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# Analytical method for determining cutting forces during orthogonal turning of C45 steel

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**Abstract.** The authors present a method for determining cutting forces during orthogonal turning of C45 steel. The method utilizes Oxley's chip formation model as well as Johnson-Cook (J-C) constitutive equation and is based on the assumption that the tool is perfectly sharp and the chip formation process is continuous. It is also assumed that the heat exchange between the workpiece, the tool and the chip is carried out by conduction with negligibly small loses caused by convection and radiation and that the thickness of the chip contacting the rake face is constant. The adoption of the above assumptions, together with the knowledge of cutting parameters (including the tool rake angle) as well as of material constants of J-C equation, allows to estimate the thermal-mechanical state of the cutting process and to determine feed and tangential components of the cutting force. Average values of feed and tangent components of the cutting force are calculated using an algorithm implemented in the Matlab environment. The method is based on iterative determination of the minimum difference between stress values in the secondary shear zone. Considered tangential and normal stress values are expressed by formulas based on Oxley's cutting mechanics and the J-C model. The cutting force components obtained in the described method have been compared with the results obtained during experimental studies and with the results obtained in computer simulations using the FEM numerical calculation method.

## Introduction

Understanding the phenomena that occur in the cutting zone during machining as well as examination of their impact on process efficiency and quality of machined parts is practically impossible without the use of appropriate models. The issue of modeling of the machining process usually relates to a mechanistic model of the cutting zone, a model of heat generation and distribution or a model of cutting edge wear. Analytical and numerical calculation models are used for forecasting the course of machining process. When describing the behaviour of the machined material at decohesion during the orthogonal turning, it is assumed that there is a slip in the Primary Shear Zone (PSZ), while in the Secondary Shear Zone (SSZ) the friction of the chip against the cutting tool face is taken into account. A good example of using analytical models of the machining process are temperature models determining the partitions of heat penetrating the chip and the workpiece. The examples that can be found in the literature include the Loewen and Shaw's model [1], Trigger and Chao's model [2], the Boothroyd's model [3] and the Oxley's model based on it [4]. The mentioned models allow estimating average temperature the in the slip zone and average contact temperature at the chip-cutting edge interface.

Numerical calculation models for simulating the machining process are a separate group used for process forecasting. Paper [5] describes the use of the ABAQUS application and the FEM method for determination of the impact of friction between the workpiece, chip and the tool on the distribution of forces, strain and temperature in the cutting zone during the turning of AISI 4340 steel. In [6], Özel used the DEFORM application for modelling the friction on the chip-cutting edge interface. The author showed the impact of the friction model used on the chip shape. Another example of using numerical calculation methods for machining process simulation is [7] where authors used the AdvantEdge application to investigate the impact of protective coatings applied on the cutting tools on heat partition in the cutting zone. Computer simulations are now considered as one of the most important aspects of research on machining processes. They allow for a safe and non-invasive view of the process, also in places inaccessible to conventional monitoring methods. Thus, they make it possible to check how individual changes introduced in the cutting zone or the process itself affect the tested output characteristics. Further, through parameter studies, computer simulations allow to analyze various solutions without the need to conduct costly research.

The aforementioned analytical methods can also be used in order to determine the cutting forces during orthogonal turning. In [8,9], the authors presented methods of determination of the cutting forces based on the known machining parameters and cutting tools geometries. The other input into the method is the workpiece mechanical properties and the constants of J-C constitutive equation. The described method for prediction of cutting forces during orthogonal turning is a less expensive and faster alternative for computer applications based on numerical calculation methods.

**Methodology**

The described approach is an alternative for numerical calculation methods that use complex calculation algorithms. The presented method is based on the chip forming model proposed by Oxley [9,10]. The principal part of the method is the algorithm which allows determination of the tangential feed components and the of the cutting force during orthogonal turning. In this paper, the method has been used to determine the cutting force components in a wider range of input parameters. The chip forming model during orthogonal turning is presented Fig. 1.

