 0.01$ and $B = 0.1$, $\omega_b = 4\pi/T$ and $\omega_t = 6\pi/T$, where T is the observation time interval length. This time interval is assumed to be $T = 0.12 \text{ s}$, i.e. such that it includes at least two periods of deformation oscillations. These parameters correspond to a maximum bending deflection equal to 1.5% of the wing semi-span, and maximum twist angle equal to $\alpha = 3^\circ$. The results shown in Fig. 1 demonstrate that, regardless the Mach number as expected, the integral formulations expressed in the FFR and BFR perfectly match, thus confirming their full equivalence.

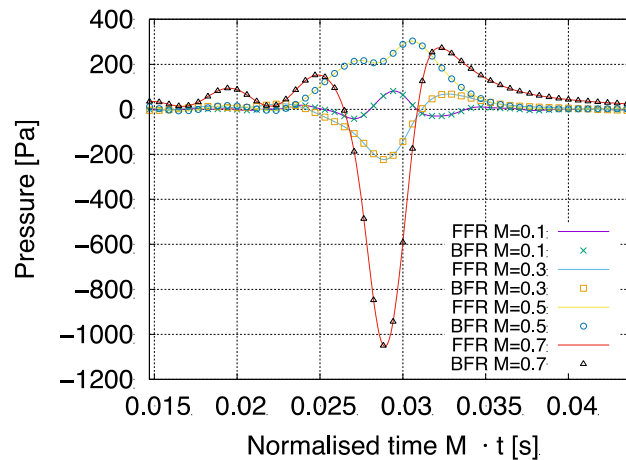


Figure 1. Comparison between time signatures of acoustic pressure evaluated by the integral formulations written in the FFR and in the BFR, at different Mach numbers.

In addition, the effect of the deformation parameters on the radiated pressure is investigated. Specifically, three different deformations are analysed: i) “deformation 1”: $A = 0.01$, $B = 0.1$, $\omega_b = 4\pi/T$ and $\omega_t = 6\pi/T$ ($w = 0.015$ and $\alpha = 3^\circ$); ii) “deformation 2”: $A = 0.02$, $B = 0.2$, and the same frequencies as in “deformation 1” ($w = 0.03$ and $\alpha = 6^\circ$); iii) “deformation 3” same amplitudes as in “deformation 1”, $\omega_b = 2\pi/T$ and $\omega_t = 3\pi/T$. For $M = 0.5$ and for the three wing deformations considered, Fig. 2 depicts the comparison between the pressure signatures evaluated through the integral formulations written in FFR and BFR. These results confirm that the predictions provided by the integral formulations expressed in the fixed and moving frames of reference perfectly coincide, regardless amplitude and frequency of deformations.

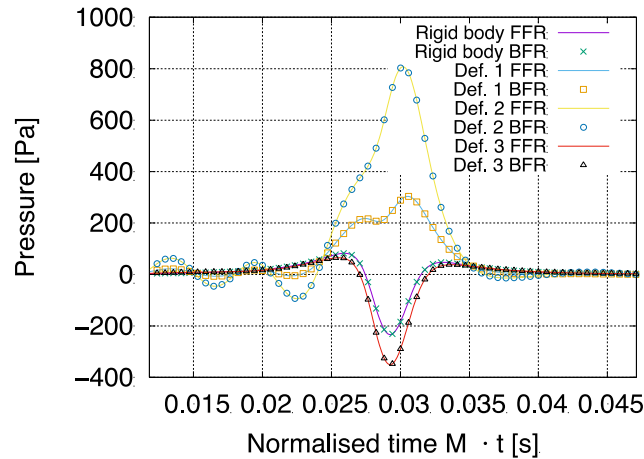


Figure 2. Acoustic pressure signature evaluated for different body deformations.

To validate the formulation for porous surfaces, a porous sphere with a radius of $R = 1 \text{ m}$ was used. A point source emitting a potential $\varphi(t) = \sin(\omega t)$, with $\omega = 5 \text{ rad/s}$, was placed at the center of the sphere (which was located at the origin of the y_1, y_2, y_3 axes). The sphere and source moved at a constant speed of 100 m/s in the opposite direction along the y_1 axis. The deformation of the sphere was described by the bending shape function $\Psi_b(\mathbf{y})$ and the Lagrangean coordinate $\mathbf{q}_b(t)$ illustrated earlier, with $A = 0.5$ and $\omega_b = 4 \text{ rad/s}$. The observer was positioned at $[-150 \text{ m}, 10 \text{ m}, 0 \text{ m}]$. Fig. 3 shows the trends in acoustic pressure reaching the observer, calculated in two ways: directly from the source and from the porous sphere. To calculate the pressure reaching the sphere emitted by the source, Bernoulli's theorem was used. From Fig. 3, it is evident that the two trends are similar enough to confirm the correctness of the formulation for porous surfaces.

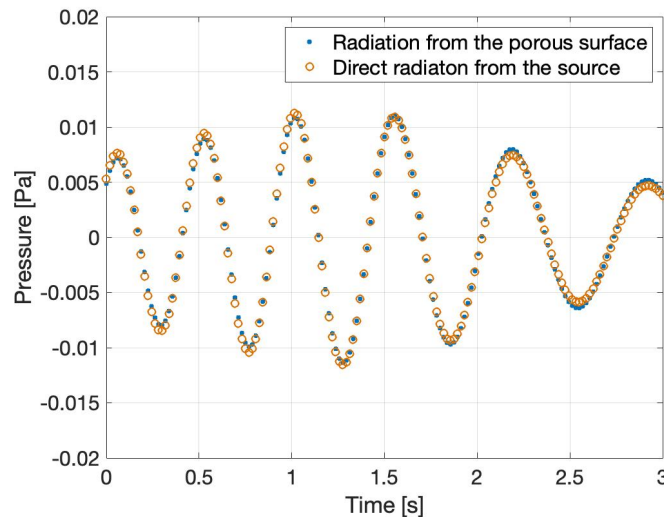


Figure 3. Comparison of acoustic pressure radiation from a point source and from a porous surface, where the porous surface always radiates the signal emitted by the point source.

The objective in the near future is to perform similar analyses to those conducted for the translating wing but for a rotor in forward flight. The numerical investigation will have a twofold purpose: to evaluate the significance of body deformation on the assessment of noise radiated by the rotor, and to discuss the advantages and disadvantages of the BFR and FFR approaches in terms of numerical performance.

Conclusions

In conclusion, two novel boundary integral formulations for the aerodynamics and aerocoustics of deformable bodies, with both porous and solid boundaries, have been presented. Through numerical investigations of a translating wing subject to bending and torsion deflections, and by validating the formulations using two reference frames, one fixed with undisturbed air and the other fixed with the undeformed body, we have confirmed the full equivalence of the formulations, regardless of the body velocity and magnitude and frequency of the deformation. Additionally, we have shown that the effect of wing deformation on acoustic radiated field is not negligible, and that the perturbation amplitude increases with the deformation amplitude. The developed formulation for porous deformable surfaces has also been validated through a comparison of direct radiation from a point source and radiation from the porous surface. The novel formulations presented in this article offer valuable insights and tools for accurately modelling the aerodynamics and acoustics of complex deformable bodies, and have the potential to enhance the design and performance of a wide range of engineering applications.

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