Determination and experimental verification of the relation between stress amplitude and vibration amplitude in the VHCF regime

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Abstract. Fatigue tests conducted on ultrasonic machines are a relatively new testing method. The operation of test rig at a load frequency of 20 kHz and the numerical way of determining the relationship between the stress amplitude and an indirect method for determining the strain in the middle part of the sample causes researchers to feel uncertain about the stress value in the specimens. Purpose of this paper is to present experimental verification of the relationship between stress amplitude and vibration amplitude in the regime of Very High Cycle Fatigue, also the calibration procedure of the ultrasonic machine is demonstrated. The paper presents the methodology based on FEM that is used to determine the relationship between the stress amplitude in the smallest cross-section of the sample and the vibration amplitude for selected geometry.

Introduction

In the process of constructing machines and structures, it is necessary to take into consideration the issues related to material fatigue. The importance of this phenomenon had been already recognized in the early 19th century, while Wöhler's first experimental work appeared in the second half of that century. So far, this work, primarily due to research capabilities, has focused on low-cycle fatigue (LCF) and high-cycle fatigue (HCF) [1]. The least recognized is the range above 10^7 cycles, the so-called Very High Cycle Fatigue (VHCF) regime.

Until the late 1990s, the main problem with VHCF research was its time-consuming nature and the resulting high cost of research. This was due to the relatively low frequencies achieved by conventional testing machines. Therefore, attempts were made to develop devices that would allow work on much higher load frequencies. In 1950s, Mason [2,3] developed a piezoelectric transducer that converts a 20 kHz electrical signal into a mechanical wave of the same frequency. This solution was tried to be used in fatigue tests but controlling the device at such high frequencies was a significant obstacle. Only the appearance of efficient computers at the end of the 20th century allowed the development of an efficient research system that reduced the time of fatigue tests by about 1000 times in comparison to conventional machines. The development of a computer control system for ultrasonic fatigue testing machines by Bathias, Wu, and Ni [4] can be considered as the moment when commercially viable VHCF research has begun.

In constructions such as car and marine engines, turbine components, high-speed railway drive elements [5] and helicopter speed reducers [6] fatigue life is greater than 10^7 cycles. For testing of this type of elements, time reduction of a single test and the relatively low energy requirements make ultrasonic fatigue testing the only economically reasonable testing method for VHCF. An ultrasonic fatigue testing system uses the phenomenon of resonance to generate stresses in the specimen. The relationship between the stress amplitude in center section of specimen for ultrasonic testing and the vibration amplitude is linear, and its determination, for each specimen design is necessary to define the stress during the test controlled by the vibration amplitude.

The aim of this paper is to present methodology of determining, using FEM, the relationship between the stress amplitude in the smallest specimen cross-section and the vibration amplitude for selected materials and geometries. The methodology of calibration of the ultrasonic fatigue machine is presented. The vibration amplitude values obtained during the calibration will allow to determine the machine's operating range, while the values obtained during the experimental verification will allow to determine the correctness of the relationship between the stress amplitude and the vibration amplitude.

Test station and method

In an ultrasonic machine (Fig. 1), a 20 kHz electrical signal produced by a generator is converted into a mechanical wave of the same frequency in a piezoceramic transducer, the amplitude of the vibration of this wave is amplified through a booster and high gain sonotrode forces the specimen to vibrate. Specimen should be designed so that its natural frequency is 20 kHz. During real-time testing, a displacement sensor measures the vibration amplitude of the specimen.

