Investigation of the complexities inherent in manufacturing near-unconstrained superplastic parts by experiments and simulation

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Abstract. Superplastic forming is a sheet metal forming process that has found widespread use in the aerospace industry. It produces parts that are free of residual stresses, dimensionally accurate, and with strains unobtainable using conventional forming methods. When combined with diffusion bonding, a phenomena where under similar processing conditions the material involved will produce a near flawless weld with itself, a variety of reinforcements and internal structures can be produced. In most superplastic parts the material is blow formed up to a die and the material takes on the dimensions of the die with small variations in thicknesses. In this work, however, we investigate a process unique to superplastic forming with diffusion bonding using four sheets. The two outer sheets are formed up to the die while the two inner sheets form a complex sandwich structure. These inner sheets are completely unsupported except at the edges of the part. Superplasticity is stress-history dependent and somewhat chaotic in nature. Therefore, the inner sheets have a large variance in geometry due to the fact that they have only limited constraints and are free to shift and translate as the forming operation progresses. Without rigid constraints on the inner sheets small variations in geometry can be magnified to create large changes in final geometry. The variances in forming are quantified using a variety of techniques to measure the major features of the process including cell wall measurements, gas pathway measurements, and computer vision-based geometry analysis. Two- and three-dimensional finite element simulations of the inner sheet forming process were used to compare the characterization results with idealized geometry. The results of the analysis provide insights into the complexities of manufacturing such internal structures.

Introduction

The subject of this research is a large area, free floating, superplastic structure that needs to form a complex and predictable pattern of interior vertical supports for two exterior sheets of titanium. The variability of this structure is much larger and less predictable than typical superplastic parts due to the stress-history dependence of superplasticity magnifying small variations in geometry to create measurable discrepancies in the finished part. Superplasticity is the ability of a metal, ceramic, or intermetallic to achieve very large strains at elevated temperatures with a controlled strain rate [1,2]. It has become a common production method in the aerospace and automotive industries due to its ability to form complex shapes with no residual stresses. The dominant mechanism for superplasticity is sliding along grain boundaries with some secondary diffusive and dislocation-based accommodation mechanisms [3].

Materials must be fine grained in order to have adequate superplastic properties and titanium is typically fine grained enough without additional processing. The grain structure of titanium, in combination with its expense and manufacturing challenges via other methods, make it an ideal material for superplastic forming. Titanium also has excellent corrosion resistance, a high strength to weight ratio, and large degree of customizability through a variety of alloying elements. There are very few weld nuggets that have superplastic properties, because the weld nugget of resistance welds or other heat-based welds have grains far too elongated to be superplastic. To date the only type of welding that can produce superplastic welds is friction stir welding. Diffusion bonding, often coupled with superplasticity, provides near indistinguishable welds when two surfaces contact each other at the correct processing conditions [4]. Superplastic forming and diffusion bonding allows for the formation of monolithic parts that have the advantages of being a single structure that reduces fasteners and part complexity [5].

Superplastic forming is most commonly used for the production of single sheet structures. Single sheet forming entails clamping the sheet in a die at the edges and blowing the sheet up to the die from one side. Multiple sheet forming also exists and has been used in production parts. Traditionally, the sheets are diffusion bonded before superplastic forming by using a pattern. The pattern is deposited using yttria powder silk screening which prevents diffusion bonding where applied [6]. Gas pathways are included into the pattern with a welded fitting at the side of the sheet. The bonded sheets are then enclosed in a die and gas formed up to it at temperature; however, there are some non-traditional multi-sheet forming patterns that are not diffusion bonded prior to forming. Numerous sheet patterns exists but the subject of this research is the Rockwell four sheet process [7]. This sheet forming pattern has a completely unsupported internal sheet structure that is free to shift and slide.

Superplastic forming is highly sensitive to material properties and processing conditions, specifically the temperature and strain rate applied; however, once the correct conditions have been dialed in, it is a very repeatable process [8]. There can be some variation in thicknesses of formed sheets, but these are mainly dependent on unavoidable variations in material properties. While the geometric constraints on the face sheets are at both the edges and the surface of the die, the core sheet constraints are only at the edges where all four sheets are clamped. These inner sheets are therefore able to move freely with forces being balanced between the cells. With the stress history dependence of superplasticity, slight variations in initial geometry can lead to large changes in formed geometry. The variability of this free-floating core is the subject of this research, both to quantify it and to provide insight into how to cope with the variability in a production setting.

The mechanical behavior of superplasticity is modeled in the form of constitutive equations governing the plastic behavior of the material. Several different forms of constitutive equations have been developed which are typically from empirical, or internal state variable forms. Empirically derived equations map the strain rate to the stress with constants derived from experiments, and have been developed for both single and multi-phase materials. Examples of empirically derived equations use the extended power law creep equation (Eq. 1) [2]:

$$\sigma = K\varepsilon^{n}\dot{\varepsilon}^{m}. \tag{1}$$

The constant K, along with the rate constants n and m, are mapped to physical experiments. Grain growth has been included in several models that add increased accuracy to the model [9]. Applications of crystal plasticity implemented in a self-consistent viscoplastic form have also been developed [10]. These equations chose the internal state variables as dislocation density and grain size. With these constitutive equations, finite element simulations of superplasticity have been implemented extensively to model the complex geometry used in the forming operation.

