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Static and free vibration analysis of laminated sandwich shell with double curvature considering the effect of transverse normal strain

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Abstract. For the static bending analysis of sandwich spherical shells, higher-order closed-form solutions are provided in the current study based on a new hyperbolic shell theory considering the effects of transverse normal strain. A shell consists of three layers wherein the top and the bottom layers (face sheets) are made up of hard material and the middle layer (core) is made up of soft material. The governing equations and associated boundary conditions of the theory are produced by employing the principle of virtual work. Semi-analytical closed-form solutions for the static problem are produced by the Navier technique for simply supported boundary conditions of the shell. The present results are compared with results that have already been published in order to confirm the accuracy and efficacy of the current higher-order hyperbolic shell theory.

Introduction

Fiber-reinforced polymer composites are the most important kind of composite material. The most significant characteristics of fiber-reinforced polymer composite materials are their high strength-to-weight and stiffness-to-weight properties. Consequently, these are being employed more frequently in a variety of technical applications. Lightweight composite panels called laminated sandwich shells have a soft inner core between two thin, stronger skins. Its function is to transfer loads from the face sheets to the core structure, and if it fails, the structure will no longer function. Inflicted stresses on sandwich structures are distributed between the face sheets and the core structure according to their material properties and thicknesses. Sandwich panels are frequently utilized in a variety of engineering fields, including civil, mechanical, aerospace, marine, and offshore. Mourtiz et al. [1] described composite shell structures are widely used in different engineering sectors for many years, including the naval, aerospace, automotive, and construction sectors, as well as sporting goods, medical devices, and many other areas. Mallikarjuna and Kant [2] provided a critical review and some results of recently developed refined theories of fiber-reinforced laminated composites and sandwiches and this review is limited to linear free vibration and transient dynamic analyses, and geometric nonlinear transient response of multilayer sandwich/fiber-reinforced composite plates. Ferreira et al. [3] presented Non-linear analysis of sandwich shells and the effect of core plasticity using first order shear deformation theory based on the finite element method and uses the Ahmad shell element with five degrees of freedom per node. Kant and Swaminathan [4] presented analytical solutions for the static analysis of laminated sandwich plates using higher order refined theory based on Navier's solution technique. The theoretical model presented by the author incorporates laminate deformations which account for the effects of transverse shear deformation. Hohe and Librescu [5] presented a nonlinear theory for doubly curved anisotropic sandwich shells with transversely compressible core using an

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advanced geometrically nonlinear shell theory of doubly curved structural sandwich panels with transversely compressible core is presented based on Kirchhoff theory. This theory accounts for dynamic effects as well as for initial geometric imperfections. Zhong and Reimerdes [6] presented stability behavior of cylindrical and conical sandwich shells with flexible core using a higher-order theory based on three-layer model and solved by numerical integration. Khare et al. [7] presented solutions for thick laminated sandwich shells using higher order theory based on closed form solutions. Closed-form formulations of 2D higher-order shear deformation theory are presented for the thermo-mechanical and free vibration analysis of simply supported, cross-ply, laminated sandwich, doubly thick curved shells. Results on static and dynamic problems of double core sandwich shell are not presented in the paper. Garg et al. [8] presented Solutions for free vibration of laminated composite and sandwich shells using higher-order closed-form. It described free vibration characteristics of simply supported, laminated cross-ply, composite, and sandwich shell panels using the various higher-order theories, which account for the effects of transverse shear strains/stresses and the transverse normal strain/stress. Results on a multilayered sandwich shell analysis are not presented by the authors. Turkin [9] presented a technique for calculating rational design parameters of a sandwich shell with account of thermal loading using nonlinear theory of thin elastic shells.

Objective of the Present Study

Based on the aforementioned literature review, it is found that the literature on the mechanics of sandwich shells considering the effects of transverse normal strain is limited. Hence, the objective of the present study is to carry out static bending analysis of laminated sandwich shells using a new hyperbolic shell theory. A semi-analytical solution for the static problem is obtained using the Navier method.