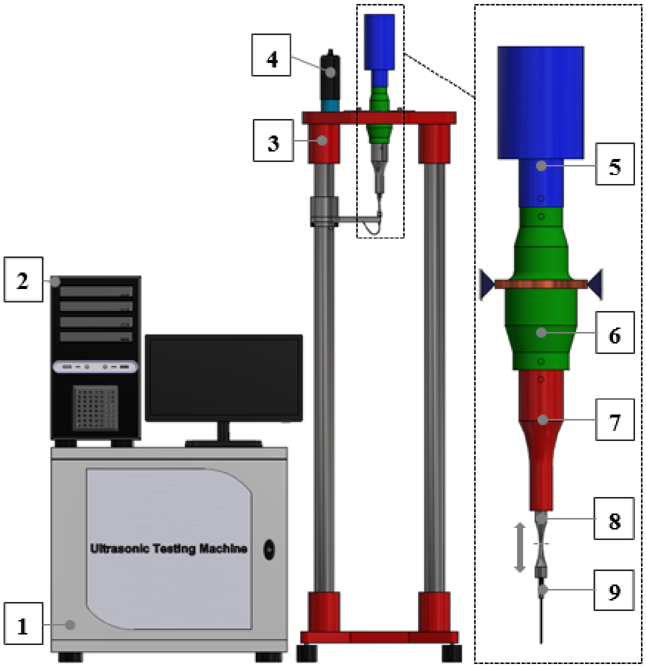


Fig. 1. Ultrasonic testing machine: 1 –generator 2.2 kW, 2 – PC unit, 3 – frame, 4 – air cooler, 5 – piezo-electric transducer, 6 – booster, 7 – sonotrode, 8 – specimen, 9 – displacement sensor

Calibration

To conduct tests on ultrasonic testing machine, it is necessary to calibrate it. Calibration allows you to determine the relation of the change in the supply voltage of the piezoceramic transducer to the amplitude of the displacement. The simplest way to perform calibration is to make a calibration rod in the form of a straight rod. To design the rod, analytical formulas can be used to define its length *l:*

$$l = \frac{1}{2f}\sqrt{\frac{E}{\rho}}\ , \tag{1}$$

where: f – frequency, E – Young modulus, ρ – density.

Analyzing the formula above, it can be seen that with increasing sample vibration frequency, the length of the sample decreases. The value of the loading frequency of 20 kHz used in most ultrasonic machines is not accidental. At 20 kHz, the specimen length dimension for metallic materials does not cause fabrication problems.

Using the analytical formulas for a simple cylindrical bar, it is possible to determine the values of displacements u, strains ε and stresses σ:

$$c = \sqrt{\frac{E}{\rho}}, \tag{2}$$

$$k = \frac{\pi}{l}, \tag{3}$$

$$\omega = \frac{\pi c}{l}, \tag{4}$$

$$u(x) = A_0 \cos(kx), \tag{5}$$

$$\varepsilon(x) = -kA_0 \sin(kx), \tag{6}$$

$$\sigma(x) = -EkA_0 \sin(kx), \tag{7}$$

where: A_0 – maximum displacement amplitude.

Fig. 2. shows examples of displacement and stress distributions along the length of a straight bar of length l.

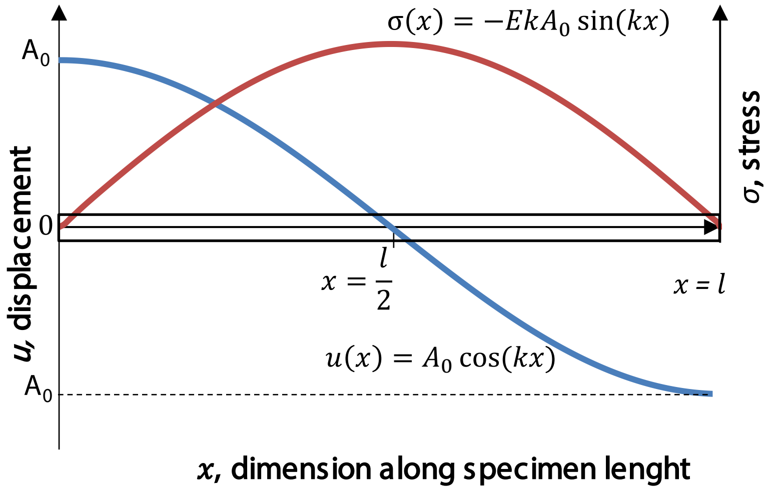


Fig. 2. Displacement and stress variation in a cylindrical bar

As it is shown above, the maximum stress of the specimen are obtained in its central section. Due to the fact that during the vibrations of the specimen, there is in a second mode of natural vibrations, then at both its ends there is an equal displacement amplitude. Using this information calibration can be made by fixing the rod in the system and using a high-resolution displacement sensor to determine the relation of the supply voltage on the amplitude of the displacement at the end of the rod. During calibration, it is important that the range of achievable vibration amplitude does not go beyond the elastic range of the material [7]. Fig. 3 shows the relationship of the supply voltage to the vibration amplitude. From the relation we can also read the range of vibrations in which the ultrasonic system is able to operate. Knowledge of the minimum and maximum amplitude of vibration, is essential to properly design a specimen for fatigue testing.

