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Static analysis of cross-ply laminated spherical shells using a new hyperbolic shear deformation theory

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Abstract. Using a new hyperbolic shear deformation theory, higher-order closed-form solutions to the static bending analysis of laminated composite spherical shells are derived in this study. The current theory accurately predicts the distributions of transverse shear stresses across the thickness of the shell. The governing equations and related boundary conditions are obtained using the principle of virtual work. The Navier type semi-analytical closed-form solutions are obtained for the simply supported boundary conditions. The results obtained using the present theory are compared with previously published results to verify the accuracy and efficiency of the present hyperbolic shear deformation theory.

Introduction

Due to their improved strength-to-weight and stiffness-to-weight ratios, laminated composite materials have seen a significant rise in a variety of engineering applications over the past several decades. They have found their way into a wide range of products, including innovative spacecraft and aircraft components, boat and scull hulls, swimming pool panels, racing car bodies, sports goods, sensor or actuator, catalysts etc. Transverse shear deformation is substantially more significant in the kinematics of thick laminated composite shells than it is in homogeneous metallic ones because advanced composite materials have low transverse shear modulus. Since 3-D elasticity solutions for the laminated shells are involve complex mathematics, laminated shell theories have been developed by researchers to make these problems mathematically more simple. Laminated shell theories are approximate in nature as they are based on assumptions and hypotheses that reduce a three-dimensional problem to a two-dimensional one. Sayyad and Ghugal [1,2] provided a thorough study of displacement-based shear deformation theories for laminated composite beams, plates, and shells. It is well-known that the classical shell theory is suitable for the analysis of thin shell only due to neglect of transverse shear deformation. Mindlin [3] considered the impact of shear deformation in the first order shear deformation theory, in which transverse shear stress is constant across the thickness of shell and does not satisfy the traction free boundary conditions at the top and bottom surfaces of the shell. Revisions to shell theories are therefore necessary in order to take transverse shear and normal deformations into account. Reddy [4] developed well-known parabolic shear deformation theory for the static and dynamic analysis of laminated composite beams, plates and shells satisfying traction free boundary conditions. Liew and Lim [5] presented the higher-order shell theory for the vibration analysis of doubly-curved shallow shells. Tornabene and Ceruti [6], have conducted different studies on the static and dynamic analysis of doubly curved shells and panels using refined shear deformation theories. Using an extended higher-order shell theory, Sayyad and Ghugal [7] found higher-order closed-form solutions for static bending and free vibration analysis of laminated composite and sandwich spherical shells. Shinde and Sayyad [8] have presented a new higher-order shear and normal deformation theory for the free vibration analysis of laminated shells. In the present work, a new hyperbolic shear deformation theory is developed for the static analysis of cross-ply laminated composite spherical shells.

Laminated Shell under Consideration

As illustrated in Fig. 1, consider a differential element of a spherical shell in the *(x, y, z)* coordinate systems where, *x* and *y* curves represents principal curvatures on the mid-plane of laminate. R_x and R_y denote the principal radii of curvature of the mid-plane along *x* and *y* axes respectively. A cross-ply laminated shell element is made up of fibrous composite materials and composed of a *N* number of layers which are perfectly bonded together. A laminate is subjected to transverse load *q (x, y)* on the top surface i.e. $z=-h/2$ because the downward *z*-direction is assumed as positive.

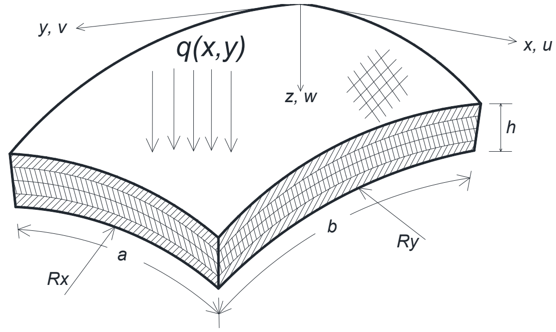

Fig. 1. Laminated shell geometry and coordinate system

Kinematics of the Present Theory

The present theory is built upon the classical shell theory and considers the effects of transverse shear and normal deformations in the in-plane and transverse displacements. Following is the displacement field assumed for the present theory.

$$\begin{aligned} u(x,y,z) &= (1+z/R_x)u_0(x,y) - z\,\partial w_0/\partial x + f(z)\theta_x(x,y), \\ v(x,y,z) &= (1+z/R_y)v_0(x,y) - z\,\partial w_0/\partial y + f(z)\theta_y(x,y), \\ w(x,y,z) &= w_0(x,y) + C_1\, g(z)\theta_z(x,y). \end{aligned} \tag{1}$$

where, *u, v, w* are the displacements of any point of the shell in *x, y, z* directions respectively. $\theta_x, \theta_y, \theta_z$ are the shear slopes in *x, y* and *z* direction respectively. u_0, v_0, w_0 are the mid-plane displacements of any point of the shell in *x, y, z* direction.

Strain Displacement Relationship

Using the linear theory of elasticity, the normal and shear strains associated with the present theory can be obtained as,

$$\begin{aligned} \varepsilon_x &= (\partial u_0/\partial x + w_0/R_x) - z\,\partial^2 w_0/\partial x^2 + f(z)\partial\theta_x/\partial x + C_1\left(f'(z)/R_x\right)\theta_z, \\ \varepsilon_y &= (\partial v_0/\partial y + w_0/R_y) - z\,\partial^2 w_0/\partial y^2 + f(z)\partial\theta_y/\partial y + C_1\left(f'(z)/R_y\right)\theta_z, \\ \varepsilon_z &= C_1 f''(z)\theta_z, \\ \gamma_{xy} &= \partial u_0/\partial y + \partial v_0/\partial x - 2z\,\partial^2 w_0/\partial x\partial y + f(z)(\partial\theta_x/\partial y + \partial\theta_y/\partial x), \\ \gamma_{xz} &= f'(z)\theta_x + C_1 f'(z)\partial\theta_z/\partial x;\ \gamma_{yz} = f'(z)\theta_y + C_1 f'(z)\partial\theta_z/\partial y, \\ f(z) &= z\cosh(\xi/2) - h/\xi\,\sinh(\xi z/h), \text{where}\ \xi = 2.634. \end{aligned} \tag{2}$$