Kinematic formulation

Fig. 1 shows a differential shell element considered in the (*x*, *y*, z) coordinate system. The *x* and *y* curves depicted here are lines of substantial curvature on the mid-plane of the laminate. The downward z-direction is seen to be positive. R_1 and R_2, respectively, stand for the primary radii of curvature of the mid-plane along the *x* and *y* axes. Layers of orthotropic composite material that are thought to be suitably linked together make up a laminate. On the top surface of a laminate, that is, $z = -h/2$, a transverse load of $q(x, y)$ is applied.

Following is the displacement field assumed for the current hyperbolic shell theory.

$$\begin{aligned} u(x,y,z) &= \left(1+\frac{z}{R_1}\right)u_0(x,y) - z\frac{\partial w_0}{\partial x} + f(z)\theta_x(x,y) \\ v(x,y,z) &= \left(1+\frac{z}{R_2}\right)v_0(x,y) - z\frac{\partial w_0}{\partial y} + f(z)\theta_y(x,y) \\ w(x,y,z) &= w_0(x,y) + f'(z)\theta_z(x,y) \end{aligned} \tag{1}$$

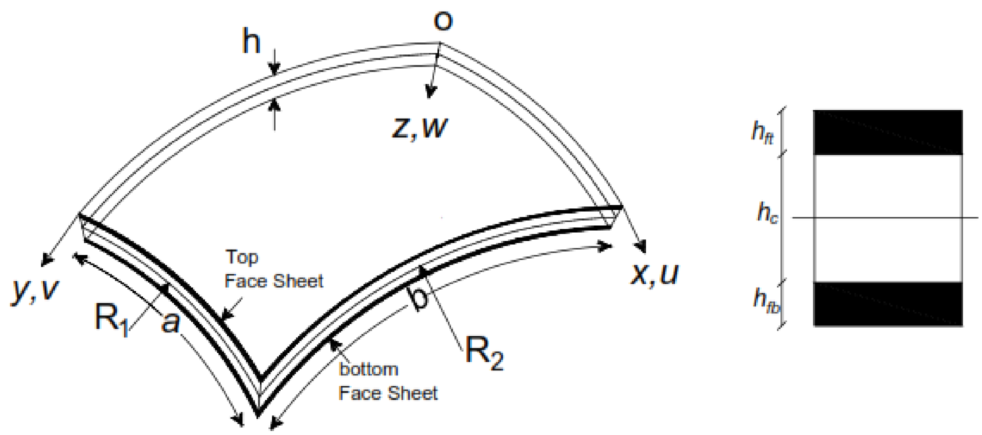

Fig. 1 Geometry and coordinates of sandwich shell under consideration

where, *u, v, w* are the displacements in *x, y, z* directions respectively; $\theta_x, \theta_y, \theta_z$ are the shear slopes in *x, y* and *z* direction respectively; u_0, v_0, w_0 are the mid-plane displacements in *x, y, z* direction respectively. Using the linear theory of elasticity, the normal and shear strains associated with the displacement field can be calculated as follows:

$$\varepsilon_x = \left(\frac{\partial u_0}{\partial x} + \frac{w_0}{R_1}\right) - z\frac{\partial^2 w_0}{\partial x^2} + f(z)\frac{\partial \theta_x}{\partial x} + \frac{f'(z)}{R_1}\theta_z$$
$$\varepsilon_y = \left(\frac{\partial v_0}{\partial y} + \frac{w_0}{R_2}\right) - z\frac{\partial^2 w_0}{\partial y^2} + f(z)\frac{\partial \theta_y}{\partial y} + \frac{f'(z)}{R_2}\theta_z$$
$$\varepsilon_z = f''(z)\theta_z$$
$$\gamma_{xy} = \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right) - 2z\frac{\partial^2 w_0}{\partial x \partial y} + f(z)\left(\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}\right) \tag{2}$$
$$\gamma_{xz} = f'(z)\theta_x + f'(z)\frac{\partial \theta_x}{\partial x}$$
$$\gamma_{yz} = f'(z)\theta_y + f'(z)\frac{\partial \theta_z}{\partial y}$$

where

$$f(z) = \left[z\cosh\left(\frac{\xi}{2}\right)\right] - \left[\left(\frac{h}{\xi}\right)\sinh\left(\frac{\xi z}{h}\right)\right],$$
$$f'(z) = \left[\cosh\left(\frac{\xi}{2}\right)\right] - \left[\cosh\left(\frac{\xi z}{h}\right)\right] \text{ where } \xi = 2.634. \tag{3}$$