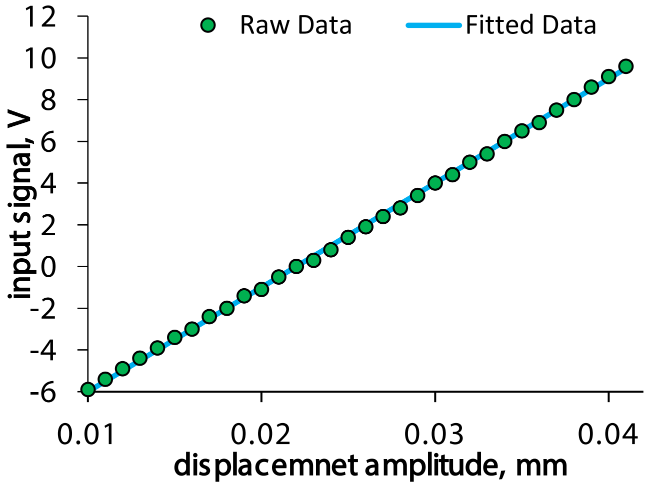


Fig. 3. Calibration curve of the ultrasonic machine presenting relation between the input voltage signal (V) to displacement amplitude (mm)

Properly performed calibration of the device enables testing. The procedure for conducting ultrasonic tests can be divided into 3 basic stages:

a) preliminary tests aimed at identification of basic properties of the tested material,
b) specimen geometry analysis using the finite element method (FEM),
c) main ultrasonic fatigue test.

The procedure is shown in Fig. 4.

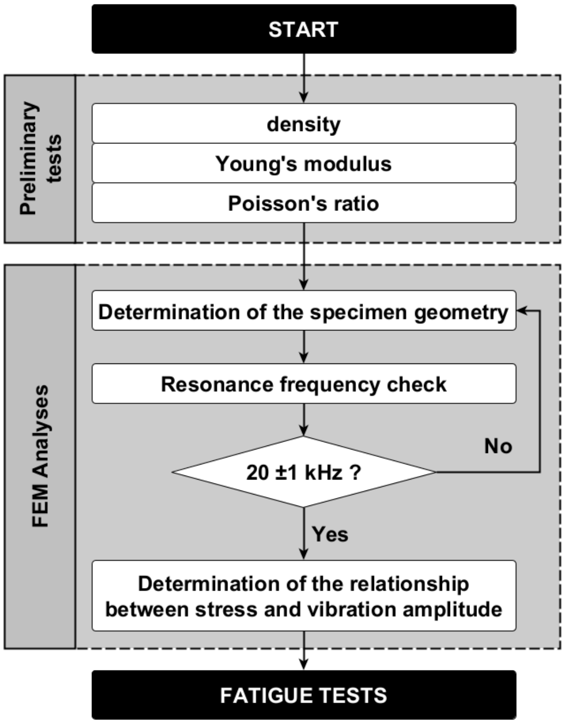


Fig. 4. Flow diagram of the test procedure

Preliminary tests include the determination of three basic strength parameters: Young's modulus (E), Poisson's ratio (v), density (ρ).

In the case of used specimen with a variable cross-section, the value of the stress amplitude is difficult to determine analytically. Therefore, in practice, the finite element method (FEM) is used for this purpose. As part of the second stage, a modal analysis is carried out using the FEM software, including the verification of the resonant frequency for the specimen geometry and the determination of the coefficient ks, which is the relationship between the vibration amplitude A_0 and the stress amplitude σ_a generated in the sample under the influence of vibration:

$$ks = \frac{\sigma_a}{A_0}. \tag{7}$$

The coefficient ks determined numerically is implemented into the test system, which after calibration calculates what voltage should be applied to the piezoceramic transducer to obtain the required vibration amplitude. In the last step of the procedure, proper fatigue tests are carried out.