Stress- Strain Relationship

Using the Hooke's law, stresses for the k^{th} layer of laminated shell can be obtained.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}. \quad (3)$$

where, $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz}$ are the stress components, $\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{xz}$ are the strain components. $(Q_{11}, Q_{12}, Q_{13}, Q_{22}, Q_{23}, Q_{33}, Q_{44}, Q_{55}, Q_{66})$ are the reduced stiffness coefficients as follows.

$$\begin{aligned} &Q_{11} = E_1(1-\mu_{23}\mu_{32})/\Delta; Q_{12} = E_1(\mu_{21}+\mu_{31}\mu_{23})/\Delta; Q_{13} = E_1(\mu_{31}+\mu_{21}\mu_{32})/\Delta; Q_{44} = G_{23}; \\ &Q_{22} = E_2(1-\mu_{13}\mu_{31})/\Delta; Q_{23} = E_2(\mu_{32}+\mu_{12}\mu_{31})/\Delta; Q_{33} = E_3(1-\mu_{12}\mu_{21})/\Delta; Q_{55} = G_{13}; Q_{66} = G_{12}. \end{aligned} \quad (4)$$

Principle of Virtual Work

$$\int_0^a\int_0^b\int_{-h/2}^{h/2} \left(\sigma_x\delta\sigma_x + \sigma_y\delta\sigma_y + \sigma_z\delta\sigma_z + \tau_{xy}\delta\tau_{xy} + \tau_{xz}\delta\tau_{xz} + \tau_{yz}\delta\tau_{yz}\right)dzdydx = \int_0^a\int_0^b q\delta w dydx. \quad (5)$$

Substituting the expressions of stresses and strains from Eqs. (2)-(4), into the Eq. (5), integrating by parts, collecting the coefficients of unknowns and setting them equal to zero, one can derive the following governing equation in terms of stress resultants as shown in Eq. (6).

$$\begin{aligned} &\boldsymbol{\delta u_0} : \partial N_x/\partial x + \partial N_{xy}/\partial y = 0; \quad \boldsymbol{\delta v_0} : \partial N_y/\partial y + \partial N_{xy}/\partial x = 0, \\ &\boldsymbol{\delta w_0} : \partial^2 M_x^b/\partial x^2 + \partial^2 M_y^b/\partial y^2 + 2\,\partial^2 M_{xy}^b/\partial x\partial y - N_x/R_x - N_y/R_y + q = 0, \\ &\boldsymbol{\delta\theta_x} : \partial M_x^S/\partial x + \partial M_{xy}^S/\partial y - Q_{xz} = 0, \\ &\boldsymbol{\delta\theta_y} : \partial M_y^S/\partial y + \partial M_{xy}^S/\partial x - Q_{yz} = 0, \\ &\boldsymbol{\delta\theta_z} : \partial S_{xz}/\partial x + \partial S_{yz}/\partial y - S_x/R_x - S_y/R_y - S_z' = 0. \end{aligned} \quad (6)$$

where,

$$\begin{aligned} &\left(N_x, N_y, N_{xy}, M_x^b, M_y^b, M_{xy}^b\right) = \int_{-h/2}^{h/2}\left[\sigma_x, \sigma_y, \tau_{xy}, z\sigma_x, z\sigma_y, z\tau_{xy}\right]dz; \\ &\left(M_x^S, M_y^S, M_{xy}^S\right) = \int_{-h/2}^{h/2}\left\{\left[f(z)\left(\sigma_x, \sigma_y, \tau_{xy}\right)\right]\right\}dz; \quad \left(Q_{xz}, Q_{yz}\right) = \int_{-h/2}^{h/2}\left\{\left[f'(z)\left(\tau_{xz}, \tau_{yz}\right)\right]\right\}dz; \\ &\left(S_x, S_y, S_{xz}, S_{yz}\right) = \int_{-h/2}^{h/2}\left[f'(z)\left(\sigma_x, \sigma_y, \tau_{xz}, \tau_{yz}\right)\right]dz; \quad \left(S_z'\right) = \int_{-h/2}^{h/2}\left\{\sigma_z\left[f''(z)\right]\right\}dz. \end{aligned} \quad (7)$$

Further substituting the expression of stress resultants from Eq. (7) into Eq. (6), the governing equations can be written in the following forms as stated in Eqs. (8)-(13).

$$\begin{aligned} &\boldsymbol{\delta u_0} : \left[A_{11}\left(\partial^2 u_0/\partial x^2 + \partial w_0/R_x\partial x\right) - B_{11}\,\partial^3 w_0/\partial x^3 + C_{11}\,\partial^2\theta_x/\partial x^2 + \left(F_{11}C_1/R_x\right)\partial\theta_z/\partial x\right] \\ &+\left[A_{12}\left(\partial^2 v_0/\partial x\partial y + \partial w_0/R_y\partial x\right) - B_{12}\,\partial^3 w_0/\partial x\partial y^2 + C_{12}\,\partial^2\theta_y/\partial x\partial y + \left(F_{12}C_1/R_y\right)\partial\theta_z/\partial x\right] \\ &+J_{13}C_1\,\partial\theta_z/\partial x + \left[A_{66}\left(\partial^2 u_0/\partial y^2 + \partial^2 v_0/\partial x\partial y\right) - 2B_{66}\,\partial^3 w_0/\partial x\partial y^2 + C_{66}\left(\partial^2\theta_x/\partial y^2 + \partial^2\theta_y/\partial x\partial y\right)\right] = 0. \end{aligned} \quad (8)$$