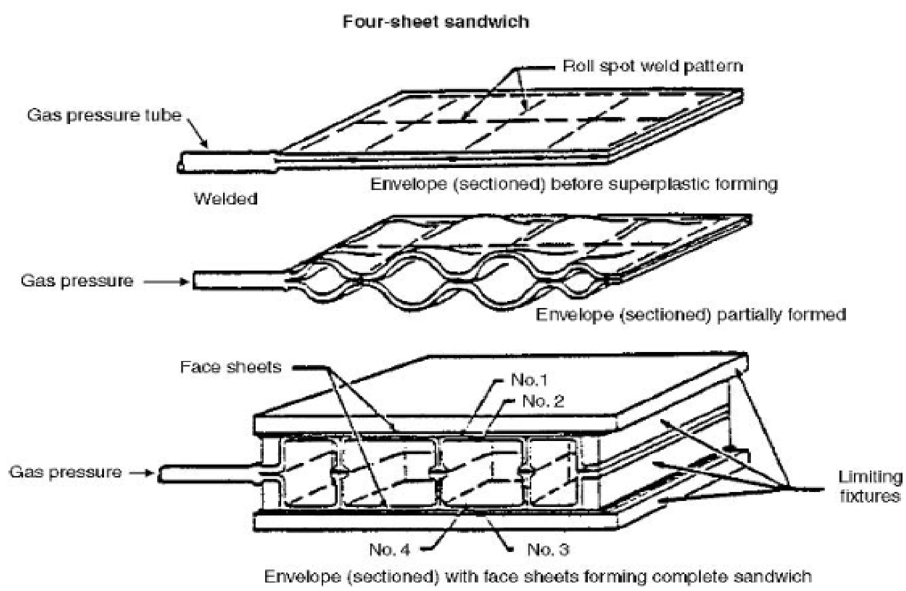
There have been many maps to the experimental using both the hyperbolic sine viscoplastic models [11] and power law creep models, and extensive studies have been conducted on both of these models for titanium. Hyperbolic sine constitutive equations provide a wider range of strain rates for highly variable parts while the power law model does a sufficient job for a smaller range of strain rates [12]. These have had very good correlation between formed parts and the production geometry provided that the material model can accurately simulate the range of processing conditions [13], and the implementation of these models is similar. The stresses are assumed to be mapped to the elastic stiffness tensor to produce the deformation tensor based on the applied pressure and temperature.

Experimental Methods

The Rockwell four sheet SPF/DB process was implemented in this research with the production of rectangular flat panel parts illustrated in Figure 1. Each part has two sheets on the outside that form up to die surfaces and two internal sheets that form a rectangular reinforcement pattern and diffusion bond to themselves and the outer sheets. In this way, an elegant and stiff sandwich structure is fabricated by simply welding a pattern to the interior sheets. The outer sheets, called face sheets, deform very little and take on the shape of the die with minimal thickness variation. The internal sheets that end up forming a rectangular grid of vertical supports are call core sheets, and are created by a rectangular grid of resistance welded lines. These lines form 1” x 2” cells and are made of evenly spaced, but separated, small welds that are not superplastic.

The separations between the core sheet welds allow gas pressure to communicate between the cells from a gas input line at one end of the part and they form an inherent defect call the gas connection path. When the core sheets are pressurized by gas, they balloon out in rectangular sections and eventually deform into each other to form vertical supports (columns) at the weld lines. At the top of each column is a triangular defect called the upper gas path. This pathway is necessary to release the gas that was used to form the face sheets to the die.

a)



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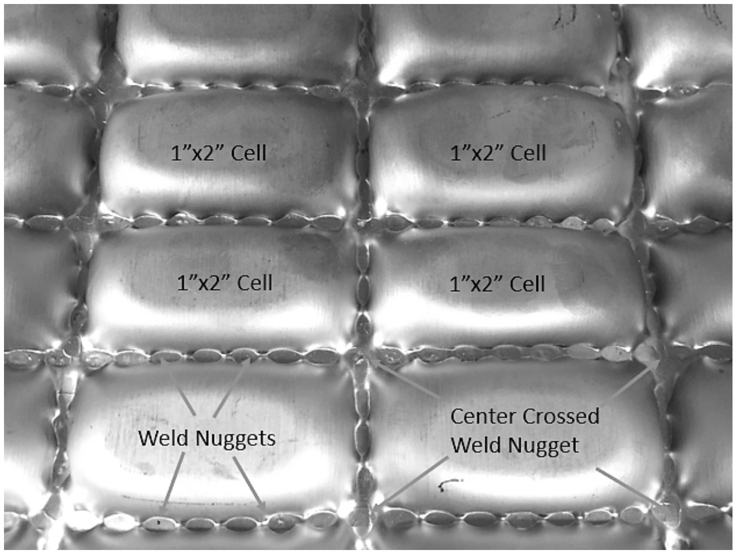

Figure 1: a) Core sheet forming process [7] begins with a resistance weld pattern then gas pressure is applied. The core sheets fold over and eventually contact each other to form a pattern of vertical supports termed columns. The core sheet forming, b) part of the way through the process is shown. The weld lines can be seen as the rectangular pattern of ellipsoidal shapes. The cells will eventually widen to contact each other at the weld nugget lines to form the columns.

To determine how controllable the core sheet formation is, and where and how the variations in geometry of a produced part can occur, both a finite element simulation was performed as well as extensive measurements on the panel geometry. The goal of the simulation was to provide an idealized case where the panel geometry would have minimal variations, and the results compared with the geometry from actual parts. While many experimental panels were produced, only four panels were used in this analysis. Each of these panels, labeled F6-F9, were fabricated with the same Ti-6Al-4V fine grain material from the same manufacturer, and at the same proprietary temperature and pressure schedule. By using the same material and process conditions one can demonstrate the variability inherent in the process. Data was collected on the major examples of variability in the core sheets: column location, column geometry, and geometry of the two different gas pathways. Thickness measurements were taken from mounted micrographs in detail and from cross sections of the cell geometry. Measurements of the two primary defects, the core gas connection holes and the upper gas pathways, were also taken. Variations in overall geometry were additionally measured.

The core sheet formation simulation used the finite element method of the core sheet forming process. An implicit formulation of ABAQUS was employed, in combination with the power law creep model, which only simulates the deformation of the superplastic core material. The deformation of the face sheets is small being at the edges of the panel and is typically not a source of significant thickness variation. Because the deformation of the weld nuggets is extremely small and entirely elastic, only the upper half of a small unit cell of the core sheet superplastic material was simulated. The weld nuggets are modeled as half-ellipsoidal cutouts from the core sheet material. The edge weld nuggets and the crossed weld nugget shown in Figure 2 are held as encastre (i.e. fixed). The interior weld nuggets are not allowed to deform in any way, through the use of a kinematic coupling of the surface nodes to a reference point, but are allowed to translate along their weld line toward or away from the center crossed weld nugget. The face sheet is modeled as a flat, analytically, rigid contact surface with minimal friction. Periodic boundary conditions are applied to the edge nodes in the form of kinematic equations that tie the displacement of each edge node with the node on the opposite side of the mesh.