When making specimens for ultrasonic testing, high quality machining is extremely important, as even a small error in geometry or too much roughness can disrupt its vibration, change its natural frequency and affect the test result. It is also important to verify that the ks factor has been determined correctly. Therefore, experimental verification of numerically determined stresses is necessary before fatigue testing.

Experimental measurement

Measurement system. On the test specimens, the strain gauge was glued at the location of the highest stresses, i.e. in the middle part of the reduced section with the smallest diameter. HBM 1-LY41-1.5/120 strain gauges with a gauge factor $k = 2$ were used in the tests.

Unfortunately, most strain gauge amplifiers have a sampling frequency value up to 20 kHz and a 20 kHz low-pass filter. Therefore, it was decided to build its own strain gauge amplifier to allow amplification in the bandwidth greater than 20 kHz. The correct operation of the circuit was checked using the function generator, which was given an electrical signal with a sinusoidal waveform, frequency 20 kHz and amplitude 0.5 V. The recorded output signal was compared with the result of the simulation of the circuit from the LTspice XVII program. The individual elements of the measurement system and the process of verifying the correct operation of the circuit are shown in Figure 5.

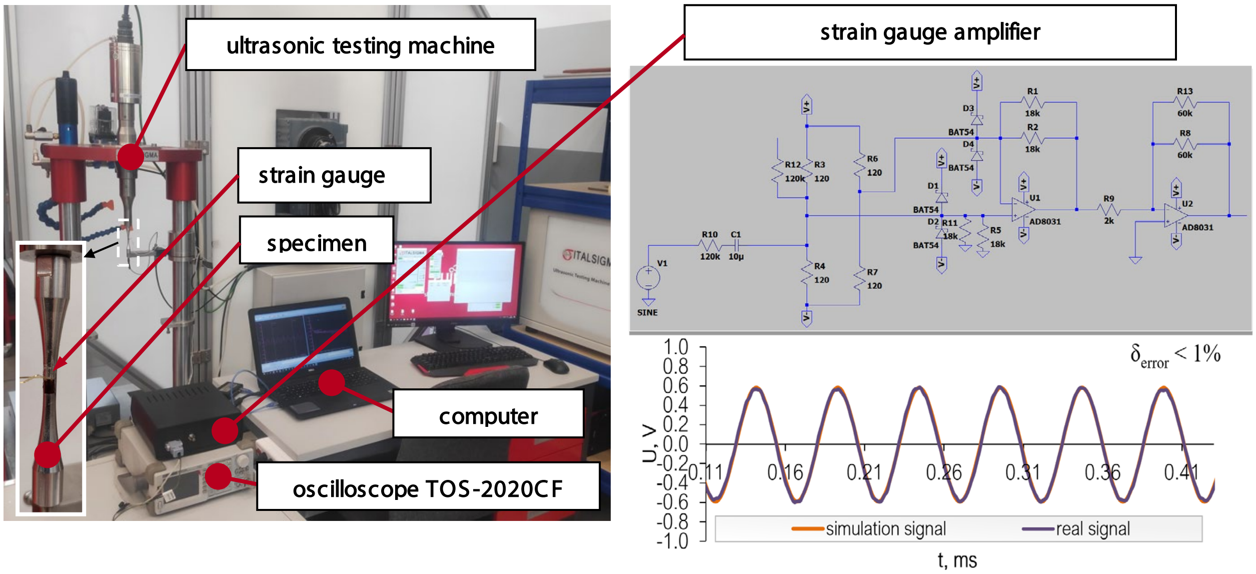


Fig. 5. Measurement system and result of circuit verification

To determine the stress amplitude σ_{atens}, the presented formula was used:

$$\sigma_{atens} = \frac{U_{pp} \cdot C_a \cdot E}{2 \cdot U_s}, \quad (8)$$

where: U_{pp} – peak to peak voltage, U_s – supply voltage, C_a – calibration constant.

Testing materials. Specimens made of two different materials and geometries were used. The material properties are presented at Table 1.