$$\begin{aligned} &\boldsymbol{\delta v_0} : \left[A_{21}\left(\partial^2 u_0/\partial x\partial y + \partial w_0/R_x\partial y\right) - B_{21}\,\partial^3 w_0/\partial x^2\partial y + C_{21}\,\partial^2\theta_x/\partial x\partial y + \left(F_{21}C_1/R_x\right)\partial\theta_z/\partial y\right] \\ &+\left[A_{22}\left(\partial^2 v_0/\partial y^2 + \partial w_0/R_y\partial y\right) - B_{22}\,\partial^3 w_0/\partial y^3 + C_{22}\,\partial^2\theta_y/\partial y^2 + \left(F_{22}C_1/R_y\right)\partial\theta_z/\partial y\right] + J_{23}C_1\,\partial\theta_z/\partial y \\ &+\left[A_{66}\left(\partial^2 u_0/\partial x\partial y + \partial^2 v_0/\partial x^2\right) - 2B_{66}\,\partial^3 w_0/\partial x^2\partial y + C_{66}\left(\partial^2\theta_x/\partial x\partial y + \partial^2\theta_y/\partial x^2\right)\right] = 0. \end{aligned} \quad (9)$$

$$\begin{aligned}
&\boldsymbol{\delta w_0}:\left[B_{11}\left(\partial^3 u_0/\partial x^3+\partial^2 w_0/R_x\partial x^2\right)-I_{11}\,\partial^4 w_0/\partial x^4+O_{11}\,\partial^3\theta_x/\partial x^3+\left(L_{11}C_1/R_x\right)\partial^2\theta_z/\partial x^2\right]\\
&+\left[B_{12}\left(\partial^2 v_0/\partial x^2\partial y+\partial^2 w_0/R_y\partial x^2\right)-I_{12}\,\partial^4 w_0/\partial x^2\partial y^2+O_{12}\,\partial^3\theta_y/\partial x^2\partial y+\left(L_{12}C_1/R_y\right)\partial^2\theta_z/\partial x^2\right]\\
&+\left[B_{12}\left(\partial^3 u_0/\partial x\partial y^2+\partial^2 w_0/R_x\partial y^2\right)-I_{12}\,\partial^4 w_0/\partial x^2\partial y^2+O_{12}\,\partial^3\theta_y/\partial x\partial y^2+\left(L_{12}C_1/R_x\right)\partial^2\theta_z/\partial y^2\right]\\
&+\left[B_{22}\left(\partial^3 v_0/\partial y^3+\partial^2 w_0/R_y\partial y^2\right)-I_{22}\,\partial^4 w_0/\partial y^4+O_{22}\,\partial^3\theta_y/\partial y^3+\left(L_{22}C_1/R_y\right)\partial^2\theta_z/\partial y^2\right]\\
&+\left[2B_{66}\left(\partial^3 u_0/\partial x\partial y^2+\partial^3 v_0/\partial x^2\partial y\right)-4I_{66}\,\partial^4 w_0/\partial x^2\partial y^2+2O_{66}\left(\partial^3\theta_x/\partial x\partial y^2+\partial^3\theta_y/\partial x^2\partial y\right)\right]\\
&-\left[A_{11}/R_x\left(\partial u_0/\partial x+w_0/R_x\right)-\left(B_{11}/R_x\right)\partial^2 w_0/\partial x^2+\left(C_{11}/R_x\right)\partial\theta_x/\partial x+\left(F_{11}C_1/R_x^2\right)\theta_z\right]\\
&-\left[A_{12}/R_x\left(\partial v_0/\partial y+w_0/R_y\right)-\left(B_{12}/R_x\right)\partial^2 w_0/\partial y^2+\left(C_{12}/R_x\right)\partial\theta_y/\partial y+\left(F_{12}C_1/R_xR_y\right)\theta_z\right]\\
&-\left[A_{12}/R_y\left(\partial u_0/\partial x+w_0/R_x\right)-\left(B_{12}/R_y\right)\partial^2 w_0/\partial x^2+\left(C_{12}/R_y\right)\partial\theta_x/\partial x+\left(F_{12}C_1/R_xR_y\right)\theta_z\right]\\
&-\left[A_{22}/R_y\left(\partial v_0/\partial y+w_0/R_y\right)-\left(B_{22}/R_y\right)\partial^2 w_0/\partial y^2+\left(C_{22}/R_y\right)\partial\theta_y/\partial y+\left(F_{22}C_1/R_y^2\right)\theta_z\right]\\
&+\left(M_{13}C_1\,\partial^2\theta_z/\partial x^2\right)+\left(M_{23}C_1\,\partial^2\theta_z/\partial y^2\right)-\left(J_{13}C_1/R_x\right)\theta_z-\left(J_{23}C_1/R_y\right)\theta_z+q=0.
\end{aligned}\qquad(10)$$

$$\begin{aligned}
&\boldsymbol{\delta\theta_x}:\left[C_{11}\left(\partial^2 u_0/\partial x^2+\partial w_0/R_x\partial x\right)-O_{11}\,\partial^3 w_0/\partial x^3+P_{11}\,\partial^2\theta_x/\partial x^2+\left(R_{11}C_1/R_x\right)\partial\theta_z/\partial x\right]\\
&+\left[C_{12}\left(\partial^2 v_0/\partial x\partial y+\partial w_0/R_y\partial x\right)-O_{12}\,\partial^3 w_0/\partial x\partial y^2+P_{12}\,\partial^2\theta_y/\partial x\partial y+\left(R_{12}C_1/R_y\right)\partial\theta_z/\partial x\right]\\
&+\left[C_{66}\left(\partial^2 u_0/\partial y^2+\partial^2 v_0/\partial x\partial y\right)-2O_{66}\,\partial^3 w_0/\partial x\partial y^2+P_{66}\left(\partial^2\theta_x/\partial y^2+\partial^2\theta_y/\partial x\partial y\right)\right]\\
&+S_{13}C_1\,\partial\theta_z/\partial x-U_{55}\theta_x-C_1U_{55}\,\partial\theta_z/\partial x=0.
\end{aligned}\qquad(11)$$