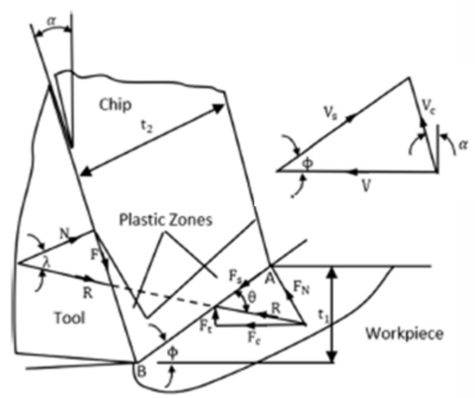


*Fig. 1. Chip formation model for orthogonal turning [11].*

Where $\alpha$ is the rake angle, $\phi$ is the shear angle, $\lambda$ is the average friction angle at tool-chip interface, $\theta$ is the angle between resultant cutting force R and primary shear zone AB. Parameters $t_1$ and $t_2$ are the depth of cut and the chip thickness respectively. V, $V_s$, and $V_c$ are the cutting velocity, shear velocity, and chip velocity, respectively. Parameter w is the cutting width that is not shown. The cutting forces are determined by recurrently changing the values: $\phi$, $C_0$ and $\delta$. $C_0$ is the ratio of shear plane length to the thickness of the PSZ -$l/\Delta s_2$ and $\delta$ is the ratio of the thickness of SSZ to chip thickness - $\Delta s_1/t_2$. The described parameters are presented in Fig. 2.

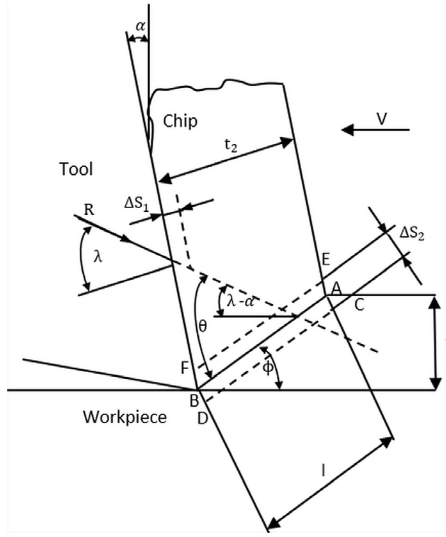


*Fig. 2. Parallel-sided shear zone model [11].*

Presented method includes calculation of stress values in the PSZ, on the sear plane AB and in the SSZ. Values of the shear flow stress ($k_{AB}$), strain ($\varepsilon_{AB}$) and strain rate ($\dot{\varepsilon}_{AB}$) on the shear plane in the PSZ are determined from the following equations:

$$k_{AB} = \frac{\sigma_{AB}}{\sqrt{3}} = \frac{1}{\sqrt{3}}(A + B\varepsilon^{n}{}_{AB})\left(1 + Cln\frac{\dot{\varepsilon}_{AB}}{\dot{\varepsilon}_0}\right)\left(1 - \left(\frac{T_{AB}-T_r}{T_m-T_r}\right)^m\right) \quad (1)$$

$$\varepsilon_{AB} = \frac{\gamma_{AB}}{\sqrt{3}} = \frac{cos\alpha}{2\sqrt{3}sin\phi\cos(\phi-\alpha)} \quad (2)$$

$$\dot{\varepsilon}_{AB} = \frac{\dot{\gamma}_{AB}}{\sqrt{3}} = C_0\frac{V_s}{\sqrt{3}l_{AB}} \quad (3)$$

The length of the shear zone ($l_{AB}$), the tool-chip interface length (h), the shear velocity (Vs), the chip velocity (Vc) and the chip thickness ($t_2$) are determined as follows:

$$l_{AB} = \frac{t_1}{sin\phi} \quad (4)$$

$$h = \frac{t_1 sin\phi}{cos\lambda\sin\phi}\left(1 + \frac{C_0 n_{eq}}{3\left(1+2\left(\frac{\pi}{4}-\phi\right)C_0 n_{eq}\right)}\right) \quad (5)$$

$$V_s = \frac{Vcos\phi}{cos(\phi-\alpha)} \quad (6)$$

$$V_c = \frac{Vsin(\phi)}{cos(\varphi-\alpha)} \quad (7)$$

$$t_2 = \frac{t_1 cos(\phi-\alpha)}{sin\phi} \quad (8)$$

Secondly, in the SSZ, the shear flow stress in the chip with von Mises criterion are determined on the basis of the equation:

$$k_{chip} = \frac{1}{\sqrt{3}}(A + B\varepsilon_{int}^{n})\left(1 + Cln\frac{\dot{\varepsilon_{int}}}{\dot{\varepsilon}_0}\right)\left(1 - \left(\frac{T_{int}-T_0}{T_m-T_0}\right)^{m}\right) \quad (9)$$

while strain ($\varepsilon_{chip}$) and strain rate ($\dot{\varepsilon_{chip}}$) are calculated as stated below:

$$\varepsilon_{chip} = \frac{\gamma_{int}}{\sqrt{3}} = 2\varepsilon_{AB} + \frac{h}{2\sqrt{3}\delta t_2} \quad (10)$$

$$\dot{\varepsilon_{chip}} = \frac{\dot{\gamma_{int}}}{\sqrt{3}} = \frac{V_c}{2\sqrt{3}\delta t_2} \quad (11)$$

Basing on [12], a portion of heat conducted to the workpiece from the shear zone (β) is expressed as:

$$\beta = 0.5 - 0.35log_{10}(R_T tan\phi) \qquad for\ 0.04 \leq R_T tan\phi \leq 10$$

$$\beta = 0.3 - 0.15 = log_{10}(R_T tan\phi) \qquad for\ R_T tan\phi\ \geq 10 \quad (12)$$

where $R_T$ is a non - dimensional thermal number.

$$R_T = \frac{\rho C_p V t_1}{K} \quad (13)$$

The average temperature in PSZ along AB is given by [12]:

$$T_{AB} = T_0 + \eta\Delta T_{sz} \quad (14)$$

$$\Delta T_{sz} = \frac{(1-\beta)F_s V_s}{\rho V t_1 w C_p} \quad (15)$$

where: $T_0$ - initial work temperature (25°C in present analysis), $\eta$ - heat partition factor (90% in present analysis), $\Delta T_{sz}$ - the temperature rise in the PSZ.

The temperature at the SSZ ($T_{int}$) is calculated using a heat partition Eq. 16.

$$T_{int} = T_0\Delta T_{sz} + \psi\Delta T_M \quad (16)$$

Where ψ - is a factor to allow for the fact that the $T_{int}$ is average value (ψ =0.9 in present analysis), $\Delta T_M$ is the maximum temperature rise in the chip occurring at the interface. By assuming a rectangular heat source at the interface Boothroyd developed the following equation to find $\Delta T_M$:

$$log_{10}\left(\frac{\Delta T_M}{\Delta T_C}\right) = 0.06 - 0.195\delta\sqrt{\frac{R_T t_2}{t_1}} + 0.5log_{10}\left(\frac{R_T t_2}{h}\right) \quad (17)$$

$$\Delta T_C = \frac{FV_c}{\rho V t_1 w C_p} \tag{18}$$

where $\Delta T_C$ is the average temperature rise in the chip.
The angles and forces in Fig. 1 are expressed by formulas:

$$\theta = arctan\left(1 + 2\left(\frac{\pi}{4} - \phi\right) - C_0 n_{eq}\right) \tag{19}$$

$$\lambda = \theta - \phi + \alpha \tag{20}$$

$$n_{eq} \approx \frac{nB\varepsilon_{AB}^n}{(A + B\varepsilon_{AB}^n)} \tag{21}$$

$$F = Rsin\,\lambda \tag{22}$$

$$N = Rcos\,\lambda \tag{23}$$

$$F_s = Rcos\big((\phi + \lambda - \alpha)\big) = k_{AB} l_{AB} w \tag{24}$$

$$N_s = Rsin(\phi + \lambda - \alpha) \tag{25}$$

$$F_c = Rcos(\lambda - \alpha) \tag{26}$$

$$F_t = Rsin(\lambda - \alpha) \tag{27}$$

where: F and N are the shear force and normal force at the tool-chip interface, respectively, $F_s$ and $N_s$ are the shear force and normal force at the PSZ, respectively, $F_c$ is the cutting force and $F_t$ is the thrust force and $n_{eq}$ is the strain hardening constant. The normal stress ($\sigma_N$) and the shear stress ($\tau_{int}$) and at the tool-chip interface are calculated from the Eq. 28 and 29:

$$\sigma_N = \frac{N}{hw} \tag{28}$$

$$\tau_{int} = \frac{F}{hw} \tag{29}$$

The normal stress $\acute{\sigma}_N$ is determined from J-C model and stress boundary conditions on the primary shear zone - Eq. 30,

$$\acute{\sigma}_N = k_{AB}\left(1 + \frac{\pi}{2} - 2\alpha - 2C_0 n_{eq}\right) \tag{30}$$