Table 1. Basic material properties of the specimens

Material	Young's modulus, E	Poisson's ratio, v	Density, ρ
	GPa	-	kg/m^3
structural steel, S355J2+N	197.27	0.27	7820
aluminum alloy, 7075 T6	72.00	0.32	2800

The geometry of the specimens (Fig. 6) and the stress values for vibration amplitude A_0 = 15 μm were determined in ABAQUS software (Fig. 7). Values of the *ks* coefficient were determined for each of the specimen using the formula (7).

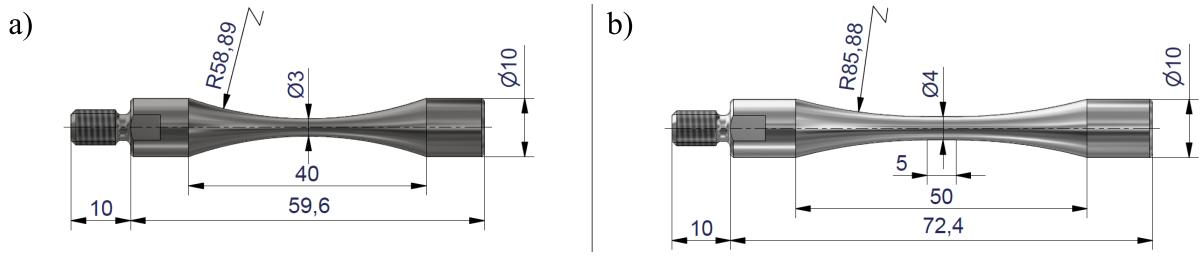
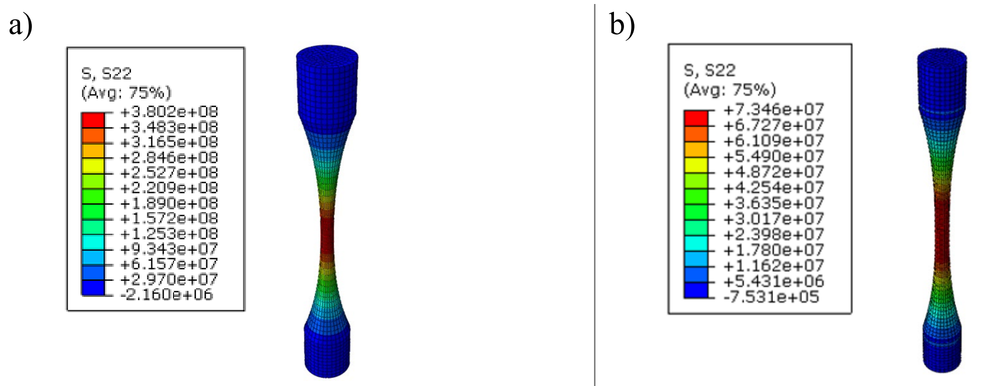


Fig. 6. Main dimensions of the specimens in mm: a) S355J2+N, b) 7075 T6



ks = 20541 MPa/mm

ks = 4867 MPa/mm

Fig. 7. Stress distribution in the axial direction of the specimens and the values of the ks coefficient: a) S355J2+N, b) 7075 T6

Results and discussion

During the tests, each specimen was put into vibration with 3 different amplitude levels. The recorded waveforms of strain changes ε at time t for S3555J2+N steel and for 7075 T6 aluminum alloy are shown in Figure 8.