The material model for this simulation uses a strain hardening model based on the power law plasticity model. Material data was used from tensile specimens elongated using the ASTM E2448-06 standard at temperature and a representative strain rate. This material model is used instead of the time hardening model typically used because it has the advantage of being more consistent during large strains. The equation for the strain hardening material model is displayed below (Eq. 2):

$$\dot{\varepsilon}^{cr} = \left(A\, q^{-n}\left[(m+1)\bar{\varepsilon}^{cr}\right]^{m}\right)^{\frac{1}{m+1}}. \tag{2}$$

It relates the equivalent creep strain rate, and the equivalent creep strain, to the deviatoric equivalent stress with some material constants. The goal of the simulation was to use idealized geometry and processing conditions, and therefore several assumptions were made. It was assumed that there is symmetry on the z-axis at the weld nuggets (i.e. the top half of each cell is identical to the bottom half), there is perfect gas distribution and pressure is applied according to a schedule equally everywhere, the material is perfectly isotropic in nature, and the welds are perfectly placed.

a)

b)

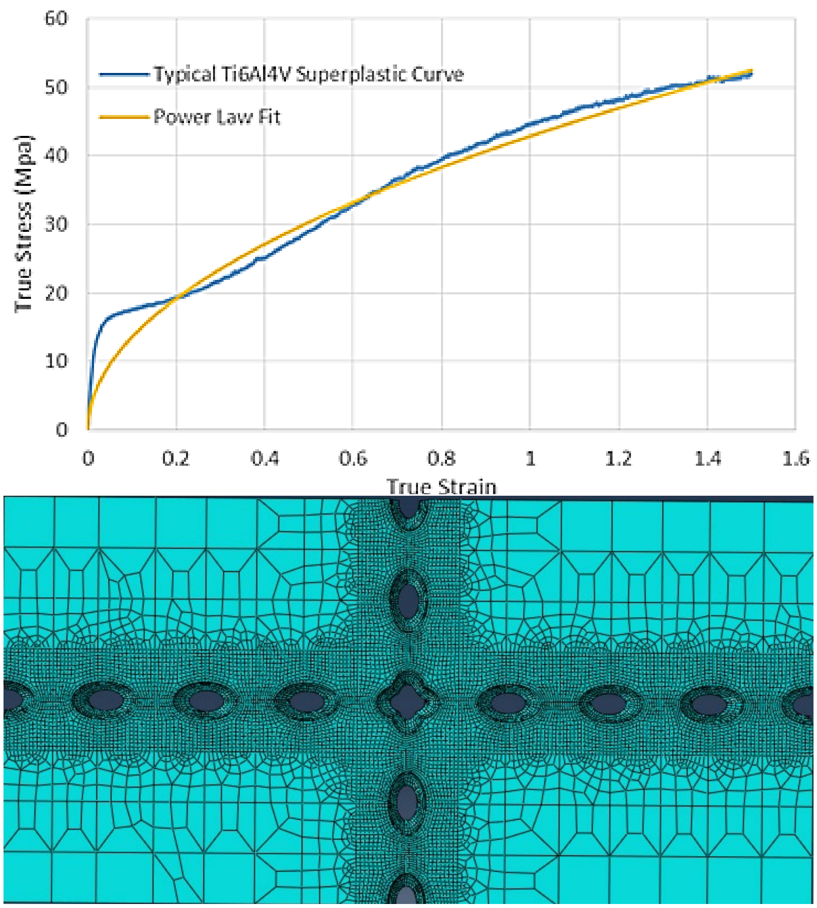

Figure 2: a) Superplastic curve with power law fit, and b) finite element mesh.

To determine the theoretical extent of the effects of different errors on the major dimension of the final product, a series of two-dimensional simulations were conducted with the same material model shown in Figure 2a. However, the mesh and formulation of the model is different from the model previously described where there were no periodic boundary conditions. The boundary conditions are such that the edges form up to vertical rigid walls, and the model is built as a set of three cells each with varying dimensions. The cells were delineated by rigid weld nuggets that were able to translate and rotate but not deform. The cells formed up to a horizontal boundary of a customizable shape which allows for various tests to be conducted. Tests conducted with this setup were to determine the effects of variations in cell dimensions, curvature of the top and bottom of the part, and effects of pressure distributions on the formed geometry.

To determine the geometric trends of the experimental panels several methods were applied and cross sections were taken of the entire sample. The samples were imaged at regular intervals and the thicknesses of each section were measured for each panel examined. Thicknesses and bulk measurements were taken using an X-Y stage microscope (MicroVu). The panels were measured for their cell-to-cell dimensions by recording the dimensions between each weld nugget which is a measure of the cell wandering that can occur in this process. Panels were also measured for the triangular gas pathway at the top of the sheet, as displayed in Figure 3a at the ends of the columns.

Formed samples were machined from each panel using an abrasive waterjet and were sectioned along the column in both the longitudinal direction and the transverse direction. Sections were taken at both the weld nugget and the gas pathway for comparison. Each column section was mounted and polished then the sample sections were imaged using the stage microscope (MicroVu) at regular intervals. Then the images were composited together to provide a macroscopic image of the sample that was thresholded, and processed by the Canny edge detection method to provide lines at the edges of the sample. The vertical location of the digitized edges were subtracted to determine the thickness distribution of the sample. The operations for the threshold and edge detection were developed by OpenCV and the code was implemented in python. Figure 3a shows an original imaged column, the edges produced with Canny edge detection are shown in Figure 3b, and a plot of the edges and resulting thickness distribution is displayed in Figure 3c.

a)

b)

c)