Stresses are calculated using the Hooke's law from the 3D elasticity problem for cross-ply laminated shells.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}^k = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{44} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}^k \tag{4}$$

where $(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz})$are the normal and shear stresses, and $(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{xz})$ are the normal and shear strains. $(Q_{11}, Q_{12}, Q_{13}, Q_{22}, Q_{23}, Q_{33}, Q_{44}, Q_{55}, Q_{66})^k$are the reduced stiffness coefficients.

$$Q_{11} = \frac{E_1(1-\mu_{23}\mu_{32})}{\Delta}; Q_{12} = \frac{E_1(\mu_{21}+\mu_{31}\mu_{23})}{\Delta}; Q_{13} = \frac{E_1(\mu_{31}+\mu_{21}\mu_{32})}{\Delta};$$
$$Q_{22} = \frac{E_2(1-\mu_{13}\mu_{31})}{\Delta}; Q_{23} = \frac{E_2(\mu_{32}+\mu_{12}\mu_{31})}{\Delta}; Q_{33} = \frac{E_3(1-\mu_{12}\mu_{21})}{\Delta};$$
$$Q_{44} = G_{23}; Q_{55} = G_{13}; Q_{66} = G_{12}; \tag{5}$$
$$\Delta = 1 - \mu_{12}\mu_{21} - \mu_{23}\mu_{32} - \mu_{13}\mu_{31} - 2\mu_{21}\mu_{32}\mu_{13}$$

Principle of Virtual Work

$$\int_0^a \int_0^b \int_{-h/2}^{h/2} \left(\sigma_x \delta\sigma_x + \sigma_y \delta\sigma_y + \sigma_z \delta\sigma_z + \tau_{xy}\delta\tau_{xy} + \tau_{xz}\delta\tau_{xz} + \tau_{yz}\delta\tau_{yz}\right) dzdydx =$$
$$\int_0^a \int_0^b \int_{-h/2}^{h/2} q(x,y)\ dzdydx \tag{6}$$

The virtual work principle can be used to generate the six variationally-consistent governing equations and boundary conditions. The resulting governing equations can be visualized as the following in terms of stress resultants.

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$$\delta u_0: \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$
$$\delta v_0: \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = 0$$
$$\delta w_0: \frac{\partial^2 M_{xx}^b}{\partial x^2} + \frac{\partial^2 M_{yy}^b}{\partial y^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\, \partial y} - \frac{N_{xx}}{R_1} - \frac{N_{yy}}{R_2} + q = 0$$
$$\delta \theta_x: \frac{\partial M_{yy}^S}{\partial x} + \frac{\partial M_{xy}^S}{\partial y} - Q_{xz}^S = 0 \quad (7)$$
$$\delta \theta_{y:}: \frac{\partial M_{yy}^S}{\partial y} + \frac{\partial M_{xy}^S}{\partial x} - Q_{yz}^S = 0$$
$$\delta \theta_z: \frac{\partial Q_{xz}^S}{\partial x} + \frac{\partial Q_{yz}^S}{\partial y} - \frac{V_{xx}^S}{R_1} - \frac{V_{yy}^S}{R_2} - V_{zz}^S = 0$$