$$\begin{aligned}
&\boldsymbol{\delta\theta_y}:\left[C_{21}\left(\partial^2 u_0/\partial x\partial y+\partial w_0/R_x\partial y\right)-O_{21}\,\partial^3 w_0/\partial x^2\partial y+P_{21}\,\partial^2\theta_x/\partial x\partial y+\left(R_{21}C_1/R_x\right)\partial\theta_z/\partial y\right]\\
&+\left[C_{22}\left(\partial^2 v_0/\partial y^2+\partial w_0/R_y\partial y\right)-O_{22}\,\partial^3 w_0/\partial y^3+P_{22}\,\partial^2\theta_y/\partial y^2+\left(R_{22}C_1/R_y\right)\partial\theta_z/\partial y\right]+\\
&+\left[C66\left(\partial^2 u_0/\partial x\partial y+\partial^2 v_0/\partial x^2\right)-2O66\,\partial^3 w_0/\partial x^2\partial y+P66\left(\partial^2\theta_x/\partial x\partial y+\partial^2\theta_y/\partial x^2\right)\right]\\
&S_{23}C_1\,\partial\theta_z/\partial y-U_{44}\left(\theta_y+C_1\,\partial\theta_z/\partial y\right)=0.
\end{aligned}\qquad(12)$$

$$\begin{aligned}
&\boldsymbol{\delta\theta_z}:-\left(F_{11}/R_x\right)\left(\partial u_0/\partial x+w_0/R_x\right)+L_{11}\partial^2 w_0/R_x\partial x^2-R_{11}\partial\theta_x/R_x\,\partial x-U_{11}C_1/R_x^2\,\theta_z\\
&-F_{12}/R_x\left(\partial v_0/\partial y+w_0/R_y\right)+L_{12}\partial^2 w_0/R_x\partial y^2-R_{12}\partial\theta_y/R_x\,\partial y-U_{12}C_1\theta_z/R_xR_y\\
&-2Y_{13}C_1\theta_z/R_x-F_{12}/R_y\left(\partial u_0/\partial x+w_0/R_x\right)+L_{12}\partial^2 w_0/R_y\partial x^2-R_{12}\partial\theta_x/R_y\,\partial x-U_{12}C_1\theta_z/R_xR_y\\
&-F_{22}/R_y\left(\partial v_0/\partial y+w_0/R_y\right)+L_{22}\partial^2 w_0/R_y\,\partial y^2-R_{22}\partial\theta_y/R_y\,\partial y-U_{22}C_1\theta_z/R_y^2-2Y_{23}C_1\theta_z/R_y\\
&-J_{13}\left(\partial u_0/\partial x+w_0/R_x\right)+M_{13}\,\partial^2 w_0/\partial x^2-S_{13}\,\partial\theta_x/\partial x-J_{23}\left(\partial v_0/\partial y+w_0/R_y\right)+M_{23}\,\partial^2 w_0/\partial y^2\\
&U_{55}\left(\partial\theta_x/\partial x+C_1\,\partial^2\theta_z/\partial x^2\right)+U_{44}\left(\partial\theta_y/\partial y+C_1\,\partial^2\theta_z/\partial y^2\right)-S_{23}\,\partial\theta_y/\partial y-Z_{33}C_1\theta_z=0.
\end{aligned}\qquad(13)$$

where,

$$\left(A_{ij},B_{ij},I_{ij},C_{ij},O_{ij},F_{ij},J_{ij}\right)=Q_{ij}\int_{-h/2}^{h/2}\left[1.0,z,z^2,f(z),zf(z),f'(z),f''(z)\right]dz;$$

$$\left(P_{ij}\right)=Q_{ij}\int_{-h/2}^{h/2}\left[f(z)\right]^2dz;\ \left(U_{ij}\right)=Q_{ij}\int_{-h/2}^{h/2}\left[f'(z)\right]^2dz;$$

$$\left(L_{ij},R_{ij},Y_{ij}\right)=Q_{ij}\int_{-h/2}^{h/2}f'(z)\left[z,f(z),f''(z)\right]dz;$$

$$\left(M_{ij},S_{ij}\right)=Q_{ij}\int_{-h/2}^{h/2}f''(z)\left[z,f(z)\right]dz;\left(Z_{ij}\right)=Q_{ij}\int_{-h/2}^{h/2}\left[f''(z)\right]^2dz.$$

Eq. (14) expresses the boundary condition associated with the present theory.

$$\begin{aligned}
&\partial u_0: N_x=0, N_{xy}=0;\ \partial v_0: N_y=0, N_{xy}=0;\\
&\partial w_0: M_x^b=0, M_y^b=0, M_{xy}^b=0, \partial w_0/\partial y=0;\\
&\partial M_x^b/\partial x=0, \partial M_y^b/\partial y=0, \partial M_{xy}^b/\partial x=0;\\
&\partial\theta_x: M_x^s=0, M_{xy}^s=0;\ \partial\theta_y: M_y^s=0, M_{xy}^s=0;\\
&\partial\theta_z: S_{xz}=0, S_{yz}=0.
\end{aligned}\tag{14}$$

The Navier Solution Method

Laminated composite spherical shells subjected to transverse load is considered for the static analysis. The top surface of the shell is subjected to a transverse load that is represented in terms of double trigonometric series.

