

Combined material model to predict flow curves of cold forging raw materials having high strain hardening exponent

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Abstract. In order to increase the accuracy of cold forging simulations, flow curves obtained by experimental compression tests are used instead of the material models existing in the software library. The parameters of Ludwik material model were determined with respect to the constructed experimental flow curves at different temperatures and strain rates. Then, the flow curves were defined into the software by using these parameters. While Ludwik model can represent the material flow curve with high accuracy at low plastic strain values, the error rate between the experimental flow curve and the Ludwik model increases at high plastic strain values. Voce material models were known to predict the flow curve of materials with high strain hardening exponents more accurately, especially at high temperature and strain values. In this study, the performance of Ludwik material model was compared to four Voce material models given in the literature and a more accurate combined material model was defined for each flow curve at different temperature and strain rates for 42CrMoS4 material. All experimental flow curves were predicted with a minimum R^2 of 0.99 and the lowest mean absolute error value with the new combined material model.

Introduction

The accuracy of cold forming process simulations using finite element analysis (FEA) software depends on the true stress -plastic strain curves used. In cases where plastic deformation is high, the stress and strain distributions are not uniform, that is, they do not show a certain trend. For this reason, it is difficult to predict the flow curves of steel materials with a high strain hardening exponent, where high plastic strain values are observed in the flow curves, using a single material model. Defining the flow curves of such materials separately according to the deformation transitions increases the accuracy of the prediction models. Plastic deformation transitions in flow curves can be listed as follows: i) the part from the beginning of plastic deformation to the maximum compression stress value, ii) after the maximum compression stress value, the material exhibits strain softening part until the actual rupture of the material. In the literature, there are models such as Hollomon [1], Ludwigson [2], Ludwik [3], Swift [4] and Voce [5] that consider the work hardening rate for material flow curve prediction. However, these models are insufficient to represent the hardening and softening behavior of materials at high plastic strain values. Guo et al. [6] suggested a model that includes both the hardening and softening phases by combining the Voce Model for Hardening [5] and a linear softening model (Voce Model with Linear Softening). The developed model was able to predict the flow stress over a wide plastic strain range. Nguyen [7] has proposed a combination of Voce hardening and Voce softening models. On the other hand, Rotpai et al. [8] proposed a new piecewise model for flow curve prediction at room and elevated temperatures in order to minimize the deviation between the predicted flow curves and the experimental flow curves. A Swift-Voce model to describe the large deformation behavior of 7050-T7451 aluminum alloy under uniaxial stress, notched stress, and pure shear operations was developed by Cao et al [9]. The combination of Swift model and the 4th order polynomial was



also proposed in order to describe the large deformation behavior of the Ti-6Al-4V alloy in the same study. In addition to predicting material flow curves with mathematical models, there are also studies carried out to predict flow curves with machine learning or regression models [10,11]. An artificial neural network model was suggested by Kocatürk et al. [10] to estimate experimental flow curves of a medium carbon steel material at different temperatures and strain rate values, and the flow curve predictions with high accuracy were obtained with this method. In another study, Aydın et al. [11] proposed a model that can obtain true stress-strain curve from experimental compression test data. Moreover, machine learning models and various regression models were used to predict the flow curves for the intermediate temperature and strain rate values where experimental flow curves are not available and promising flow curves were estimated. In this study, flow curves at different temperatures and strain rates, which were constructed by using the compression test results of a medium carbon alloy steel material, were employed. The parameters of Ludwik Model, Voce Model for Hardening [7], Voce Model for Softening [5], Voce Model with Linear Softening [2] and Voce Model for Hardening and Softening [5] were determined using the "curve_fit" function in the Scipy library on the Python programming language for every temperature and strain rate values. Voce Model for Hardening and Softening best-predicted the flow curve up to maximum compression stress point while Voce model with linear softening and Ludwik Model best-predicted the part from maximum compression stress to the maximum loading point. A new combined material model based on plastic strain subintervals was proposed for the flow curves at different temperatures and strain rates.

