

## Coupling machine learning and synthetic image DIC-based techniques for the calibration of elastoplastic constitutive models

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**Keywords:** Constitutive Model Calibration, Elastoplasticity, Machine Learning, DIC

**Abstract.** Today, most design tasks are based on simulation tools. However, the success of the simulation depends on the accurate calibration of constitutive models. Inverse-based calibration methods, such as the Finite Element Model Updating and the Virtual Fields Method, have been developed for identifying constitutive parameters. These methods are based on mechanical tests that allow heterogeneous strain fields under the “Material Testing 2.0” paradigm in which digital image correlation plays a vital role. Although these methods have been proven effective, constitutive model calibration is still a complex task. A machine learning approach is developed and implemented to calibrate elastoplastic constitutive models for metal sheets, using datasets populated with finite element simulation results of strain field data from mechanical tests. Feature importance analysis is conducted to understand the importance of the different input features and to reduce the computational cost related with model training. Synthetic image DIC-based techniques were coupled with the numerically generated database, enabling the construction of a virtual experiments database that accounts for sources of uncertainty that can influence experimental DIC measurements. A robustness analysis of the methodology is performed for the boundary conditions of the test.

### Introduction

The numerical simulation of sheet metal forming processes has become an essential tool to obtain high-performance components for automotive and aerospace industries. The quality of numerical simulations depends on the quality of the constitutive modelling that describes the material behaviour, which is related to the type of constitutive laws and the strategy to identify their parameters. Inverse identification strategies based on Finite Element Model Updating (FEMU) and Virtual Fields Method (VFM) have been proposed (e.g. [1–3]), which make use of full-field measurements of heterogeneous strain field data collected from non-standardized mechanical tests. However, the computational time and a complex implementation process are still major drawbacks of such identification strategies [4–5]. More recently, machine learning based approaches have been explored to identify material parameters (e.g. [6–8]), showing to be a promising alternative to both FEMU and VFM [3]. The authors of the current work have previously proposed a machine learning approach to identify material parameters (isotropic hardening law + orthotropic yield criterion), using a numerically generated database populated with finite element simulation results of cruciform tensile tests and the XG-Boost algorithm [6]. In this work, synthetic image DIC-based techniques are coupled with the numerically generated database, enabling the construction of a virtual experiments database that accounts for sources of uncertainty that can influence experimental DIC measurements. A robustness analysis on the ML approach is performed for the boundary conditions of the finite element model. In particular, the orientation of the material axes

was varied with reference to the global axes system of the cruciform test to emulate specimen misalignments that may occur in material testing.

## Methodology and Implementation

### General Approach.

Fig. 1 schematizes the proposed approach for calibrating elastoplastic constitutive models. First, a database was established using numerically generated features (i.e. strain fields and loads) obtained from FEA simulations of an heterogeneous mechanical test. Different combinations of constitutive parameters were considered while maintaining the same sample geometry and boundary conditions; a Python script was created to enhance the whole process and automatize the generation of the database. Then, a feature importance step was performed to evaluate how useful the data is at predicting the constitutive parameters. A virtual experiments database was created from the FEA-generated database, where synthetic images were generated and processed via DIC. The virtual experiments database was then split into training (90%) and testing (10%) sets. The training set was used to establish an ML model for predicting the material parameters from the strain fields and loads. Finally, the predictive performance of the ML model was evaluated using the testing set.

### Numerical model.

The mechanical test selected for this study is the biaxial tensile test on a cruciform sample. The sample geometry was designed in a previous work, enabling heterogeneous stress and strain fields and a wide range of stress and strain paths that are commonly observed in sheet metal forming processes [9]. Fig. 2(a) shows the geometry and the dimensions of the cruciform sample in the sheet plane. The numerical simulation model only considers one fourth of the sample (see grey region in Fig. 2(a)) due to symmetries in the boundary conditions, sample geometry and material behaviour. Moreover, plane stress conditions and a constant thickness of the sheet equal to 1 mm are assumed. Fig. 2(b) shows the boundary conditions and finite element mesh of the numerical model. Symmetry boundary conditions were prescribed on the 0x and 0y axes ( $u_x = u_y = 0$  mm), and displacement boundary conditions were applied to the nodes located at the ends of both arms of the sample to promote equal displacements along both 0x and 0y axes ( $u_x = u_y = 2$  mm). The numerical model is discretized with a regular mesh made of 405 CPS4R elements (bilinear shape functions and reduced integration). All FEA simulations were performed using ABAQUS CAE software [10]. Each simulation was carried out for twenty equally spaced time-steps, where the force and the strain field ( $\epsilon_{xx}$ ,  $\epsilon_{yy}$ , and  $\epsilon_{xy}$ ) was obtained for each time-step.