$$q(x,y)=q_0\sin\alpha x\sin\beta y. \tag{15}$$

where, $\alpha=\pi/a, \beta=\pi/b$ and q_0 represents the intensity of the load. The unknown variables $u_0, v_0, w_0, \theta_x, \theta_y, \theta_z$ are assumed in the following double trigonometric forms, which satisfy the simply supported boundary conditions exactly:

$$\begin{Bmatrix} u_0\\ v_0\\ w_0\\ \theta_x\\ \theta_y\\ \theta_z \end{Bmatrix}=\begin{Bmatrix} u_{mn}\cos\alpha x\sin\beta y\\ v_{mn}\sin\alpha x\cos\beta y\\ w_{mn}\sin\alpha x\sin\beta y\\ \theta_{xmn}\cos\alpha x\sin\beta y\\ \theta_{ymn}\sin\alpha x\cos\beta y\\ \theta_{zmn}\sin\alpha x\sin\beta y \end{Bmatrix}. \tag{16}$$

where, $u_{mn}, v_{mn}, w_{mn}, \theta_{xmn}, \theta_{ymn}, \theta_{zmn}$ are the unknown parameters to be determined. Now, substituting Eq. (16) into Eqs. (8)-(13), can be written in following compact form for the static analysis of laminated composite spherical shell as given in Eq. (17).

$$[K]_{6\times6}\{\Delta\}_{6\times1}=\{f\}_{6\times1}. \tag{17}$$

where, $[K]$ is the stiffness matrix, $\{\Delta\}$ and $\{f\}$ are the matrix of unknown displacements and matrix of force vectors respectively. Elements of stiffness matrix are written in Eq. (18).

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\begin{aligned}
&K_{11} = -A_{11}\alpha^2 - A_{66}\beta^2, \quad K_{12} = -A_{12}\alpha\beta - A_{66}\alpha\beta,\\
&K_{13} = A_{11}/R_x\,\alpha + A_{12}/R_y\,\beta + B_{11}\alpha^3 + B_{12}\alpha\beta^2 + 2B_{66}\alpha\beta^2,\\
&K_{14} = -C_{11}\alpha^2 - C_{66}\beta^2,\ K_{15} = -C_{12}\alpha\beta - C_{66}\alpha\beta,\\
&K_{16} = \left(F_{11}/R_x + F_{12}/R_y + J_{13}\right)C_1\alpha,\ K_{22} = -A_{22}\beta^2 - A_{66}\alpha^2,\\
&K_{23} = \left(A_{12}/R_x + A_{22}/R_y\right)\beta + B_{12}\alpha^2\beta + B_{22}\beta^3 + 2B_{66}\alpha^2\beta,\\
&K_{24} = -C_{12}\alpha\beta - C_{66}\alpha\beta, K_{25} = -C_{66}\alpha^2 - C_{22}\beta^2,\\
&K_{26} = \left(F_{12}/R_x + F_{22}/R_y + J_{23}\right)C_1\beta,\\
&K_{331} = -I_{11}\alpha^4 - I_{22}\beta^4 - 2\alpha^2\beta^2\left(I_{12} + 2I_{66}\right)\\
&K_{332} = -2\alpha^2\left(B_{11}/R_x + B_{12}/R_y\right) - 2\beta^2\left(B_{12}/R_x + B_{22}/R_y\right) - A_{11}/R_x^2 - A_{22}/R_y^2 - 2A_{12}/R_xR_y,\\
&K_{33} = K_{331} + K_{332} \qquad (18)\\
&K_{34} = O_{11}\alpha^3 + O_{21}\alpha\beta^2 + 2O_{66}\alpha\beta^2 + \alpha C_{11}/R_x + \alpha C_{12}/R_y,\\
&K_{35} = O_{22}\beta^3 + O_{12}\alpha^2\beta + 2O_{66}\alpha^2\beta + \beta C_{12}/R_x + \beta C_{22}/R_y,\\
&K_{361} = C_1\left(-\alpha^2\left(L_{11}/R_x + L_{12}/R_y\right) - \beta^2\left(L_{12}/R_x + L_{22}/R_y\right) - M_{13}\alpha^2\right)\\
&K_{362} = C_1\left(-M_{23}\beta^2 - F_{11}/R_x^2 - F_{22}/R_y^2 - 2F_{12}/R_xR_y - J_{13}/R_x - J_{23}/R_y\right),\\
&K_{36} = K_{361} + K_{362},\\
&K_{44} = -P_{11}\alpha^2 - P_{66}\beta^2 - U_{55},\ K_{45} = -P_{12}\alpha\beta - P_{66}\alpha\beta,\\
&K_{46} = C_1\alpha\left(R_{11}/R_x + R_{12}/R_y + S_{13} - U_{55}\right), K_{55} = -P_{66}\alpha^2 - P_{22}\beta^2 - U_{44},\\
&K_{56} = C_1\beta\left(R_{12}/R_x + R_{22}/R_y + S_{23} - U_{44}\right),\\
&K_{66} = C_1^2\left(-U_{55}\alpha^2 - U_{44}\beta^2 - U_{11}/R_x^2 - 2U_{12}/R_xR_y - U_{22}/R_y^2 - 2W_{13}/R_x - 2W_{23}/R_y - Z_{33}\right).
\end{aligned}
$$

Numerical Result and Discussion

The following material properties stated in Eqs. (19) and (20) are considered to obtain the static deformation quantities of isotropic and orthotropic laminated spherical shells using the current hyperbolic shell theory.

$$E_1 = E_2 = E_3 = 210\ \text{GPa}, G_{13} = G_{23} = G_{12} = G = E/2(1+\mu),\ \mu_{12} = \mu_{32} = \mu_{31} = \mu = 0.3. \qquad (19)$$

$$E_1/E_2 = 25,\ E_3/E_2 = 1,\ E_3/E_2 = 1, G_{12}/E_2 = G_{13}/E_2 = 0.5,\ G_{23}/E_2 = 0.2,\ \mu_{12} = \mu_{13} = \mu_{23} = 0.25. \qquad (20)$$