Values of the shear angle $\phi$ and the strain-rate constant $C_0$ are determined when the differences between $\tau_{int}$ and $k_{chip}$ as well as $\sigma_N$ and $\sigma_0$, respectively, are minimal. Also the strain-rate constant $\delta$ is determined when the calculated $F_c$ is minimal. With determined shear angle $\phi$, and strain rate constants $C_0$ and $\delta$, the machining forces are calculated. A detailed scheme of the algorithm for determining the tangential ($F_c$) and the feed ($F_f$) components of the cuttning force has been presented in the article [12]. The results obtained by means of the presented method were compared with the results of experimental studies as well as with the results obtained from computer simulations based on FEM (Finite Element Method).

**Laboratory Test**

Laboratory tests included performing a series of orthogonal turning tests. The workpiece was a AISI1045 steel pipe with outer diameter of D=50 mm and wall thickness of d=3mm. The test stand consisted of a TUJ 50 conventional lathe and a measuring track enabling the measurement of cutting forces, which consisted the following elements:

- Kistler (Winterthur, Switzerland) 9257B piezoelectric dynamometer together with a Kistler 9403 tool holder (sensitivity: $F_f$: -7.70 pC/N; $F_p$: -7.82 pC/N; $F_c$: -3.71 pC/ N; threshold: <0.01 N) attached to the lathe support,
- Kistler multichannel charge amplifier type 5019B with a set of filters (scale: $F_f$, $F_p$: 50 N/V, $F_c$: 200 N/V),
- PC with DynoWare Software (Kistler) for archiving and analysis of cutting force components.

The measuring stand with the parameters given above made it possible to measure the components of the total cutting force with a frequency of 1000 Hz and with the following inaccuracies: $F_f$, $F_p$: ±0.25 N, $F_c$: ±1N. The schematic diagram of the test stand is shown in Figure 3. The figure also shows the distribution of cutting force components occurring in the process of orthogonal turning.

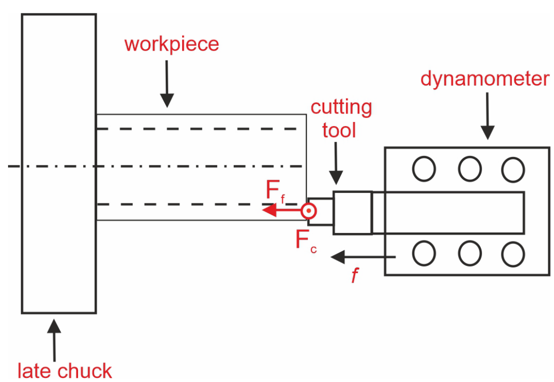


*Fig. 3. The schematic diagram of the test stand.*

The chemical composition of the AISI 1045 steel in accordance with the PN-EN ISO 683-1:2018-09 standard is presented in Table 1.

*Table 1. The percentage chemical composition of AISI 1045 steel.*

| Symbol | C | Si | Mn | Cr | Ni | Mo | Cu | S | P |
|---|---|---|---|---|---|---|---|---|---|
| | 0.42-0.50 | 0.10-0.40 | 0.50-0.80 | ≤0.30 | ≤0.30 | ≤0.10 | ≤0.30 | ≤0.40 | ≤0.40 |

The parameters variability range was chosen on the basis of catalogue data and is presented in Table 2.

*Table 2. Values of cutting parameters.*

| Cutting parameters | Parameter values | | |
|---|---|---|---|
| f [mm/rev] | 0.1 | | |
| V [m/min] | 100 | 200 | 300 |
| Rake angle α [deg] | -12 | -6 | 0 |

The tools used in the tests, shown in Fig. 4, were made of P10 carbide without a protective coating. They had a flat rake surface and the length of the cutting edge was 5 mm.