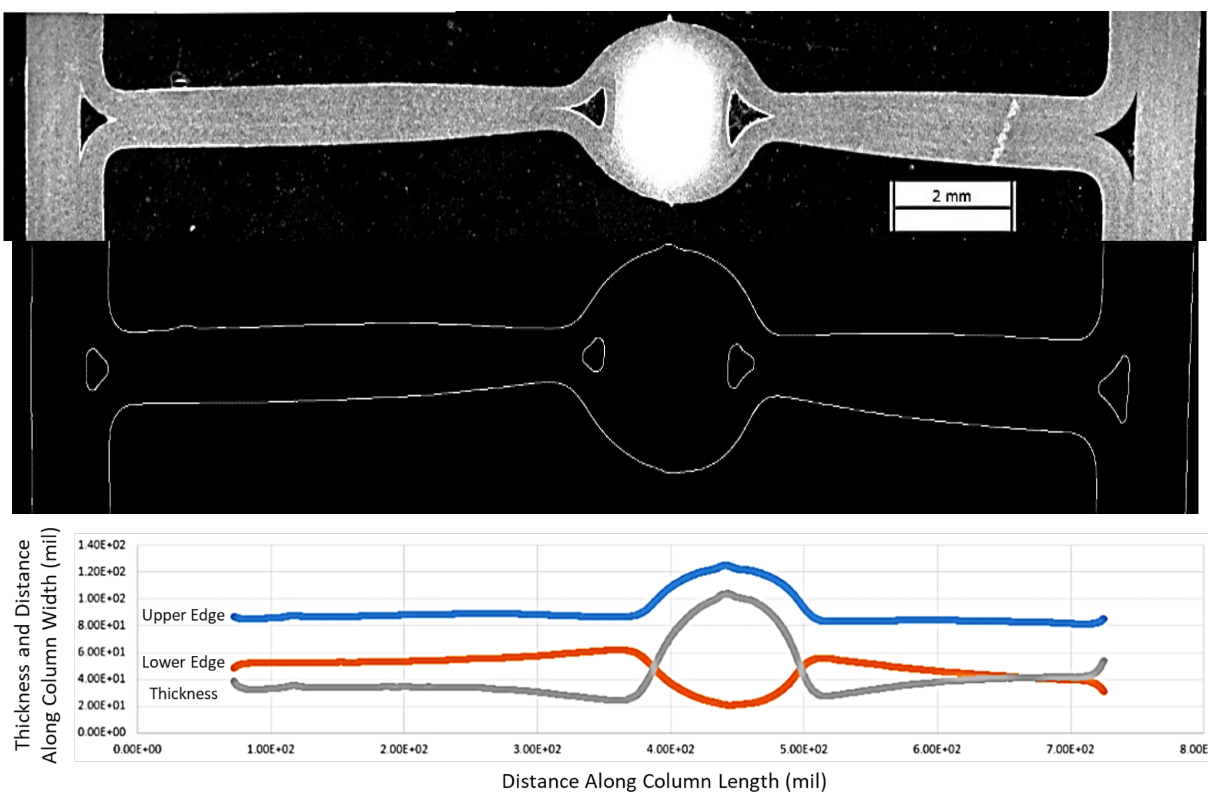

Figure 3: Edge detection of a mounted sample; a) the mounted sample macrograph, b) processed image with canny edge detection, and c) thickness distribution plot after pixel locations are subtracted.

The gas connection pathways in between the different cells have some variation in both size and shape. Samples were measured for the area and aspect ratio of the pathways. The measurements were taken using calibrated images. A chroma key was introduced behind the gas pathways then images were taken. The images were calibrated with a measured length of a single weld nugget. The pathways were digitally isolated from the titanium, separated into a group of pixels representing an individual pathway, then measured using the number of pixels and the weld nugget calibration. Figure 4 has images of the dataset for panels F6 and F9 along with an explanation of the algorithm.

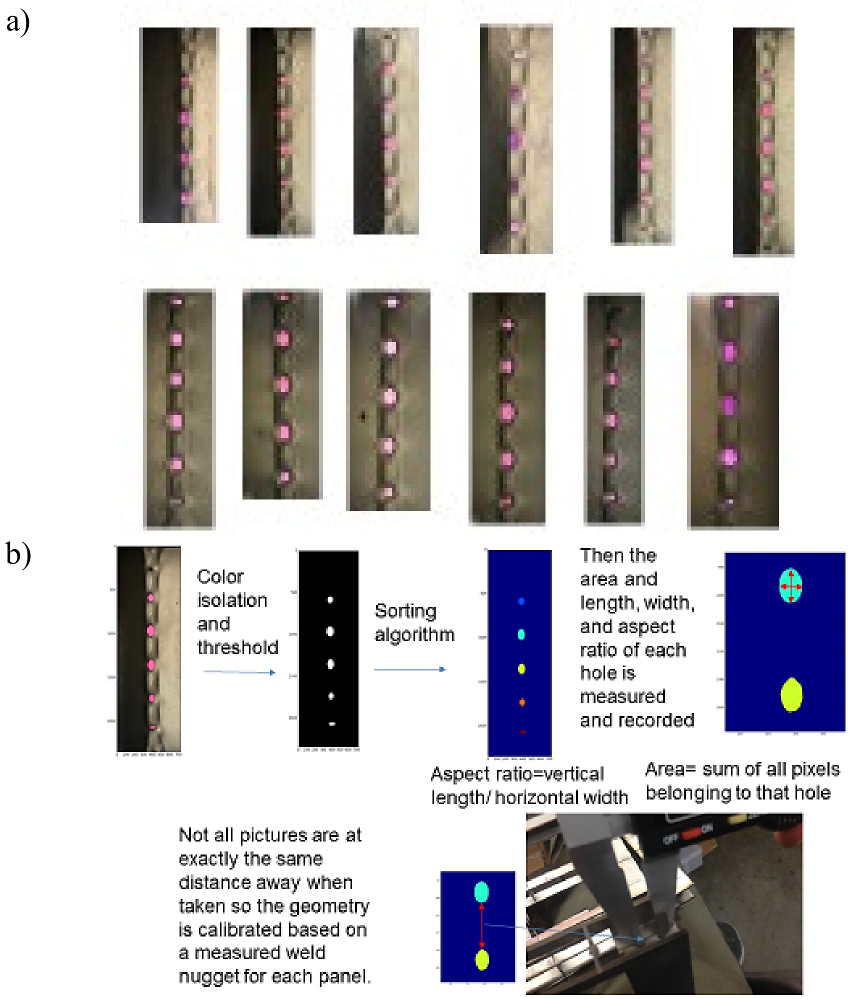

Figure 4: Gas connection pathway measurement process with a) the chroma key dataset for panels F6 and F9, and b) an explanation of the algorithm.

