

Micro-mechanical study of damage evolution in isotropic metallic materials

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Keywords: Damage, Finite-Element, Unit-Cell

Abstract. Virtual tests enable the expansion of the knowledge base accessible by direct experimentation. In particular, the role of microstructure on damage can be investigated using unit-cell models for porous materials. Additionally, it is of great interest to assess the role of the loading history on the response. In this paper, we present a dedicated user-defined element (U.E.L.) that was developed and implemented in the finite element (F.E.) code, ABAQUS. Verification of the capabilities of the U.E.L. is provided. The simulation results presented provide insights into the effect of J_3 on the mechanical response and porosity evolution.

Introduction

Assessment of damage of polycrystalline metallic materials is a major concern for any engineering application. Consequently, there have been intense efforts to gain understanding of the dissipative mechanisms leading to damage and ultimately failure. While advancement in diagnostic techniques e.g., X-ray tomography, has led to improved understanding of damage evolution, data is limited to a few loading paths. Numerical studies are needed to provide interpretation of data as well as for gaining insights into the damage evolution for loadings that are inaccessible to direct mechanical testing.

Beginning with the pioneering works of Needleman and collaborators (see [1]) micro-mechanical finite-element (F.E.) studies using a cylindrical unit-cell were conducted. However, for analysis of the response under full 3-D loadings of isotropic materials with randomly distributed voids in three perpendicular directions, the unit-cell should be initially cubic see [2]–[5]. For a comprehensive survey of the literature on plasticity-damage couplings for isotropic materials with matrix obeying von Mises, Tresca, and the isotropic form of the CPB criterion [6], the reader is referred to the monograph [5]. For example, for a porous material, with matrix exhibiting tension-compression asymmetry, the F.E. results obtained using cubic unit-cells showed the importance of the matrix tension-compression asymmetry on the rate of void growth and confirm the predictions of the dilatational model of Cazacu-Stewart [7]. Moreover, anisotropy and creep in porous crystals were investigated with cubic unit-cells in [4] and the results were further explained using an analytical model for porous crystals in [5]. The F.E. results provide insights on the effects of stress triaxiality and Lode parameter on void evolution for ductile single crystals in the dislocation creep.

In this paper, we present results of a micro-mechanical finite-element (FE) study on cubic unit-cells performed using the commercial code ABAQUS [8]. To be able to investigate separately the influence of the invariants of the applied stress on porosity evolution, a user element routine (UEL) has been developed. The FE unit cell model and constitutive assumption is presented in Section 2 and the UEL developed is described in Section 3. Verification and validation of the U.E.L. conducted as well as results concerning the effects of J_3 on the dilatational response for tensile loadings are presented in Section 4. Conclusions of the study are presented in Section 5.

General Numerical Setup

It is assumed that the porous solid contains a regular array of initially spherical voids. The inter-void spacing is considered to be the same in any direction. Thus, the unit cell is initially cubic with edge lengths $2 C_0$ and contains a single void of initial radius r_0 at its center (see Fig. 1 a.). The void volume fraction is defined as:

$$f = \frac{V_{void}}{V_{cell}} = \frac{V_{cell} - V_{matrix}}{V_{cell}} = 1 - \frac{V_{matrix}}{V_{cell}} \quad (1)$$

so the initial void volume fraction writes as:

$$f_0 = \frac{\pi}{6} \left(\frac{r_0}{C_0} \right)^3 \quad (2)$$

Let \mathbf{u} denote the incremental displacement between the current and reference configuration, and \mathbf{t} the prescribed Cauchy stress vector, defined on the current configuration. Symmetry conditions are imposed on the coordinate planes. To simulate the constraints of the surrounding material, we need to enforce that the faces of the unit cell, which are initially parallel to the coordinate planes, remain planes and are shear free. Given the void geometry and the symmetry conditions, only $1/8^{\text{th}}$ of the unit-cell must be analyzed. The F.E. mesh considered in the calculations presented hereafter consists of 22,000 3-D tetrahedral ABAQUS C3D10 elements and is shown in Fig. 1 b. The coordinate axes are principal directions for the stress experienced by the porous material. The normal components, $\Sigma_1, \Sigma_2, \Sigma_3$ i.e. the tractions over the current areas of the RVE faces are calculated as follows:

$$\Sigma_1 = \frac{1}{C_2 \cdot C_3} \int_0^{C_3} \int_0^{C_2} t_1 dx_2 dx_3, \Sigma_2 = \frac{1}{C_1 \cdot C_3} \int_0^{C_3} \int_0^{C_1} t_2 dx_1 dx_3, \Sigma_3 = \frac{1}{C_1 \cdot C_2} \int_0^{C_2} \int_0^{C_1} t_3 dx_1 dx_2 \quad (3)$$

C_i with $i=1...3$ denote the current cell dimensions are calculated using: $C_i = C_0 + U_i(t)$, $i=1...3$.

The overall principal strains and the von Mises macroscopic effective strain are calculated as:

$$E_1 = \ln \left(\frac{C_1}{C_0} \right), E_2 = \ln \left(\frac{C_2}{C_0} \right), E_3 = \ln \left(\frac{C_3}{C_0} \right), E_e = \sqrt{\frac{2}{3} (E_1^2 + E_2^2 + E_3^2)} \quad (4)$$

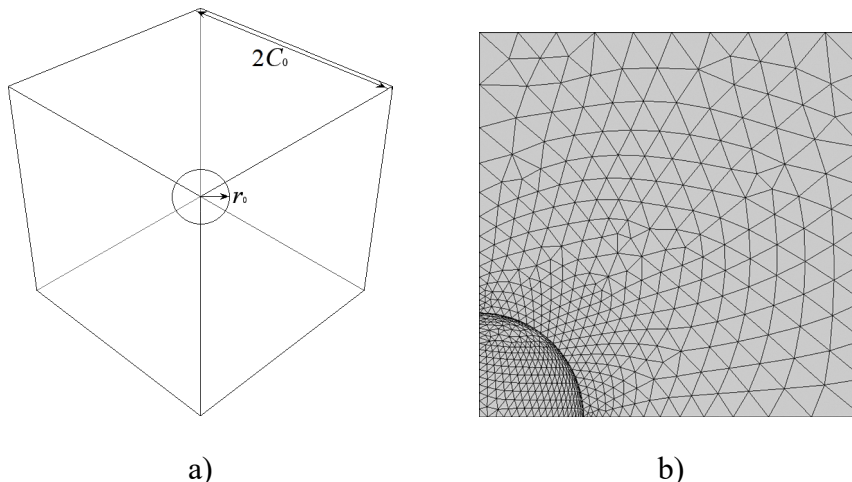


Fig. 1. Depiction of a) unit-cell with single centrally located spherical void and fully dense matrix and b) FE mesh of (1/8th) of the unit-cell.

The time histories of the displacements, $U_i(t)$, $i=1...3$ are determined by the analysis in such a way that the overall Cauchy stresses $\Sigma_1, \Sigma_2, \Sigma_3$ experienced by the porous material follow a prescribed proportional loading history. Such prescribed proportional loading are of interest for assessing the effects of the stress state (stress invariants) on the response of porous solids and damage evolution. Let us recall that for isotropic materials, the stress state effects are uniquely determined by the following stress invariants: the mean stress, $\Sigma_m = \frac{1}{3}(\Sigma_1 + \Sigma_2 + \Sigma_3)$, and the 2nd and 3rd invariants of the deviatoric stress tensor J_2 and J_3 ; the stress triaxiality is defined $T_\Sigma = \frac{\Sigma_m}{\Sigma_e}$,