The non-dimensional form stated in Eq. (21) is used to present the numerical results in tabular as well as graphical form.

$$
\begin{aligned}
&\bar{u}\left(0,b/2,h/2\right) = h^2E_3u/q_0a^3, \quad \bar{w}\left(a/2,b/2,0\right) = 100h^3E_3w/q_0a^4,\\
&\left(\bar{\sigma}_x,\bar{\sigma}_y\right)\left(a/2,b/2,h/2\right) = \left(h^2/q_0a^2\right)\left(\sigma_x,\sigma_y\right),\ \bar{\tau}_{xy}\left(0,0,h/2\right) = h^2\tau_{xy}/q_0a^2, \qquad (21)\\
&\bar{\tau}_{xz}\left(0,0,h/2\right) = h\tau_{xz}/q_0a,\ \ \bar{\tau}_{yz}\left(0,0,h/2\right) = h\tau_{yz}/q_0a.
\end{aligned}
$$

Table 1 *Non-dimensional displacement and stresses in isotropic spherical shell under the sinusoidal mechanical load (a/h=10, $R_x = R_y = R$).*

R/a	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
5	Present	0.0434	2.6020	0.1437	0.1437	0.1225	0.2076	0.2076
	Shinde and Sayyad [8]	0.0508	2.6099	0.1490	0.1490	0.1228	0.2272	0.2272
	Reddy [4]	0.0506	2.6472	0.1519	0.1519	0.1225	0.2130	0.2130
	Mindlin [3]	0.0502	2.6262	0.1506	0.1506	0.1230	0.2137	0.2137
50	Present	0.0383	2.9357	0.1901	0.1901	0.1093	0.2340	0.2340
	Shinde and Sayyad [8]	0.0454	2.9407	0.1967	0.1967	0.1096	0.2559	0.2559
	Reddy [4]	0.0456	2.9572	0.1962	0.1962	0.1102	0.2379	0.2379
	Mindlin [3]	0.0452	2.9310	0.1944	0.1944	0.1109	0.2385	0.2385
100	Present	0.0377	2.9386	0.1918	0.1918	0.1078	0.2342	0.2342
	Shinde and Sayyad [8]	0.0447	2.9435	0.1985	0.1985	0.1081	0.2562	0.2562
	Reddy [4]	0.0450	2.9598	0.1979	0.1979	0.1088	0.2381	0.2381
	Mindlin [3]	0.0446	2.9336	0.1961	0.1961	0.1095	0.2387	0.2387
∞	Present	0.0371	2.9395	0.1935	0.1935	0.1062	0.2343	0.2343
	Shinde and Sayyad [8]	0.0441	2.9445	0.2001	0.2001	0.1065	0.2563	0.2563
	Reddy [4]	0.0444	2.9607	0.1994	0.1994	0.1074	0.2382	0.2382
	Mindlin [3]	0.0440	2.9345	0.1976	0.1976	0.1080	0.2387	0.2387
	Pagano [9]	0.0443	2.9425	0.1988	0.1988	---	0.2383	0.2383

Table 2 *Non-dimensional displacement and stresses in ($0^0/90^0$) spherical shell under the sinusoidal mechanical load (a/h=10, $R_x = R_y = R$).*

R/a	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
5	Present	0.0097	1.1141	0.6515	0.0760	0.0691	0.1044	0.1266
	Shinde and Sayyad [8]	0.0151	1.1200	0.6450	0.0759	0.0692	0.0930	0.1230
	Reddy [4]	0.0151	1.1164	0.6530	0.0754	0.0694	0.0823	0.1382
	Mindlin [3]	0.0148	1.1096	0.6262	0.0747	0.0686	0.0839	0.1402
50	Present	0.0160	1.2125	0.7388	0.0854	0.0552	0.1245	0.1269
	Shinde and Sayyad [8]	0.0098	1.2186	0.7354	0.0856	0.0552	0.1159	0.1192
	Reddy [4]	0.0100	1.2148	0.7424	0.0847	0.0555	0.1170	0.1230
	Mindlin [3]	0.0096	1.2070	0.7116	0.0840	0.0546	0.1189	0.1250
100	Present	0.0163	1.2133	0.7410	0.0857	0.0541	0.1251	0.1264
	Shinde and Sayyad [8]	0.0095	1.2194	0.7378	0.0858	0.0542	0.1168	0.1184
	Reddy [4]	0.0096	1.2156	0.7447	0.0850	0.0545	0.1186	0.1216
	Mindlin [3]	0.0092	1.2078	0.7138	0.0842	0.0536	0.1205	0.1235
∞	Present	0.0165	1.2136	0.7428	0.0858	0.0530	0.1258	0.1258
	Shinde and Sayyad [8]	0.0091	1.2197	0.7398	0.0860	0.0531	0.1176	0.1176
	Reddy [4]	0.0092	1.2158	0.7466	0.0851	0.0534	0.1201	0.1201
	Mindlin [3]	0.0088	1.2081	0.7156	0.0843	0.0525	0.1220	0.1220
	Pagano [9]	--	1.2250	0.7302	0.0886	0.0535	0.1210	0.1250

The non-dimensional displacements and stresses for an isotropic, ($0^0/90^0$) and ($0^0/90^0/0^0$) laminated spherical shells for aspect ratio 10 and *R/a*=5, 50, 100, ∞ subjected to sinusoidal loading are shown in Tables 1 through 3 using the present theory. The numerical findings are compared with higher-order shear deformation theories that have been previously published. Additionally, first-order shear deformation theory [3] and Pagano's exact elasticity solution [9] are used to compare the displacements and stresses for plate obtained using the present theory. As can be seen from Table 1, the displacement and stresses obtained by the present theory are in good agreement with those other higher-order theories. For plates, there is a less percentage of inaccuracy in the precise and present theory results. Due to the influence of normal and shear deformations, the findings obtained using the present theory are also superior to those obtained using other higher order theories in the case of ($0^0/90^0$) and ($0^0/90^0/0^0$) laminated spherical shells. For ($0^0/90^0$) and ($0^0/90^0/0^0$) laminated composite spherical shells exposed to sinusoidal load, through-the-thickness distributions of in-plane and transverse shear stresses are shown in Figs. 2 and 3.