Results

The finite element simulation results were highly symmetric and uniform, which showed very little of the variations that are typical of fabricated panels. In Figure 5, the fully formed simulation is shown, but subsequent figures in this section displaying finite element simulation results were mirrored about the z-axis and replicated along the x and y axes. There are some trends that were exhibited by the experimental results that are detailed in this section, and they are compared with the thickness distributions and the gas pathway statistics.

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a) b)

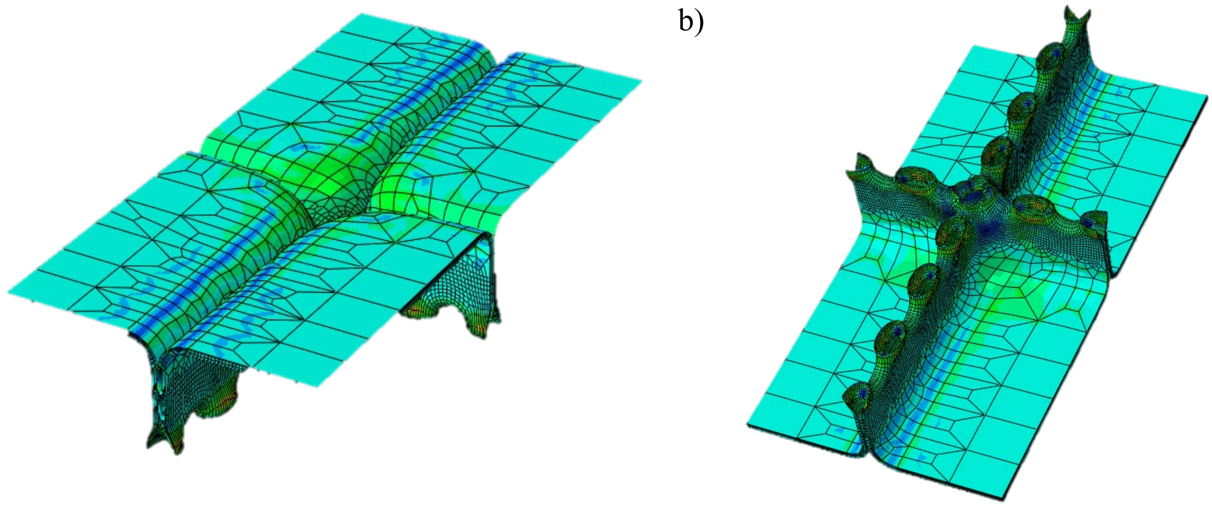

Figure 5: Finite element simulation showing a) the top contacting the face sheets, and b) the internal weld nuggets.

Thinning of the columns is a major problem for this SPF/DB structure because a weakened column can provide a significant failure point. The thickness distributions developed from the edge detection of the mounted samples provide insight into the variance that can develop in finished parts. There is a significant difference between the thickness that occurs at the weld nuggets, and the thicknesses that occurs at the gas connections; therefore, the thickness distributions were consolidated into two groups and analyzed separately. The thickness distributions are best summarized by the plots in Figure 6 which show all the measured thicknesses and the extent of the variability can be observed.

Thickness distributions were also computed for the finite element simulation for comparison for which there was almost no variation in the thickness. The measurements were taken at the midpoint at the center of the weld column and the center of the path column. A standard deviation of 1.2% was observed for the twenty thickness measurements taken at the center column in both the weld nugget area and gas pathway. This shows that with perfect geometry and conditions, there should not be any variation in the column thicknesses. However, as seen in Figure 6, there is a significant amount.

The analysis revealed that there is not always an even stress distribution or perfect weld nugget placement. Length mismatch was also computed between the upper and lower portions of the columns. Ideally the length of both the top and bottom of a column should be equal placing the weld nugget or gas path directly in the center between the two face sheets. This gives the minimum loading to weld nugget area which is beneficial due to the relatively large number of stress concentrations located there. Unequal lengths in the column also leads to unequal thicknesses and can lead to improper processing conditions at thinned sections in the column. The column length mismatch averaged 7.8% of the gas path columns and 9.8% of the weld nugget columns.