Materials and Method

The flow curves were obtained from the experimental compression test results of 42CrMoS4 medium carbon alloy steel material. To obtain the flow curves from the experimental compression test data of the materials, the model suggested by Aydın et al. [11] was used. Compression tests were carried out at different temperatures and strain rates in accordance with ASTM E9 standard [12]. Compression tests were performed with a ZWICK universal tensile/compression testing machine for temperatures at 25, 100 and 200°C and strain rates of 0.001 and 0.275 s⁻¹. Before each compression test, MoS₂ based lubricant was used for test plates in order to minimize the friction effects. A model was developed in the Python programming language to compare the performance of the material models used to predict the experimental flow curves obtained. In this model, the material models are defined firstly, then the model coefficients that give the best prediction are determined with the curve fitting method according to the defined plastic strain intervals, and the performances of the models are reported according to the coefficients obtained. The determined coefficients, R² and mean absolute error (MAE) were used for the performance comparison of the obtained flow curves. Functions in the Numpy library were used to calculate the performance criteria. The "curve_fit" function in the Scipy library was used to find the model parameters according to the experimental flow curves. Finally, the Matplotlib library was used to plot the resulting flow curves. The material models used to predict flow curves are defined below. In these models, σ represents the true stress, ε the true plastic strain, n is the hardening exponent, and $\sigma_0, \beta, K, A, B, C, D, k$ represent the material coefficients, respectively.

Ludwik model [9]:

$$\sigma = \sigma_0 + K\varepsilon^n \quad (1)$$

Voce hardening model [12]:

$$\sigma = \sigma_0(1 - Ae^{-\beta\varepsilon}) \quad (2)$$

Voce softening model [12]:

$$\sigma = \sigma_0(1 - Ae^{\beta\epsilon}) \quad (3)$$

Voce linear softening model [8]:

$$\sigma = \sigma_0(1 - Ae^{-\beta\epsilon}) - k\epsilon \quad (4)$$

Voce hardening and softening model [3]:

$$\sigma = \sigma_0 + A(1 - e^{-B\epsilon}) + C(1 - e^{D\epsilon}) \quad (5)$$

Comparison of Ludwik and Voce Models

In this section, the flow curve prediction performances of Ludwik and Voce material models in the true plastic strain range of $[0, 1.8]$, which is the whole plastic strain range for 42CrMoS4 material, were compared for 3 different temperatures and 2 different strain rates. The results obtained are given in Table 1. In order to compare the models fairly, the "curve_fit" function in the Scipy library is used with initial parameter values for all models. The Voce Hardening and Softening model best predicted the flow curves at all temperatures and strain rate value of 0.001 s^{-1} with a minimum R^2 value of 0.97. For 25°C and 0.275 s^{-1} strain rate, the Voce Linear Softening model best predicted the flow curve with $0.97 R^2$, while the Ludwik model best predicted the flow curves at 100 and 200°C , 0.275 s^{-1} strain rate with 0.65 and $0.78 R^2$, respectively. Flow curve predictions obtained at 100°C and strain rate values of 0.001 and 0.275 s^{-1} were shown in Fig. 1 for the plastic strain range of $[0, 1.8]$.

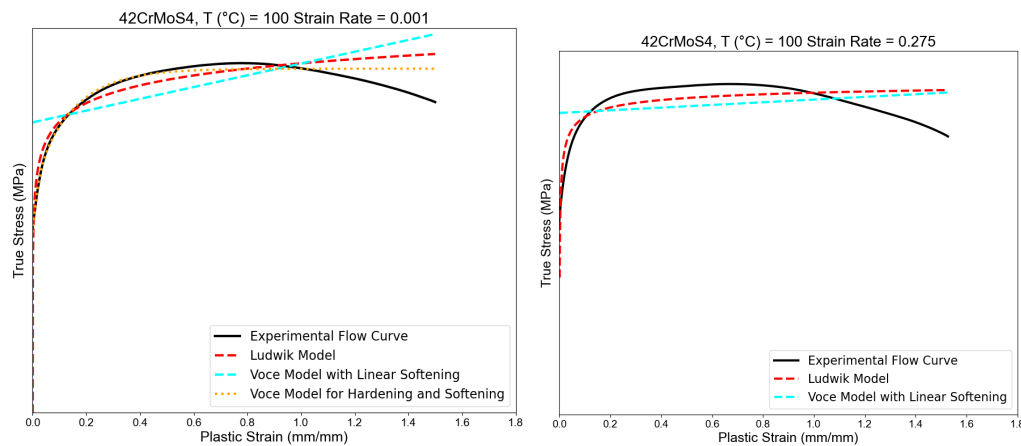


Fig. 1. Flow curve predictions in the plastic strain range of $[0, 1.8]$ for 100°C , 0.001 and 0.275 s^{-1} strain rates.

Table 1. Model comparison for plastic strain range of $[0, 1.8]$.