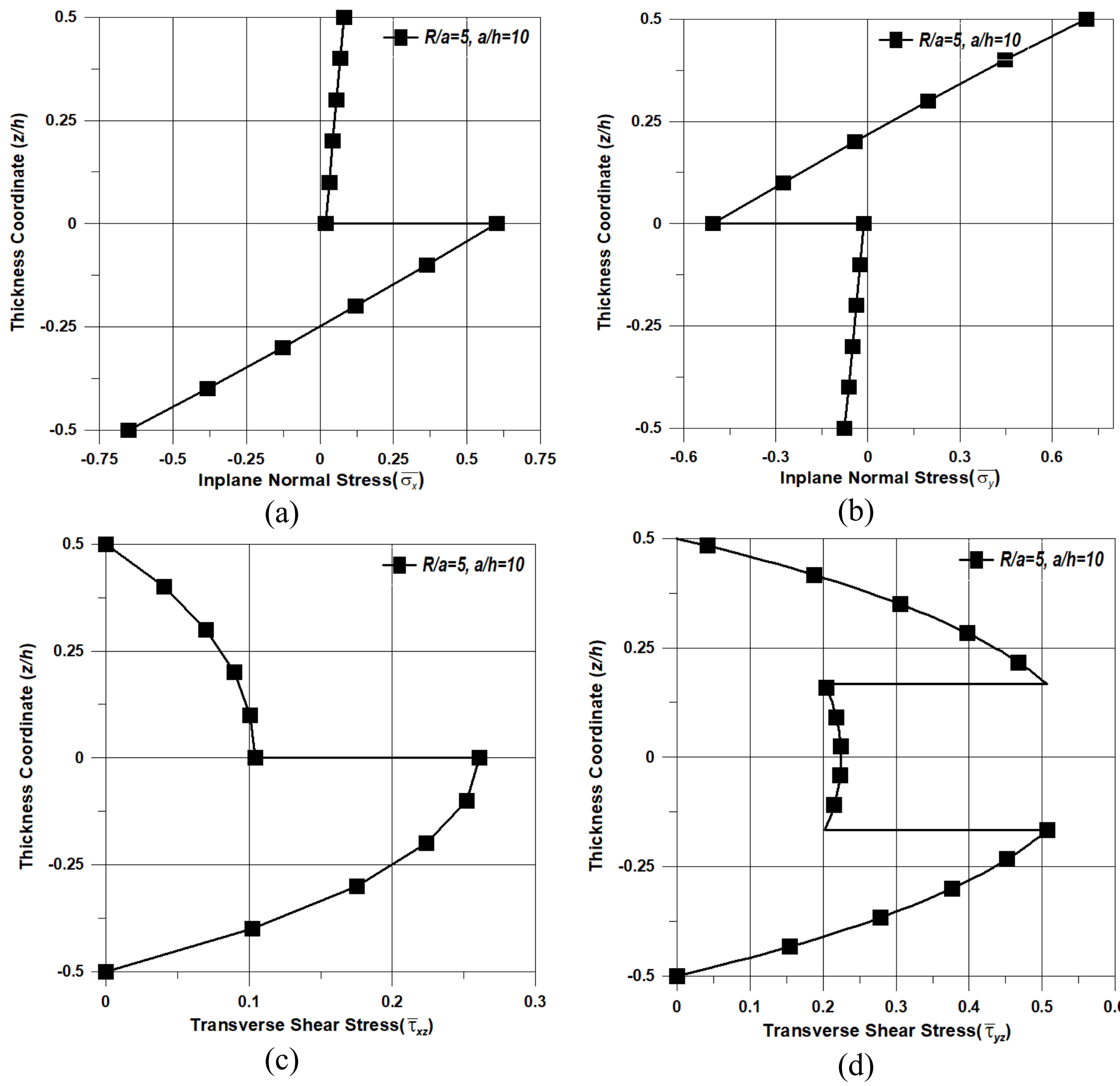

Fig. 2 Stress distributions in ($0^0/90^0$) laminated composite spherical shells

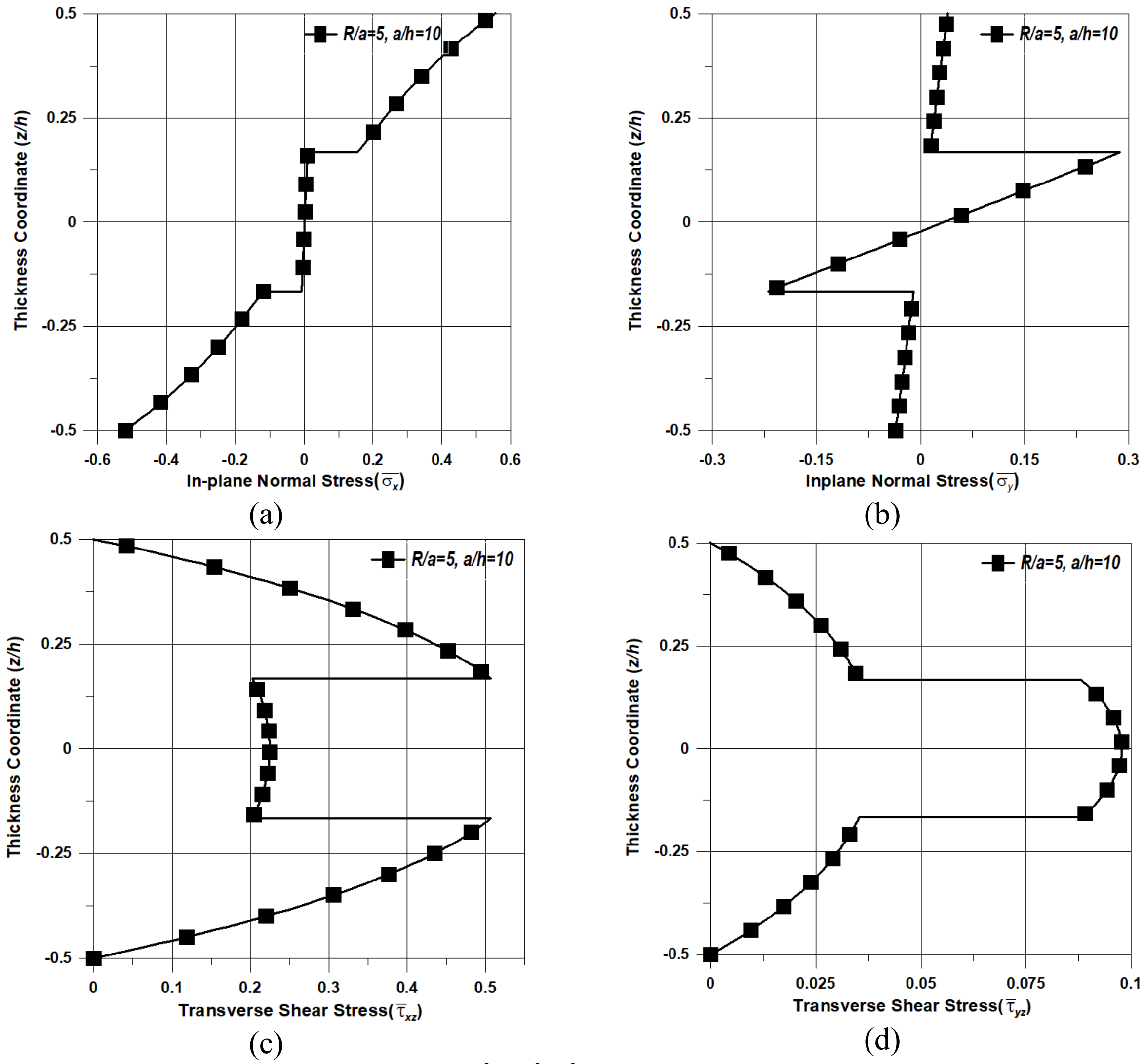

Fig. 3 Stress distributions in ($0^0/90^0/0^0$) laminated composite spherical shells

Conclusions

For the static analysis of laminated composite spherical shells, a new hyperbolic shell theory is developed in this paper. To account for the impact of transverse shear and normal deformations, the present theory is expanded with the introduction of hyperbolic shape functions in terms of thickness coordinates. The kinematics of the present theory takes into consideration of traction-free boundary conditions on the top and bottom surfaces of the shell as well as realistic distribution of the transverse shear stresses throughout the thickness of the shell. Using the principle of virtual work, the governing equations and accompanying boundary conditions are obtained. Using Navier's solution method, higher-order closed form solutions for static analysis of simply supported spherical shells are presented. By taking both curvature radii as infinite, these solutions also apply to plates. The numerical findings for the plate using the present hyperbolic shell theory are in good agreement with the exact elasticity solutions, demonstrating the validity and accuracy of the present theory.