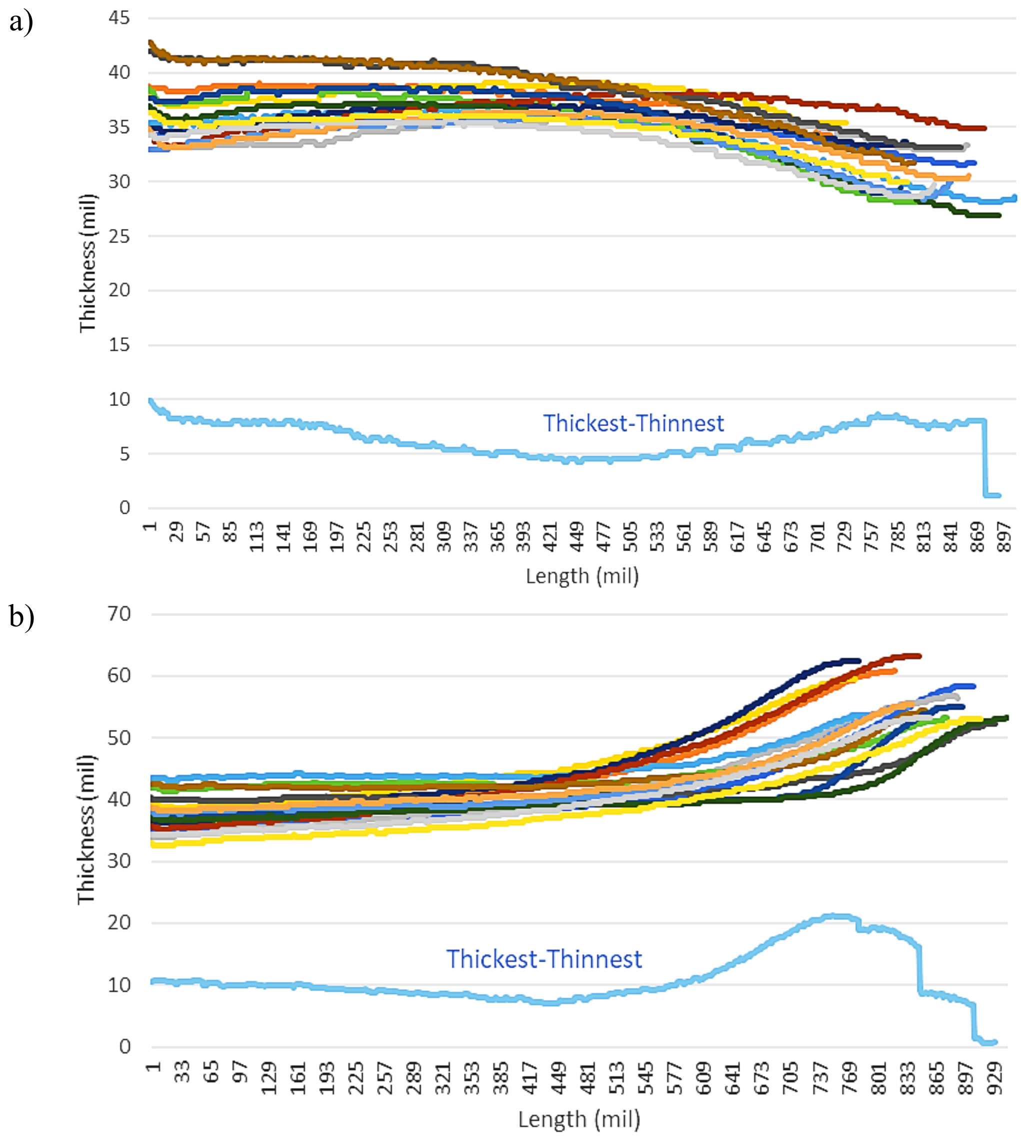

Figure 6: a) Weld nugget column thickness, and b) gas pathway column thickness. There is a large difference between the thickest column versus the thinnest column. These columns support the face sheets and provide the dominant failure points for the finished part. Getting a column that is undersized can be detrimental to the performance of the part.

The gas pathways represent a major defect that is inherent to the core sheet formation. Gas must be able to transfer between the cells in some way but the regular defects can be problematic. They also can be highly inconsistent in size and shape depending on the local loading during forming, which makes them a hard feature to predict. Structurally, the gas pathway and weld nugget locations are not as important as the upper gas path and thickness variations in the columns near the face sheets since the stresses are minimized at the center of the sandwich during bending.

Weld nugget placement is computer controlled and highly accurate in both distance between welds and weld quality. The finite element simulation predicted some key features in the gas pathway geometry, such as the trend of large pathways in the center of the cell, and horizontally shrunken pathways closer to the ends of the cell, as depicted in Figure 7. That is a result of the material transitioning toward the ends to form the perpendicular columns, which in turn causes the weld nuggets to translate, and as a result distorts and sometimes close off the gas pathways at the ends of the cell. From image analysis, the gas pathway dimensions have an average of the 2.055

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mm^2 with a deviation of 1 mm^2. The finite element simulation had gas pathways that were significantly larger with an average area of 5.24 mm^2 and a similar relative deviation of 2.33 mm^2.

a)
b)

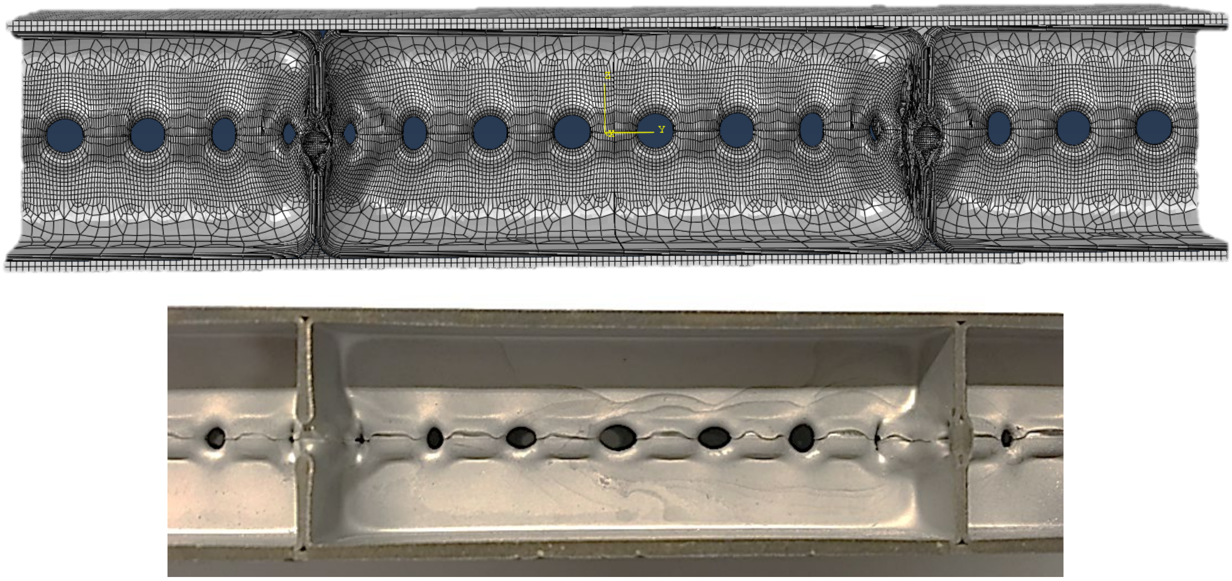


Figure 7: a) FEM results of the gas pathway geometry, and b) actual geometry of F6. The center of the cell has the largest gas pathways which become more distorted as they reach the edges of the cell; perfect gas path geometry is uncommon.

Cell wandering, the process of the column locations not lining up with the initial weld nugget placement, is also an issue in these SPF/DB structures. The variations in the cell size are a natural effect of the stress history dependence of superplasticity. Ideally the cell widths should have a value of 50.8 mm, and in the finite element simulation they do. Despite the precision weld placement, there was still variability in the cell widths. The cells tended to wander with a variation of 1.7% of their average length, as displayed in Figure 8.