Temperature (°C)	Strain Rate	Model Name	Pl. Str. Interval	R ²	MAE
25	0.001	Ludwik	0.0-1.8	0.900	1117751
25	0.001	Voce with Linear Softening	0.0-1.8	0.440	2825240
25	0.001	Voce with Hardening and Softening	0.0-1.8	0.980	446045
25	0.275	Ludwik	0.0-1.8	0.600	9942
25	0.275	Voce with Linear Softening	0.0-1.8	0.970	2822
100	0.001	Ludwik	0.0-1.8	0.870	1019870
100	0.001	Voce with Linear Softening	0.0-1.8	0.390	2332653
100	0.001	Voce with Hardening and Softening	0.0-1.8	0.970	438462
100	0.275	Ludwik	0.0-1.8	0.650	7555
100	0.275	Voce with Linear Softening	0.0-1.8	0.050	12002
200	0.001	Ludwik	0.0-1.8	0.880	893179
200	0.001	Voce with Linear Softening	0.0-1.8	0.400	2079574
200	0.001	Voce with Hardening and Softening	0.0-1.8	0.970	370809
200	0.275	Ludwik	0.0-1.8	0.780	6440
200	0.275	Voce with Linear Softening	0.0-1.8	0.190	12061

In order to obtain closer results to the experimental flow curves for 42CrMoS4 material, and to identify models that predict the hardening and softening phases during plastic deformation more accurately, the model predictions for the lower plastic strain ranges were also compared. In order to determine the model that best predicts material behavior up to the maximum compression stress value, the flow curve prediction performances of Ludwik and Voce material models in the true plastic strain range $[0, 0.6]$ were compared for 3 different temperatures and 2 different strain rates. Flow curve predictions obtained at 100 °C and strain rate values of 0.001 and 0.275 s⁻¹ were shown in Fig. 2 for the plastic strain range of $[0, 0.6]$. The results obtained for the true plastic strain range of $[0, 0.6]$ were tabulated in Table 2. The Voce Hardening and Softening model best predicted the flow curves at all temperatures and strain rate values with a minimum R² value of 0.999.

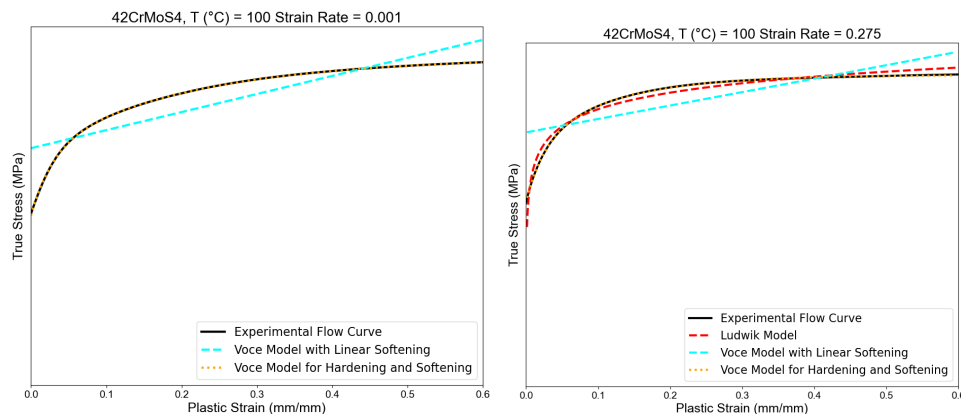


Fig. 2. Flow curve predictions in the plastic strain range of $[0, 0.6]$ for 100°C, 0.001 and 0.275 s⁻¹ strain rates.

Table 2. Model comparison for plastic strain range of [0, 0.6].

Temperature(°C)	Strain Rate	Model Name	Pl. Str. Interval	R ²	MAE
25	0.001	Voce with Linear Softening	0.0-0.6	0.800	1121473
25	0.001	Voce for Hardening and Softening	0.0-0.6	0.999	9083
25	0.275	Ludwik	0.0-0.6	0.980	1642
25	0.275	Voce with Linear Softening	0.0-0.6	0.660	6054
25	0.275	Voce for Hardening and Softening	0.0-0.6	0.999	86
100	0.001	Voce with Linear Softening	0.0-0.6	0.780	922398
100	0.001	Voce for Hardening and Softening	0.0-0.6	0.999	11076
100	0.275	Ludwik	0.0-0.6	0.970	1474
100	0.275	Voce with Linear Softening	0.0-0.6	0.630	5177
100	0.275	Voce for Hardening and Softening	0.0-0.6	0.999	87
200	0.001	Voce with Linear Softening	0.0-0.6	0.750	849120
200	0.001	Voce for Hardening and Softening	0.0-0.6	0.999	22864
200	0.275	Voce with Linear Softening	0.0-0.6	0.730	4821
200	0.275	Voce for Hardening and Softening	0.0-0.6	0.999	42