where

$$\left(N_{xx}, N_{yy}, Ns_{xy}, M_{xx}^b,\ M_{yy}^b, M_{xy}^b\right) = \int_{-h/2}^{h/2} [\sigma_x, \sigma_y, \tau_{xy}, z\sigma_x,\ z\sigma_y, z\tau_{xy}]\ dz$$
$$\left(M_{xx}^S, M_{yy}^S, M_{xy}^S\right) = \int_{-h/2}^{h/2} \{[f(z)(\sigma_x, \sigma_y, \tau_{xy})]\}\ dz$$
$$\left(Q_{xz}^S, Q_{yz}^S\right) = \int_{-h/2}^{h/2} \{\ [f'(z)(\tau_{xz}, \tau_{yz})]\}\ dz \quad (8)$$
$$\left(V_{xx}^S, V_{yy}^S\right) = \int_{-h/2}^{h/2} \{\ [f'(z)(\sigma_x, \sigma_y)]\}\ dz$$
$$(V_{zz}^S) = \int_{-h/2}^{h/2} \{[f''(z)]\sigma_z\ \}\ dz$$

Closed-Form Solution

The following solution form is assumed for the unknown variables in accordance with Navier's solution process, and it precisely satisfies the boundary conditions that are easily supported.

$$(u_0,\ \theta_x) = (u_{mn}, \theta_{xmn})\cos\alpha x\ \sin\beta y$$
$$\left(v_0,\ \theta_y\right) = \left(v_{mn}, \theta_{ymn}\right)\sin\alpha x\ \cos\beta y \quad (9)$$
$$(w_0,\ \theta_z) = (w_{mn}, \theta_{zmn})\sin\alpha x\ \sin\beta y$$

where, $u_{mn}, \theta_{xmn}, v_{mn}, \theta_{ymn}, w_{mn}, \theta_{zmn}$ are the unknown coefficients. The expression for the transverse sinusoidal load is expressed as,

$$q(x, y) = q_0 \sin\alpha x \sin\beta y \quad == \quad (10)$$

where $\alpha = \pi/a,\ \beta = \pi/b$, and q_0 is the maximum intensity of the sinusoidal load. Substituting Eqs. (9) – (10) into Eq. (7) and the resultant equations are expressed in matrix form as,

$$[K]\{\Delta\} = \{f\} \quad (11)$$

where $[K]$, $\{f\}$ and $\{\Delta\}$ represent the stiffness matrix, force vector and the vector of unknowns. The elements of these matrices are as follows.