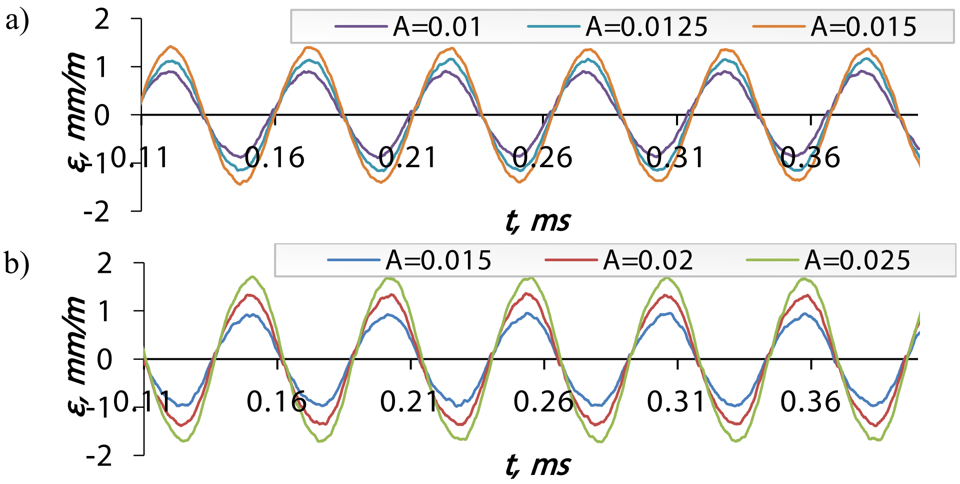


Fig. 8. Changes in strain values for individual vibration amplitudes of steel (a) and aluminum alloy (b) specimen

The specimens were loaded for a short time to minimize the risk of strain gauge damage. Stresses for given vibration amplitudes were calculated using formula (7) and (8). The results and percent error are presented in Table 2. The results in the form of a diagram are shown on the Fig. 9.

Table 2. Basic material properties of the specimens

Parameters	S355J2+N			7075 T6		
	I	II	III	I	II	III
A_0, mm	0.015	0.02	0.025	0.01	0.0125	0.015
σ, MPa	73.01	97.34	121.68	205.41	256.76	308.12
σ_{tens}, MPa	72.75	96.59	119.84	197.29	248.45	303.20
$\delta = \frac{\sigma - \sigma tens}{\sigma} \cdot 100, \%$	0.35	0.77	1.51	3.95	3.24	1.59

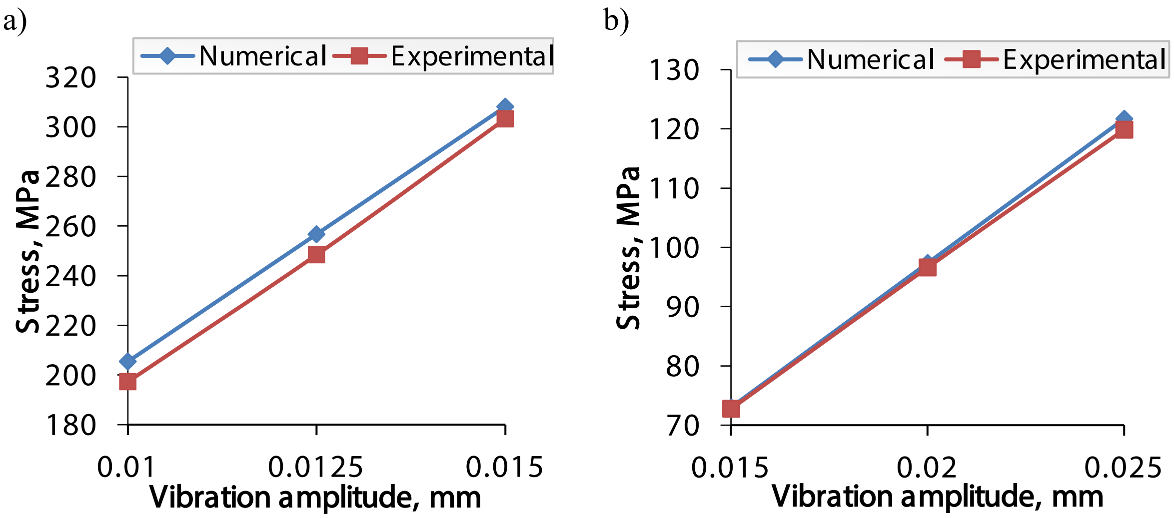


Fig. 9. Vibration amplitude as a function of stress determined in the experiment and numerical calculations for: a) S355J2+N, b) 7075 T6

The results of the stress from the experimental measurements for steel have a small percentage mistake. The designed steel specimen has an hourglass shape, and the maximum stress occur in the center of the specimen, so probably the apparent difference in stress value is due to averaging it over the base length of the strain gauge. To verify this, the change in strain along the length of the test specimens for A_0 = 15 µm was determined using FEM software (Fig. 10). For aluminum, the experimental results and numerical results were very similar. The specimen in the central part has a cylindrical section of 5 mm, which causes a uniform distribution of strain along this length, so that averaging over the length of the strain gauge does not have such an impact on the result.