In order to determine the model that can predict the hardening and softening behavior of 42CrMoS4 material more accurately, model predictions were compared. For this purpose, the flow curve prediction performances of Ludwik and Voce material models in the plastic strain range of [0.6, 1.8] were compared for three different temperatures and 2 different strain rates. The results were given in Table 3. Flow curve predictions obtained at 100 °C and strain rate values of 0.001 and 0.275 s⁻¹ were shown in Fig. 3 for the plastic strain range of [0.6, 1.8]. The Voce Linear Softening model with a minimum R² of 0.78 obtained the best prediction for the flow curves with strain rate of 0.001 s⁻¹ and for all temperature values. The Voce Linear Softening model also obtained the best prediction with 0.92 R² for the flow curve at 200°C and 0.275 s⁻¹ strain rate. The best predictions for both flow curves at 25 and 100°C and 0.275 s⁻¹ strain rate were obtained with the Ludwik model with 0.999 R².

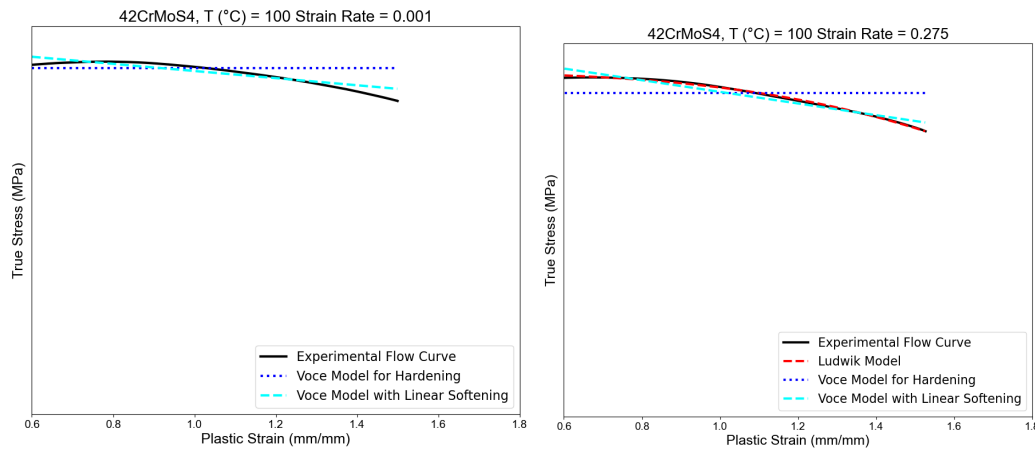


Fig. 3. Flow curve predictions in the plastic strain range of $[0.6, 1.8]$ for 100°C , 0.001 and 0.275 s^{-1} strain rates.

Table 3. Model comparison for plastic strain range of $[0.6, 1.8]$.

Temperature($^{\circ}\text{C}$)	Strain Rate	Model Name	Pl. Str. Interval	R^2	MAE
25	0.001	Voce with Linear Softening	0.6-1.8	0.850	117884
25	0.275	Ludwik	0.6-1.8	0.999	157
25	0.275	Voce for Hardening	0.6-1.8	0.000	4152
25	0.275	Voce with Linear Softening	0.6-1.8	0.980	617
100	0.001	Voce for Hardening	0.6-1.8	0.000	334923
100	0.001	Voce with Linear Softening	0.6-1.8	0.810	165377
100	0.275	Ludwik	0.6-1.8	0.999	205
100	0.275	Voce for Hardening	0.6-1.8	0.000	3482
100	0.275	Voce with Linear Softening	0.6-1.8	0.930	900
200	0.001	Voce for Hardening	0.6-1.8	0.000	298707
200	0.001	Voce with Linear Softening	0.6-1.8	0.780	159582
200	0.001	Voce for Hardening and Softening	0.6-1.8	0.000	298707
200	0.275	Voce for Hardening	0.6-1.8	0.000	2607
200	0.275	Voce with Linear Softening	0.6-1.8	0.920	734

According to the flow curve prediction results, while the Voce Hardening and Softening model best predicted the material behavior up to the maximum stress point, the Voce Linear Softening and Ludwik models best predicted the hardening and softening behaviors observed after the maximum compression stress value. In order to determine the transition points for these stages, analyzes were carried out to find the model that gives the best estimate in each interval by dividing the whole plastic strain range into 9 equal parts with a true plastic strain range of 0.2. As a result, a new combined material model was determined for each flow curve, which predicts the flow

curves very close to the experimental data. The performances of the combined models that give the best predictions at each determined subinterval of true plastic strain range for the flow curves at each temperature and strain rate value were reported in Table 4. Flow curve predictions obtained with the combined model for strain rate values of 0.001 and 0.275 at 100°C were shown in Fig. 4.