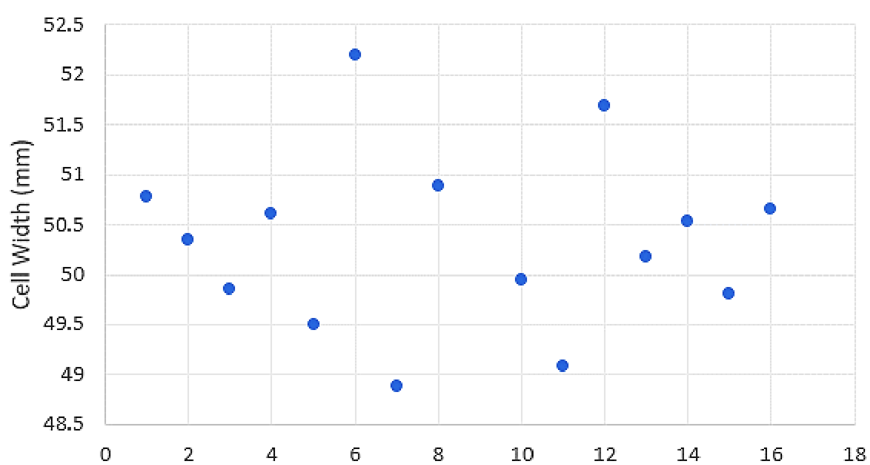


Figure 8: Cell width scatter (50.8 mm ideal).

The effects of upper gas pathway dimensions are arguably the most important characteristic of all the geometric variations in the core sheet. This is because the triangular pathway is directly at diffusion bonds in both the column and the core-face sheet interface. It is also a major stress path in compression, and especially bending where the stresses are maximized at the upper and lower surface of the part, and where failure always occurs at the largest upper gas pathway. The size of the upper gas path in the finite element simulation only varied at the crossed weld nugget area. However, in the experimental panel the trend was to have the smallest pathways closest to the gas inlet, and they get larger as a function of distance from the gas inlet. This is because the material must deform plastically to transmit gasses and the gas inlet is on one side giving the area closest to the gas inlet more time to form than the material far away from the inlet. This has the possible effect of having the failure point be a function of the gas inlet location depending on how the part is loaded. Figure 9 shows the measurements of gas pathway widths along a trimmed panel section.

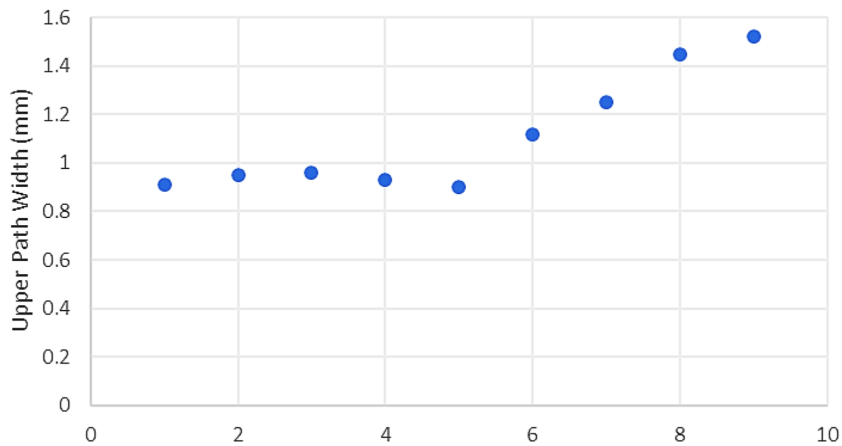


Figure 9: Upper gas pathway width along a trimmed panel section.

Discussion

The purpose of this research was to highlight the unique challenges in producing a consistent part when using an edge supported internal superplastic structure, quantify the extent of the variability in such a structure, and provide insight into possible ways to mitigate problems that could arise. Superplastic forming is a stress history dependent process as is all plasticity. A load causes a change in dimensions of a material which can in turn create larger loads. The difficulties arise in the four-sheet process due to the fact that the internal structures are load balanced and not geometrically constrained. Imperfect geometry causes changes in loading and changes in forming over time.

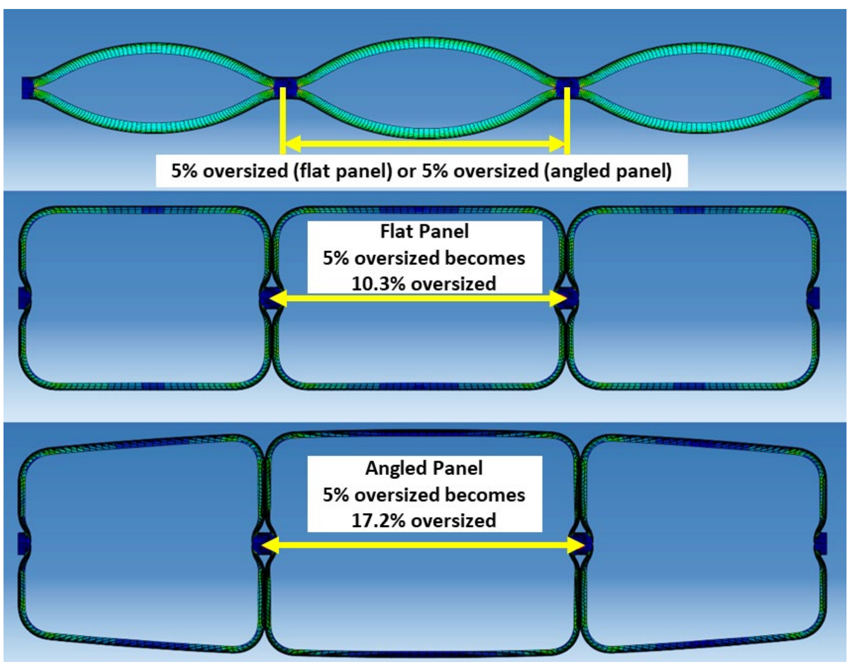
To illustrate this mechanism, an analysis of cell wander was performed using a two-dimensional simulation and can be viewed in Figure 10. The simulation has four weld nugget areas encompassing three cells along the smaller cross section of the rectangular cell with a dimension of 25.4 mm between cells. The center cell has displaced weld nuggets relative to the outer cells. Both a flat panel and a panel with an expanded (angled) center section were analyzed for normal precise weld nugget location and an error in weld nugget location. The flat panel, with perfect geometry, has perfectly balanced loads on the weld nugget and no increase in error. When the weld nuggets are moved 5% closer together in the center than they should be, the error is increased. This is because the outer cells end up forming quicker since they have a larger surface area exposed to the forming pressure, and therefore a larger stress.