$$K_{11} = -A_{11}\alpha^2 - A_{66}\beta^2, \quad K_{12} = K_{21} = -A_{12}\alpha\beta - A_{66}\alpha\beta,$$
$$K_{13} = K_{31} = \frac{A_{11}}{R_1}\alpha + \frac{A_{12}}{R_2}\alpha + B_{11}\alpha^3 + B_{12}\alpha\beta^2 + 2B_{66}\alpha\beta^2,$$
$$K_{14} = K_{41} = -C_{11}\alpha^2 - C_{66}\beta^2, \; K_{15} = K_{51} = -C_{12}\alpha\beta - C_{66}\alpha\beta,$$
$$K_{16} = K_{61} = \left[\frac{F_{11}}{R_1}\alpha + \frac{F_{12}}{R_2}\alpha + D_{13}\alpha\right], \; K_{22} = -A_{22}\beta^2 - A_{66}\alpha^2,$$
$$K_{23} = K_{32} = B_{22}\beta^3 + [B_{12} + 2B_{66}]\alpha^2\beta + \left[\frac{A_{12}}{R_1} + \frac{A_{22}}{R_2}\right]\beta$$
$$K_{24} = K_{42} = -C_{21}\alpha\beta - C_{66}\alpha\beta, K_{25} = K_{52} = -C_{22}\beta^2 - C_{66}\alpha^2,$$
$$K_{26} = K_{62} = \left(D_{23} + \frac{F_{21}}{R_1} + \frac{F_{22}}{R_2}\right)\beta,$$
$$K_{33} = -(H_{11}\alpha^4 + H_{22}\beta^4) - 2\alpha^2\beta^2(H_{12} + 2H_{66}) - 2\alpha^2\left(\frac{B_{11}}{R_1} + \frac{B_{12}}{R_2}\right) - 2\beta^2\left(\frac{B_{12}}{R_1} + \frac{B_{22}}{R_2}\right)$$
$$-\left(\frac{2A_{12}}{R_1R_2} + \frac{A_{11}}{R_1^2} + \frac{A_{22}}{R_2^2}\right),$$
$$K_{34} = K_{43} = I_{11}\alpha^3 + I_{21}\alpha\beta^2 + 2I_{66}\alpha\beta^2 + \frac{C_{11}}{R_1}\alpha + \frac{C_{21}}{R_2}\alpha,$$
$$K_{35} = K_{53} = I_{12}\alpha^2\beta + I_{22}\beta^3 + 2I_{66}\alpha^2\beta + \frac{C_{12}}{R_1}\beta + \frac{C_{22}}{R_2}\beta,$$
$$K_{36} = K_{63} = \begin{Bmatrix} -K_{13}\alpha^2 - K_{23}\beta^2 - \frac{D_{13}}{R_1} - \frac{D_{23}}{R_2} - \left(\frac{J_{11}}{R_1} + \frac{J_{12}}{R_2}\right)\alpha^2 - \left(\frac{J_{21}}{R_1} + \frac{J_{22}}{R_2}\right)\beta^2 \\ -\left(2\frac{F_{12}}{R_1R_2} + \frac{F_{22}}{R_2^2} + \frac{F_{11}}{R_1^2}\right) \end{Bmatrix},$$
$$K_{44} = -L_{11}\alpha^2 - L_{66}\beta^2 - O_{55}, K_{45} = K_{54} = -(L_{12} + L_{66})\alpha\beta, \tag{11}$$
$$K_{46} = K_{64} = \left(N_{13} - O_{55} + \frac{M_{11}}{R_1} + \frac{M_{12}}{R_2}\right)\alpha,$$
$$K_{55} = -L_{66}\alpha^2 - L_{22}\beta^2 - O_{44},$$
$$K_{56} = K_{65} = \left(-O_{44} + N_{23} + \frac{M_{21}}{R_1} + \frac{M_{22}}{R_2}\right)\beta,$$
$$K_{66} = \left(-O_{55}\alpha^2 - O_{44}\beta^2 - S_{33} + 2\frac{P_{23}}{R_2} - 2\frac{P_{13}}{R_1} - \frac{O_{11}}{R_1^2} - 2\frac{O_{12}}{R_1R_2} - \frac{O_{22}}{R_2^2}\right),$$

where

$$\left(A_{ij}, B_{ij}, H_{ij}, C_{ij}, F_{ij}\, I_{ij}\right) = Q_{ij}\int_{-h/2}^{h/2}[1.0, z, z^2, f(z), f'(z), zf(z)]\, dz;$$
$$L_{ij} = Q_{ij}\int_{-h/2}^{h/2}\{[f(z)]^2\}\, dz; O_{ij} = Q_{ij}\int_{-h/2}^{h/2}\{[f'(z)]^2\}\, dz;$$
$$\left(D_{ij}, S_{ij}, P_{ij}\right) = Q_{ij}\int_{-h/2}^{h/2}\{[f''(z)], [f''(z)]^2, [f'(z)]\}\, dz; \tag{12}$$
$$\left(K_{ij}, N_{ij}\right) = Q_{ij}\int_{-h/2}^{h/2} f''(z)[z, f(z)]\, dz;$$
$$\left(J_{ij}, M_{ij}\right) = Q_{ij}\int_{-h/2}^{h/2} f'(z)[z, f(z)]\, dz;$$

Numerical Result and Discussion

To carry out the static analysis of the sandwich spherical shells, the following material characteristics stated in Eq. (13)-(14) and non-dimensional parameters stated in Eq. (15) are taken into account.