$$\text{with } \Sigma_e = \sqrt{3J_2}$$

U.E.L. Subroutine

The U.E.L. is a 2-node, 6 D.O.F. ($u(i)$, $i=1...6$), truss element fixed to the R.V.E. at the corner node across the body diagonal from the void. Node 1 of the U.E.L. is the corner node of the R.V.E. and node 2 is free and receives a displacement boundary condition in the x_1, x_2 , and x_3 directions. Depending on loading, the “control direction” changes such that U.E.L. change in length is compatible with the sign of the desired applied force. The U.E.L. has 3 input parameters: stress triaxiality T_Σ , sign of third invariant S_{J_3} , and spring stiffness k_{prop} . The U.E.L. calculates the force applied to node 1 (corner node of R.V.E.) in the x_1, x_2, x_3 directions based on a displacement applied to node 2. U.E.L. change in length is calculated first, stress ratio ρ to ensure the prescribed T_Σ is calculated next, then effective spring stiffnesses in the x_1, x_2, x_3 based on the change in length and stress ratios are calculated, and finally the U.E.L. stiffness matrix and force vectors are constructed.

For example, for overall axisymmetric loading, $\Sigma_1 = \Sigma_3$ and $\Sigma_1 = \rho \cdot \Sigma_2$ with ρ constant. these steps are written out as follows:

1. Calculate the change in length in the 3 orthogonal directions:

$$\Delta u_1 = u(1) - u(4), \Delta u_2 = u(2) - u(5), \Delta u_3 = u(3) - u(6) \tag{5}$$

2. Calculate the necessary stress ratio ρ to impose prescribed stress triaxiality:

$$\rho = \frac{3T_\Sigma - S_{J_3}}{3T_\Sigma + 2 \cdot S_{J_3}} \tag{6}$$

3. Calculate effective stiffness of the 3 orthogonal springs based on load scenario:

- Control direction along x_2 :

$$k_{eff}^{(1)} = \rho k_{eff}^{(2)} \frac{A_1 \Delta u_2}{A_2 \Delta u_1}, k_{eff}^{(2)} = k_{prop}, k_{eff}^{(3)} = \rho k_{eff}^{(2)} \frac{A_3 \Delta u_2}{A_2 \Delta u_3} \tag{7}$$

- Control direction along x_1 :

$$k_{eff}^{(1)} = k_{prop}, k_{eff}^{(2)} = \frac{k_{eff}^{(1)} A_2 \Delta u_1}{\rho A_1 \Delta u_2}, k_{eff}^{(3)} = \rho k_{eff}^{(1)} \frac{A_3 \Delta u_1}{A_1 \Delta u_3} \tag{8}$$

4. Calculate the stiffness matrix of the U.E.L.
5. Calculate the force vector of the U.E.L.

The forces from the force vector are applied to the corner node of the R.V.E. producing incremental displacements $U_i(t)$. Every node on each periodic face has its respective normal displacement defined as the incremental displacement, $U_i(t)$ in the periodic boundary condition.

In the next section, we further verify and use the developed subroutine. For this purpose, we assume that the plastic behavior of the fully dense matrix is governed by the von Mises yield criterion with associated plastic flow rule, i.e. the effective stress is defined as:

$$\bar{\sigma} = \sqrt{3J_2} \tag{9}$$

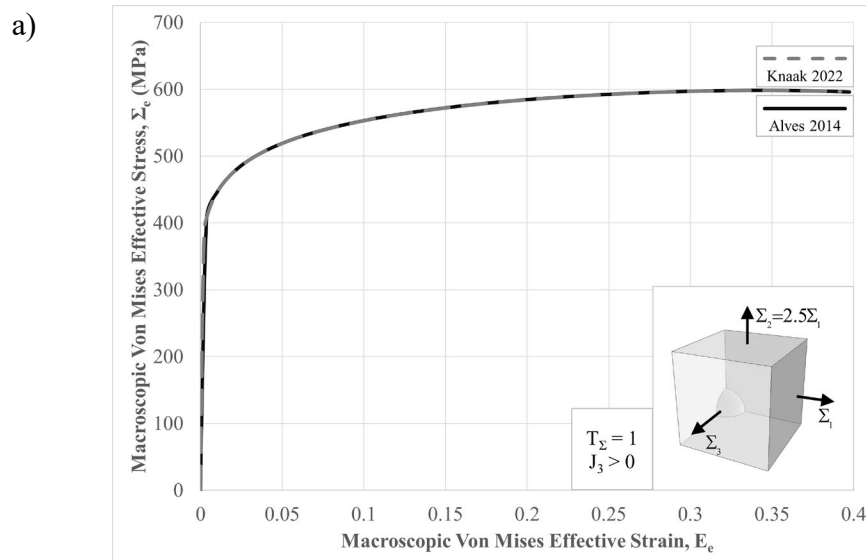
It is assumed that hardening is isotropic and governed by the equivalent plastic strain, $\bar{\epsilon}^p$ defined as the work-equivalent measure of the effective stress $\bar{\sigma}$. A Swift -type law is considered, so in the plastic regime:

$$\bar{\sigma} = A(\epsilon_0 + \bar{\epsilon}^p)^n \tag{10}$$

where A , ϵ_0 , and n are material parameters. The parameter values used in the FE calculations are as follows: elasticity parameters are Young’s Modulus $E = 200$ GPa and Poisson’s coefficient $\nu = 1/3$; the hardening parameters are: $A = 728.22$ MPa, $\epsilon_0 = 0.0025$, and $n = 0.1$. The simulations are conducted using the built-in von Mises material implementation in Abaqus [8]. The unit-cell has initial edge length $2 \cdot C_0 = 12$ mm, and initial void radius $r_0 = 1.625$ mm (see Fig.1), which correspond to an initial porosity of 1.04%.

U.E.L. Verification, Validation and Results

Tensile axisymmetric loadings corresponding to fixed stress triaxiality $T_\Sigma = 1$ were compared with results from the literature [2]. Note that the stress-strain and void evolution predictions match nearly exactly throughout the entire loading path and provide excellent agreement of the U.E.L. subroutine for tensile loadings with $J_3 > 0$ (Fig. 2) or at $J_3 < 0$ (Fig. 3).



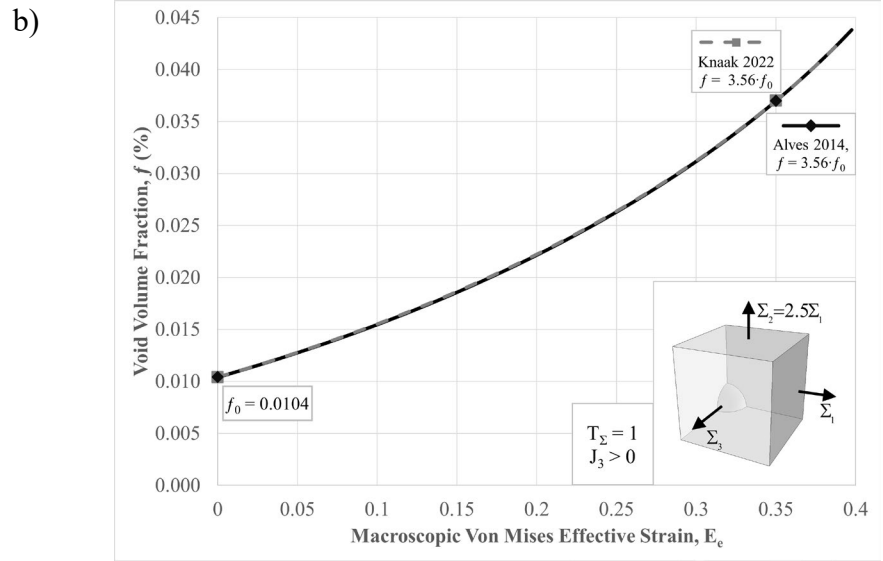
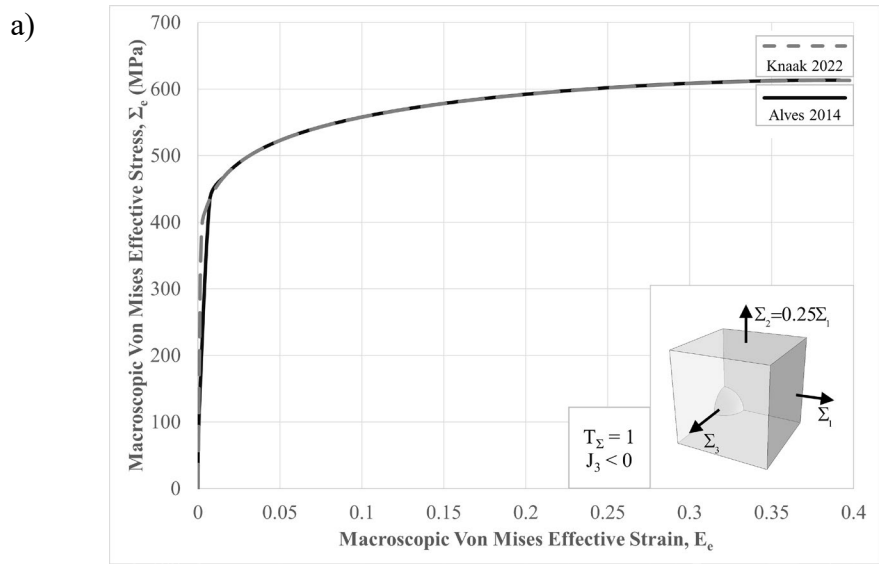