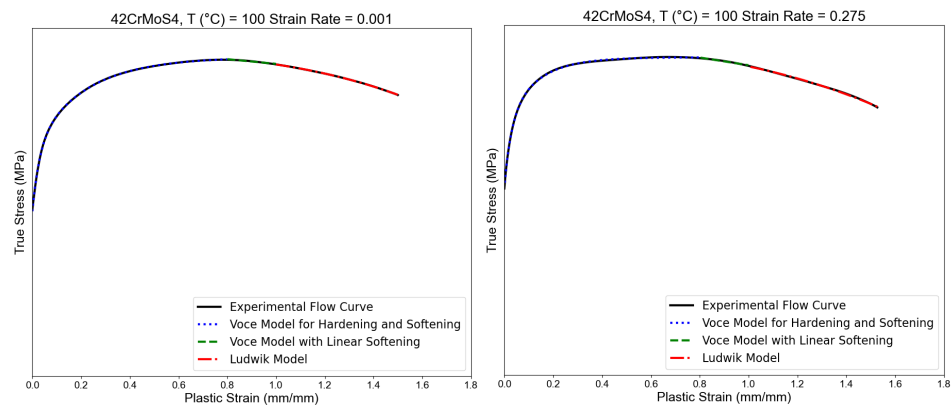


Fig. 4. Combined model flow curves of 42CrMoS4 in the plastic strain range of [0, 1.8] for 100°C, 0.001 and 0.275 s⁻¹ strain rates.

Table 4. Combined model performances for 42CrMoS4 material based on plastic strain range.

Temperature (°C)	Strain Rate	Model Name	Pl. Str. Interval	R ²	MAE
25	0.001	Voce for Hardening and Softening	0.0-0.8	0.999	44344
25	0.001	Voce with Linear Softening	0.8-1.0	0.993	1080
25	0.001	Ludwik	1.0-1.8	0.999	709
25	0.275	Voce for Hardening and Softening	0.0-0.6	0.9999	86
25	0.275	Voce with Linear Softening	0.6-1.0	0.968	119
25	0.275	Ludwik	1.0-1.8	0.999	40
100	0.001	Voce for Hardening and Softening	0.0-0.8	0.999	16048
100	0.001	Voce with Linear Softening	0.8-1.0	0.963	3838
100	0.001	Ludwik	1.0-1.8	0.999	830
100	0.275	Voce for Hardening and Softening	0.0-0.8	0.9999	145
100	0.275	Voce with Linear Softening	0.8-1.0	0.981	17
100	0.275	Ludwik	1.0-1.8	0.999	41
200	0.001	Voce for Hardening and Softening	0.0-0.8	0.999	41638
200	0.001	Voce with Linear Softening	0.8-1.0	0.946	4050
200	0.001	Ludwik	1.0-1.8	0.999	1993
200	0.275	Voce for Hardening and Softening	0.0-0.8	0.999	63
200	0.275	Voce with Linear Softening	0.8-1.0	0.987	12
200	0.275	Ludwik	1.0-1.8	0.999	14

Summary

Within the scope of the study, flow curves obtained from experimental compression test results of medium carbon alloyed 42CrMoS4 steel were used. The coefficients of both material models were determined for different temperature and strain rate values. In order to fit the curve with model equations, "curve_fit" function in the Scipy library has used on Python programming language. The performances of different material models were also examined by dividing the total plastic strain range into subintervals, and combined piecewise material models were proposed according to the true plastic strain values for each temperature and strain rate value. Flow curve prediction performances of Ludwik material model and four different Voce material models were compared. In the comparisons, the flow curve up to the maximum compression stress point was best predicted by the Voce Hardening and Softening model, while the Ludwik and Voce Linear Softening models best predicted the part after the maximum compression stress point to the breaking point. For the flow curves at different temperatures and strain rates, the new combined material model depending on plastic strain ranges were proposed by using Ludwik and four different Voce models. All experimental flow curves were predicted with a minimum R^2 of 0.99 and lower absolute error values with the new combined material model while Ludwik model predicted the flow curves with a maximum R^2 of 0.90.

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