Face Sheet Properties

$$\frac{E_1}{E_2} = 25, \frac{E_3}{E_2} = 1, \frac{G_{12}}{E_2} = \frac{G_{13}}{E_2} = 0.5, \frac{G_{23}}{E_2} = 0.2, \mu_{12} = \mu_{13} = \mu_{23} = 0.2 \tag{13}$$

Core Properties

$$\boldsymbol{E_1 = E_2 = 0.04,\ E_3 = 0.5,\ G_{13} = G_{23} = 0.06, G_{12} = 0.016,} \atop \boldsymbol{\mu_{12} = \mu_{32} = \mu_{31} = 0.25} \tag{14}$$

and

$$\begin{gathered} \bar{u}\left(0,\frac{b}{2},\frac{z}{h}\right) = \frac{h^2 E_3}{q_0 a^3} u, \quad \bar{w}\left(\frac{a}{2},\frac{b}{2},\frac{z}{h}\right) = \frac{100 h^3 E_3}{q_0 a^4} w, \\ (\bar{\sigma}_x, \bar{\sigma}_y)\left(\frac{a}{2},\frac{b}{2},\frac{z}{h}\right) = \frac{h^2}{q_0 a^2}(\sigma_x, \sigma_y), \ \ \bar{\tau}_{xy}\left(0,0,\frac{z}{h}\right) = \frac{h^2}{q_0 a^2}\tau_{xy} \\ \bar{\tau}_{zx}\left(0,\frac{b}{2},\frac{z}{h}\right) = \frac{h}{q_0 a}\tau_{zx}, \ \ \bar{\tau}_{yz}\left(\frac{a}{2},0,\frac{z}{h}\right) = \frac{h}{q_0 a}\tau_{yz}. \end{gathered} \tag{15}$$

where, E_3 is modulus of elasticity of the middle layer i.e. the core of the sandwich shell.

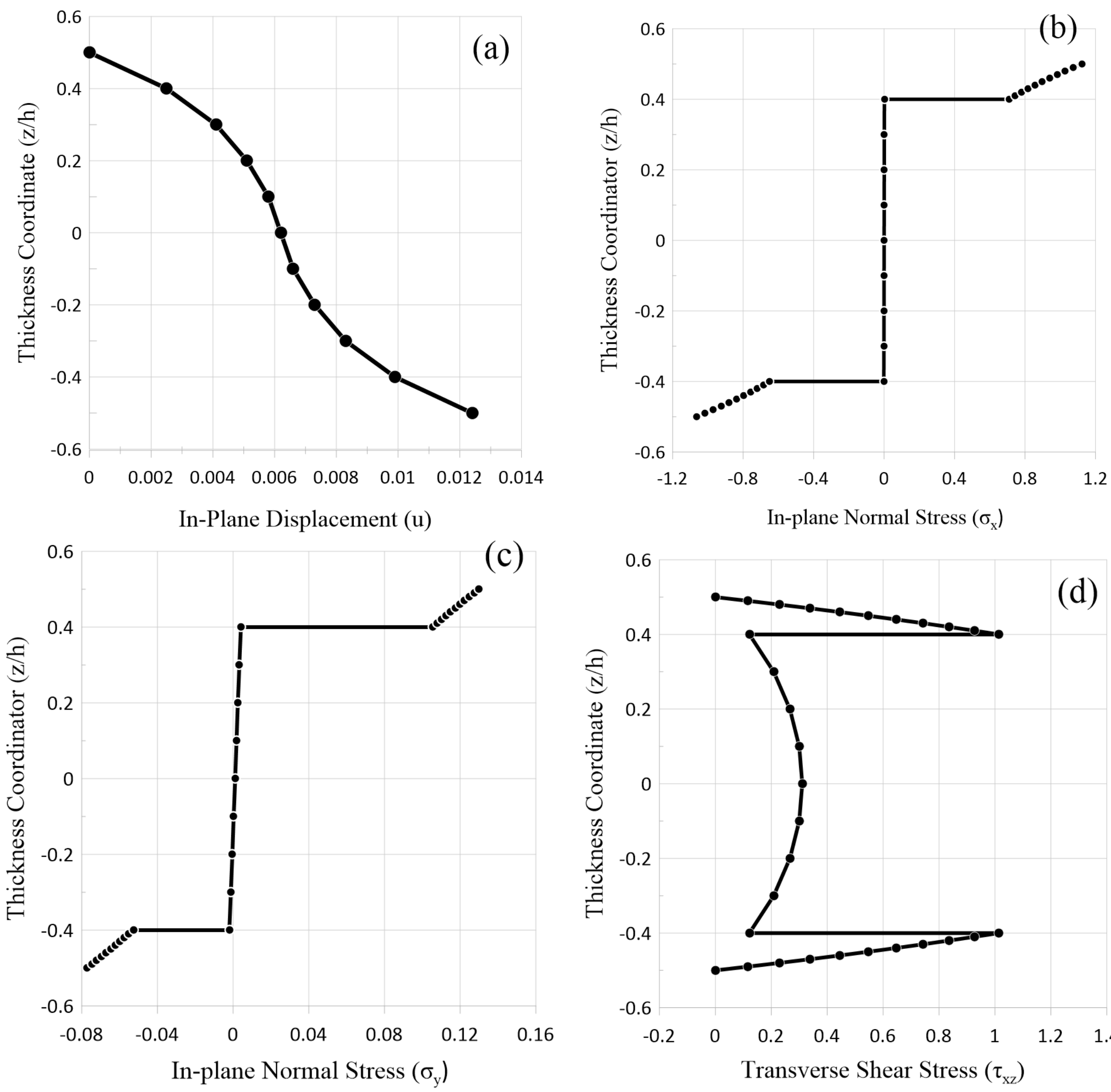
Sandwich plate consists of face sheets with thickness of $0.1h$ and the core with thickness $0.8h$ where h is the total thickness of the sandwich shells. The material properties used for the sandwich shell are mentioned in Eq. (13)-(14) whereas non-dimensional parameters are stated in Eq. (15). Table 1 shows the comparison of non-dimensional displacements and stresses obtained using the current theory with those presented by third-order theory of Reddy [10], first-order theory of Mindlin [13] and Exact elasticity solution by Pagano [12] wherever applicable. Through-the-thickness distributions of displacements and stresses are plotted in Fig. 2. The numerical results are presented for R/a = 5, 10, 20, 50, 100 and ∞. Table 1 reveals that the current theory predicts the numerical results in good agreement with existing literature. Transverse shear stresses are discontinuous at the layer interface because those are calculated using constitutive relations. To get the stress continuity at the layer interface, those must be calculated using the equilibrium equations of the theory of elasticity.