Fig. 2. Comparison of axisymmetric loading corresponding to fixed stress triaxiality $T_{\Sigma} = 1$ with $J_3 > 0$ obtained with the UEL and results in [2] for a) macroscopic effective stress-strain b) void volume fraction vs. macroscopic effective strain.



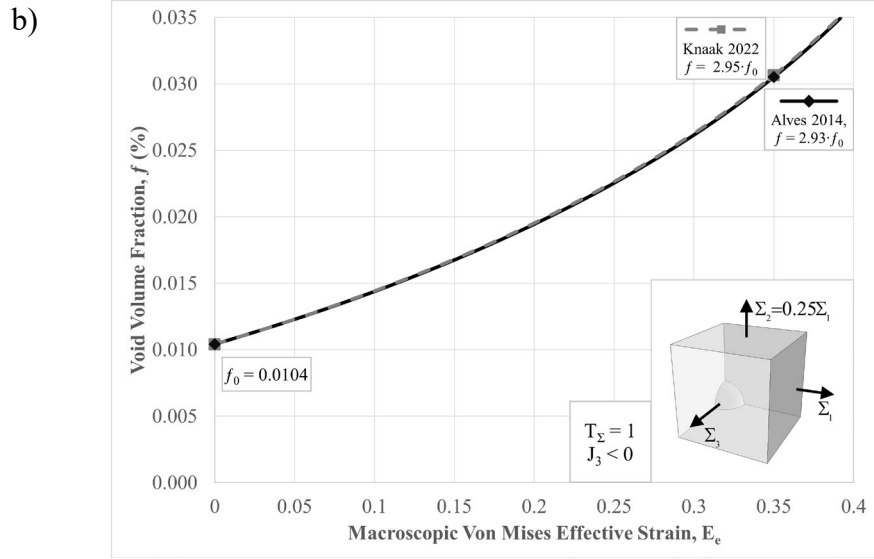
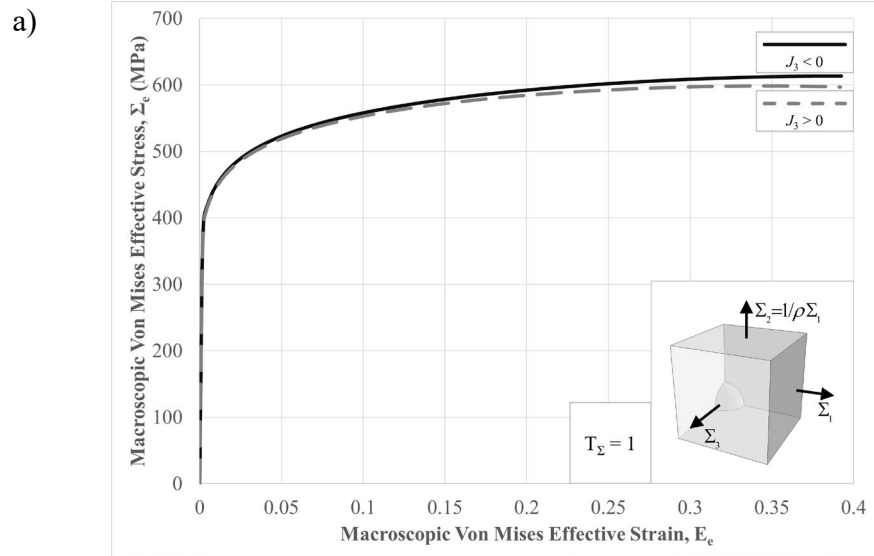