*Fig. 4. The tools used in the tests.*

**Simulation Researches**

Computer simulation of the cutting process was carried out using FEM. The simulation was carried out from the moment of first contact of the tool and the workpiece and was continued until a steady state was obtained, that is, until stable stress and temperature values were observed. Both the workpiece and the cutting edge were discretized. Stress of the tool material was calculated using a software package based on a numerical model of the chip formation process, the so-called two-dimensional Lagrange model. In the initial stage of building the material model of the workpiece, three sets of material constants were used. Tables 3 and 4 below present constans of J-C flow stress model and thermo physical properties of AISI 1045 steel, respectively.

*Table 3. Constans of J-C flow stress model for AISI 1045 steel.[10,13]*

| Set number | A(MPa) | B(MPa) | C | n | m | $T_0$[°C] | $T_m$[°C] |
|---|---|---|---|---|---|---|---|
| 1 | 553.1 | 600.8 | 0.0134 | 0.234 | 1 | 25 | 1460 |
| 2 | 731.63 | 518.7 | 0.00571 | 0.94054 | 0.3241 | 25 | 1460 |
| 3 | 546.83 | 609.35 | 0.01376 | 0.94053 | 0.2127 | 25 | 1460 |

*Table 4. Thermo physical properties of AISI 1045 steel.[10]*

| Specific heat, S (J/kgK) | Thermal conductivity, $\lambda_c$ (W/mK) | Density ρ (kg/m$^3$) |
|---|---|---|
| 420+0.504$T$ | 52.61-0.0281$T$ | 7850 |

After verifying the results and comparing them with empirical data, set no. 1 was selected for further research. This set of constants was also used in the analytical method for determining the cutting forces described in this article.

**Results**

Fig. 5 includes mean values of cutting forces determined by means of:

- experiment: $F_{c\_EXP}$, is the tangential component, $F_{f\_EXP}$ is the feed component,
- simulation (FEM): $F_{c\_FEM}$, $F_{f\_FEM}$,
- analytical method: $F_{c\_CALC}$, $F_{f\_CALC}$.

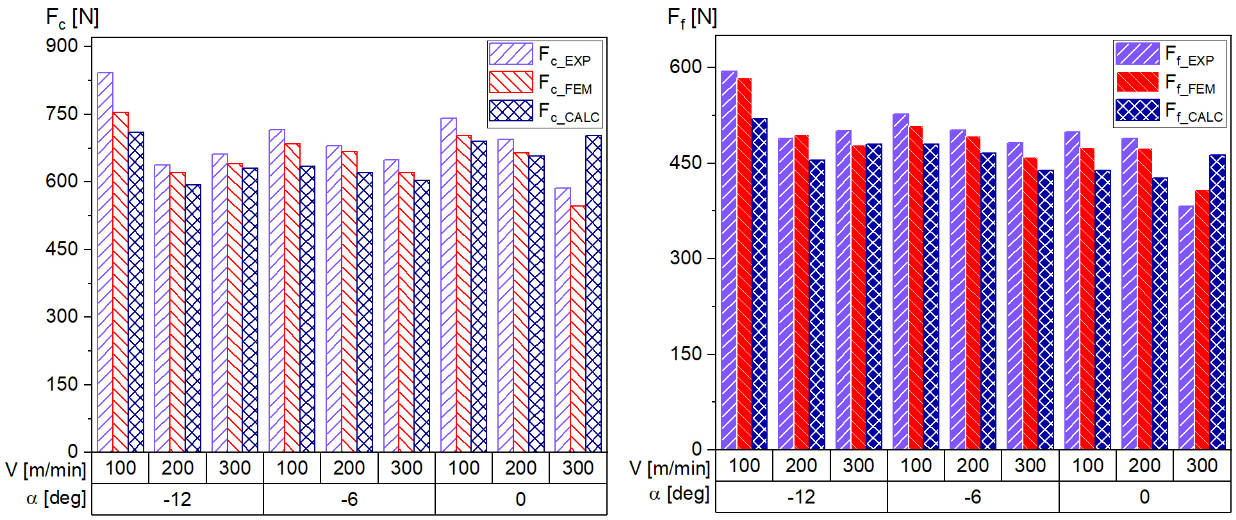


*Fig. 5. Mean values of cutting forces determined by various means.*

Values of the relative error for the results obtained by means of the FEM and analytical methods as well as for the results obtained from the laboratory test are presented in Figure 6 and were determined according to the formulas:

$$\Delta F_{c\_FEM}=abs(F_{c\_EXP}-F_{c\_FEM})/F_{c\_EXP}*100\%$$

$$\Delta F_{f\_FEM}=abs(F_{f\_EXP}-F_{f\_FEM})/F_{f\_EXP}*100\%$$

(31)

$$\Delta F_{c\_CALC}=abs(F_{c\_EXP}-F_{c\_CALC})/F_{c\_EXP}*100\%$$

$$\Delta F_{f\_CALC}=abs(F_{f\_EXP}-F_{f\_CALC})/F_{f\_EXP}*100\%$$

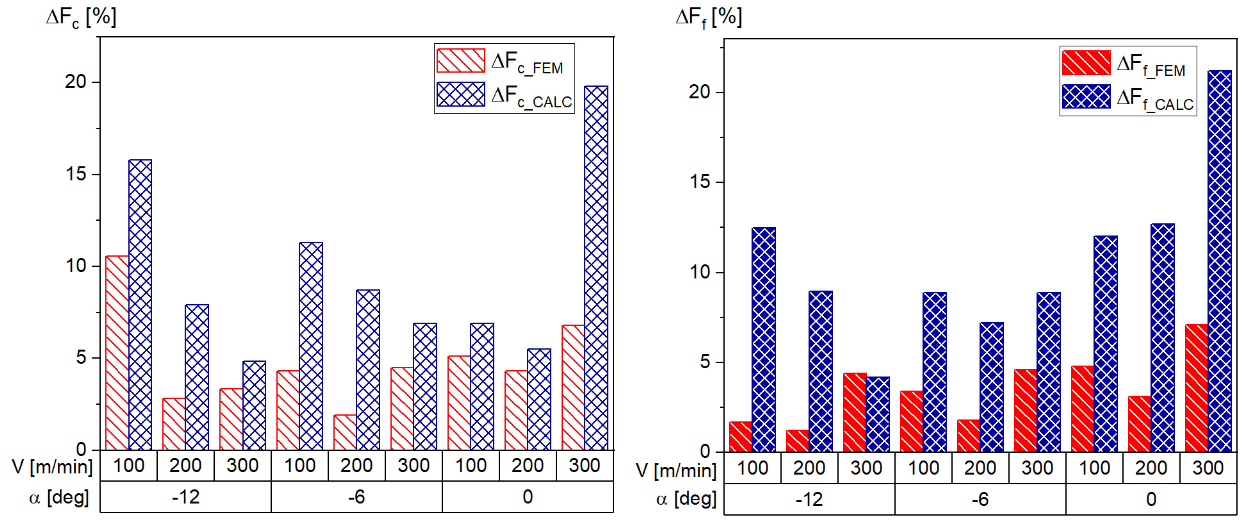


*Fig. 6. Relative error for obtained results.*

## Summary

The presented analytical method for determination of cutting forces during orthogonal turning is characterized by a low calculational complexity and can be an alternative for very complex numerical programs. The input parameters include cutting parameters, tool geometry and the constants of J-C constitutive equation. The analysis of forces obtained in experimental studies allows the following conclusions to be drawn:

- The cutting forces obtained using the FEM simulation have a good conformity with the forces obtained experimentally. The maximum relative error between the results is 10.56% for the $F_c$ component and 7.1% for the $F_f$ component;
- The cutting force values used in the analytical method are closest to the experimental results for the rake angle of $\alpha$=-6°. In this case, the maximum relative error at V=100 m/min is 11.3% for $F_c$ and 8.9% for $F_f$;
- In the remaining trials, where rake angles were -12° and 0°, respectively, the relative error between the results obtained experimentally and with the analytical method was greater;
- The maximum relative error for angle $\alpha$ =-12 and V =300 m/min was $\Delta F_{c\_CALC}$=19.8%, $\Delta F_{f\_CALC}$=21.2%.

  The proposed analytical method is sensitive to the values of the constants of J-C constitutive equation.

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