Table 1 *Non-dimensional displacements and stresses of three-layer (0^0/core/0^0) sandwich spherical shells under sinusoidal load (a = 10h, $R_1 = R_2 = R$).*

R/a	Theory	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
5	Present ($\varepsilon_z \neq 0$)	0.0124	0.9965	1.0655	0.0772	0.0926	0.3108	0.0558
	Reddy [10] ($\varepsilon_z = 0$)	0.0131	1.0063	1.0733	0.0745	0.0932	0.2956	0.0486
	Mindlin [13] ($\varepsilon_z = 0$)	0.0109	0.7122	1.0147	0.0607	0.0715	0.3096	0.0384
10	Present ($\varepsilon_z \neq 0$)	0.0099	1.0152	1.1002	0.0921	0.0805	0.3166	0.0569
	Reddy [10] ($\varepsilon_z = 0$)	0.0102	1.0250	1.1081	0.0891	0.0812	0.3011	0.0495
	Mindlin [13] ($\varepsilon_z = 0$)	0.0088	0.7215	1.0385	0.0708	0.0628	0.3137	0.0389
20	Present ($\varepsilon_z \neq 0$)	0.0085	1.0199	1.1128	0.0993	0.0739	0.3181	0.0571
	Reddy [10] ($\varepsilon_z = 0$)	0.0086	1.0298	1.1207	0.0962	0.0747	0.3025	0.0497
	Mindlin [13] ($\varepsilon_z = 0$)	0.0077	0.7238	1.0471	0.0757	0.0582	0.3147	0.0390
50	Present ($\varepsilon_z \neq 0$)	0.0076	1.0213	1.1247	0.1035	0.0698	0.3185	0.0572
	Reddy [10] ($\varepsilon_z = 0$)	0.0077	1.0312	1.1267	0.1003	0.0707	0.3029	0.0498
	Mindlin [13] ($\varepsilon_z = 0$)	0.0070	0.7245	1.0512	0.0786	0.0553	0.3150	0.0390
100	Present ($\varepsilon_z \neq 0$)	0.0073	1.0215	1.1205	0.1048	0.0685	0.3186	0.0572
	Reddy [10] ($\varepsilon_z = 0$)	0.0073	1.0314	1.1284	0.1017	0.0693	0.3029	0.0498
	Mindlin [13] ($\varepsilon_z = 0$)	0.0068	0.7246	1.0524	0.0795	0.0544	0.3150	0.0390
Plate	Present ($\varepsilon_z \neq 0$)	0.0069	1.0215	1.1220	0.1062	0.0671	0.3186	0.0572
	Reddy [10] ($\varepsilon_z = 0$)	0.0070	1.0315	1.1300	0.1030	0.0679	0.3029	0.0498
	Mindlin [13] ($\varepsilon_z = 0$)	0.0066	0.7246	1.0535	0.0805	0.0534	0.3151	0.0390
	Exact [12]	0.0071	1.1002	1.1518	0.1098	0.0706	0.2997	0.0526