Table 3 *Non-dimensional displacement and stresses in ($0^0/90^0/0^0$) spherical shell under the sinusoidal mechanical load (a/h=10, $R_x = R_y = R$).*

R/a	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
5	Present	0.0124	0.6713	0.5538	0.0394	0.0384	0.2246	0.0391
	Shinde and Sayyad [8]	0.0112	0.6972	0.5455	0.0360	0.0399	0.3005	0.0395
	Reddy [4]	0.0108	0.6769	0.5218	0.0352	0.0388	0.3508	0.1109
	Mindlin [3]	0.0098	0.6025	0.4780	0.0311	0.0346	0.3658	0.1018
50	Present	0.0088	0.7062	0.5659	0.0399	.0287	0.2362	0.0411
	Shinde and Sayyad [8]	0.0079	0.7345	0.5923	0.0396	0.0299	0.3165	0.0416
	Reddy [4]	0.0075	0.7121	0.5662	0.0385	0.0290	0.3691	0.1167
	Mindlin [3]	0.0069	0.6303	0.5153	0.0338	0.0257	0.3826	0.1064
100	Present	0.0086	0.7064	0.5652	0.0398	0.0280	0.2363	0.0411
	Shinde and Sayyad [8]	0.0077	0.7348	0.5935	0.0397	0.0293	0.3167	0.0416
	Reddy [4]	0.0073	0.7124	0.5674	0.0386	0.0283	0.3692	0.1167
	Mindlin [3]	0.0067	0.6305	0.5163	0.0339	0.0252	0.3828	0.1064
∞	Present	0.0084	0.7065	0.5644	0.0397	0.0274	0.2364	0.0411
	Shinde and Sayyad [8]	0.0074	0.7349	0.5946	0.0398	0.0286	0.3167	0.0416
	Reddy [4]	0.0071	0.7125	0.5684	0.0387	0.0277	0.3693	0.1168
	Mindlin [3]	0.0065	0.6306	0.5172	0.0340	0.0246	0.3828	0.1065
	Pagano [9]	--	0.7528	0.5898	0.0418	0.0289	0.3570	0.1200

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