Initially, the tension in the larger cells creates a slight tension in the small center cell that expands it, decreasing the error. However, the outer cells expand outwards first and contact the face sheet which creates a horizontal compression of the center cell since it is smaller and exhibits a smaller horizontal force on the weld nuggets. The weld nuggets relocate to compensate for this differential. By the time the columns are half formed everything has balanced out leaving the center cell even smaller than the weld nugget placement. The 1.27 mm (5%) error in weld nugget placement becomes 2.62 mm (10.3%) after forming. For the angled panel, the center cell is taller than the outer cells and therefore always horizontally expands compressing the outer cells. The offset weld nuggets are 1.27 mm (5%) further apart than desired and this ends up expanding the cell by 4.37 mm (17.2%).

This cell wandering, as well as other geometric defects, are exacerbated in any case where there is uneven loading in the cells. Small defects in material properties, temperature gradients, providing gas pressure from one side of the part, and even gravity can all have effects on the final formed geometry. As shown in Figure 10b and Figure 10c, while flat panels simply increase error, panels with features such as the angle panel are going to have different behavior. Flat panels are not the only shape that the four-sheet process is meant for and it can be molded into a variety of complex contoured shapes for aerospace structures. A complex die surface can also increase the variation in cell geometry by changing the loading conditions in each cell.

Migration in the cells of a production part full of complex contours are going to vary significantly more than a flat panel, and there are techniques that can mitigate some of the variation issues. Weight savings gained from removing fasteners and making a monolithic structure can be utilized to increase the thickness and robustness of the SPF/DB component. This would not decrease the variability of the geometry, but does ensure that there is more than enough material in any location to compensate for complications that may arise due to geometry. Precision placement of the core sheet welds was an additional technique to prevent changes in geometry. Weld placement can be used to compensate for complex curvatures that could cause cell wandering and column thinning.

a)



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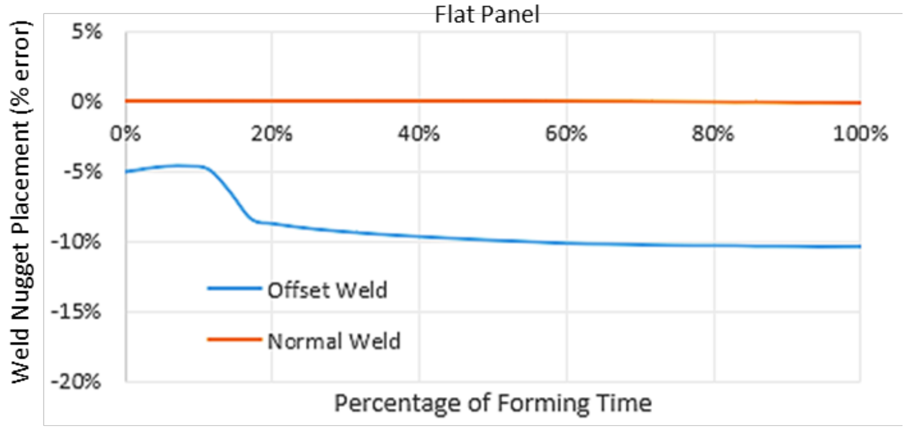

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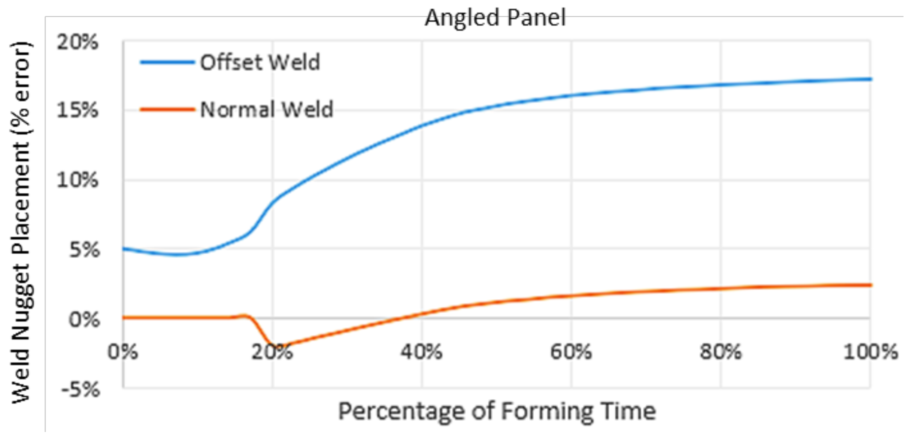

Figure 10: a) Three short (25.4 mm) cell formation in 2D with an undersized/oversized center cell. The flat panel causes an increase in weld error while the angled panel tends to expand the center increasing the error if the welds are oversized but decreasing if undersized. Tracking distance between the center weld nuggets produces the error plots for the b) flat panel, and c) angled panel. These show the predominant source of error occurs near the start of the forming cycle where the partially formed cells are significantly different in size.

Summary

Superplasticity when combined with diffusion bonding is an elegant way to produce high strength metal sandwich parts. The panels can be significantly more cost effective than alternatives; however, there can be challenges in production. This research examined a SPF/DB four sheet design that has a nearly unsupported internal reinforcement structure. The lack of constraints and the extent of the forming strain allow for a large variability in the formed geometry. The internal sheet forming was simulated with the finite element method and compared with the experimental measurements to show that there is significantly more variability in the produced part. This was explained as the effect of large strains combined with superplasticity, a stress history dependent process, which serves to amplify small variations in initial geometry. The effects of this variability can be mitigated by more precise initial geometry, better control over material properties, and recognizing that certain areas of a part are going to have significantly more variation than others. These techniques can be applied to any large plasticity cases where there is loosely constrained forming.

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