Conclusions

In this work, a new hyperbolic higher-order shell theory is used to obtained closed-form solutions for the static analysis of a sandwich spherical shell under sinusoidal loading. To take into account the effect of cross-sectional deformation, the theory is formulated by adding hyperbolic kinematic function in terms of thickness coordinates. Using the principle of virtual work, the governing equation and the associated boundary condition are obtained. Navier's solution method is used as solution technique. Based on the numerical results and discussion it is concluded that the present theory accurately predict the static response of sandwich spherical shells under transverse loading.



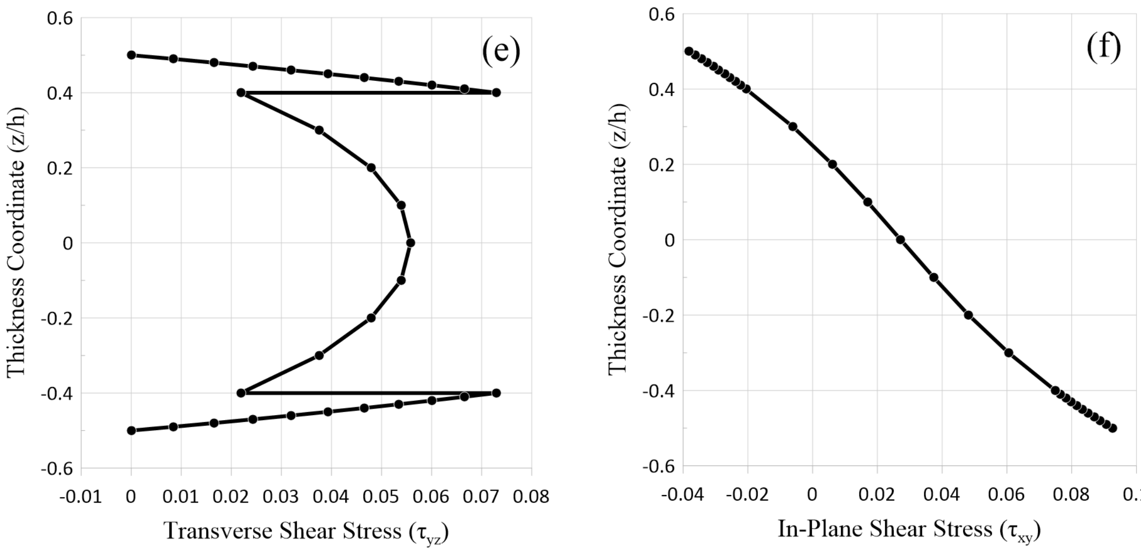


Fig. 2 Through-the-thickness plots of displacements and stresses (R/a = 5, a/h = 10)

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