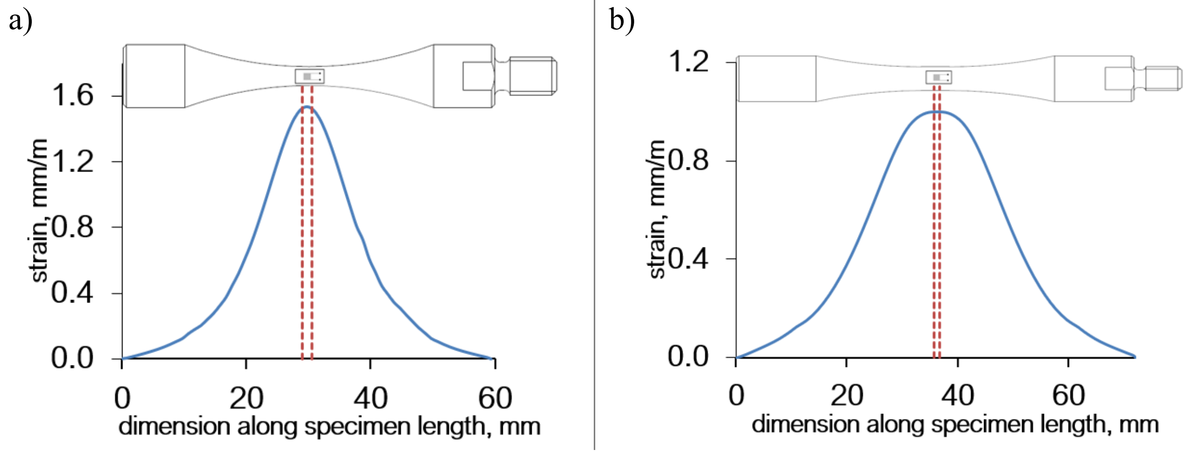


Fig. 10. Dependence of strain changes along the axis of the specimen for: a) S355J2+N, b) 7075 T6

Conclusion

The presented work is a part of the verification and validation of the research procedure implemented in the Laboratory for Research on Materials and Structures in the Bydgoszcz University of Science and Technology. To sum up the above work:

1. The conducted tests confirmed that the specimens were prepared correctly and the determined *ks* relations are correct.
2. Proper calibration of the ultrasonic machine is necessary to carry out fatigue tests.
3. The presented strain gauge system allows measurement of strain for frequencies of 20 kHz.
4. Measuring the strain of hourglass-shaped specimens using the strain gauge method, the accuracy of the result decreases due to their averaging over the length of the strain gauge base. It should also be remembered that for strain gauge measurements it is necessary to apply the strain gauge precisely in the proper location.

References

[1] Kocańda S., Szala J., Basic of fatigue calculation. Polish Scientific Publishers PWN, Warsaw 1997.

[2] Mason W.P., Piezoelectric crystals and their application to ultrasonics. Van Nostrand, New York 1950.

[3] Mason W. P., Internal Friction and Fatigue in Metals at Large Strain Amplitudes, J Acoust Soc Am, Vol. 28, No. 6, p. 1207, 1956. https://doi.org/10.1121/1.1908595

[4] Bathias C., Ni J., Wu T., An Automatic Ultrasonic Fatigue Testing System for Studying Low Crack Growth at Room and High Temperatures. STP 13973S, ASTM International, 1994.

[5] Bathias C., Paris C., Gigacycle fatigue in mechanical practice. New York, Marcel Dekker, 2004. https://doi.org/10.1201/9780203020609

[6] Shaniavski A. A., Skvortsov G. V., Crack growth in the gigacycle fatigue regime for helicopter gears, Fatigue Fract Eng Mater Struct, Vol. 22, No. 7, pp. 609–619, 1999. https://doi.org/10.1046/j.1460-2695.1999.00188.x

[7] Swacha P., Lipski A., Cracking of S355J2+N Steel in the High-Cycle and Very-High-Cycle Fatigue Regimes, Int J Fatigue, p. 107388, 2023. https://doi.org/10.1016/j.ijfatigue.2022.107388