Fig. 3. Comparison of axisymmetric loading corresponding to fixed stress triaxiality $T_{\Sigma} = 1$ with $J_3 < 0$ obtained with the UEL and results in [2] for a) macroscopic effective stress-strain b) void volume fraction vs. macroscopic effective strain.

Fig. 4 shows a comparison of the stress-strain response predictions and void evolution predictions for loadings at same triaxiality $T_{\Sigma} = 1$ with $J_3 > 0$ and $J_3 < 0$, respectively.



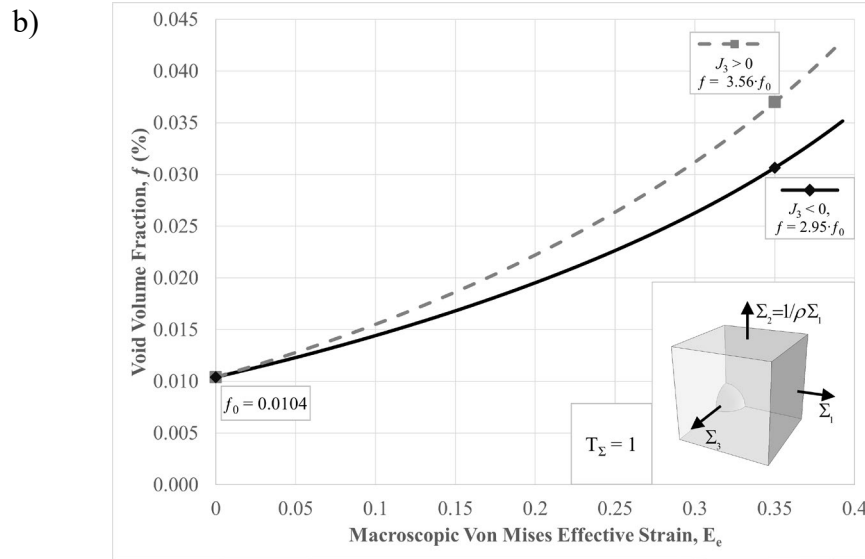


Fig. 4. Comparison of axisymmetric loadings corresponding to fixed stress triaxiality $T_{\Sigma} = 1$ with $J_3 > 0$ and $J_3 < 0$ predictions in a) macroscopic effective stress-strain response and b) void volume fraction vs. macroscopic effective strain.

Stress-strain response shows the response for loading corresponding to $J_3 < 0$ is stiffer than the response at $J_3 > 0$. This leads to a faster void growth for loading corresponding $J_3 > 0$ than the response for $J_3 < 0$. It can be concluded that there is a significant influence of J_3 (principal stress ordering) on stress-strain response and void evolution for a porous material governed by the von Mises yield criterion.

Summary

The knowledge gained experimentally needs to be complemented with systematic investigations in which the role of each stress invariant can be isolated. To be able to assess the influence of the loading history, and the relative influence of J_3 on both the stress-strain response and porosity evolution, it is essential that during the simulations the triaxiality remains constant. For this purpose, a user element routine was developed for the commercial finite-element code ABAQUS. The verification of the accuracy of the implementation was performed. Most importantly, the F.E. results show that even for isotropic porous von Mises materials, the third invariant, J_3 , influences the response.

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