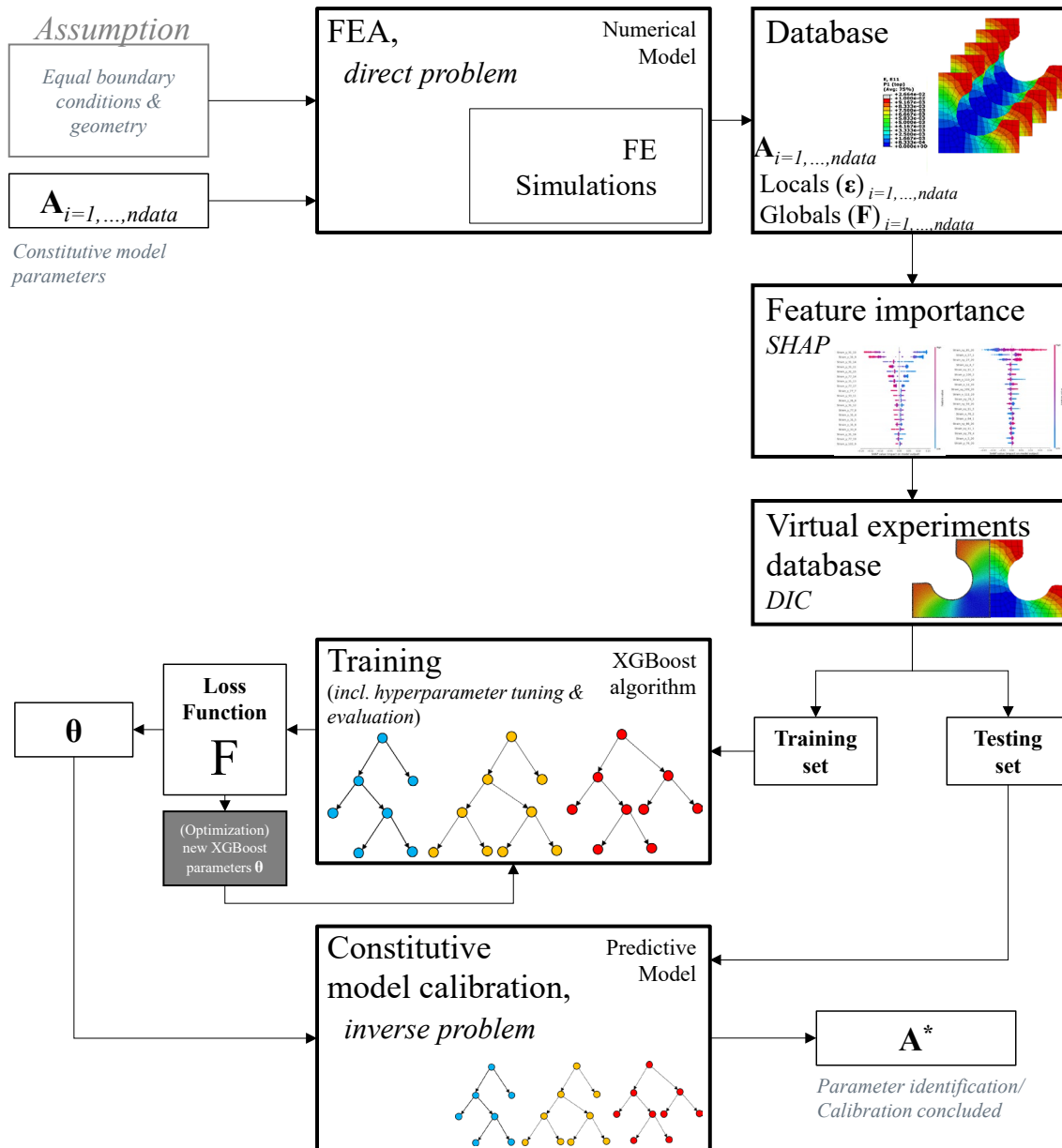


Fig. 1. Flowchart of the proposed identification approach.

The material constitutive model assumes an isotropic elastic behavior, described by the Hooke’s law (with Young’s modulus  $E = 210$  GPa and Poisson’s ratio  $\nu = 0.3$ ), and an orthotropic plastic behavior, described by Hill’48 yield criterion with isotropic hardening described by Swift law under an associated flow rule. Under plane stress conditions, the Hill’48 yield criterion can be written as follows:

$$(F + H)\sigma_{yy}^2 + (G + H)\sigma_{xx}^2 - 2H\sigma_{xx}\sigma_{yy} + 2N\tau_{xy}^2 = Y^2 \quad (1)$$

where  $F$ ,  $G$ ,  $H$  and  $N$  are anisotropy coefficients,  $\sigma_{xx}$ ,  $\sigma_{yy}$  and  $\tau_{xy}$  are the components of the Cauchy stress tensor in the material axes system of the metal sheet, and  $Y$  is the yield stress.

The condition  $G+H=1$  (i.e.  $\sigma_{xx} = Y$ ) was assumed, which corresponds to the following relationships:

$$F = \frac{r_0}{r_{90}(r_0 + 1)}; G = \frac{1}{r_0 + 1}; H = \frac{r_0}{r_0 + 1}; N = \frac{1}{2} \frac{(r_0 + r_{90})(2r_{45} + 1)}{r_{90}(r_0 + 1)} \quad (2)$$

where  $r_0$ ,  $r_{45}$  and  $r_{90}$  are the Lankford ratios obtained at  $0^\circ$ ,  $45^\circ$  and  $90^\circ$  w.r.t. the rolling direction of the sheet, respectively. The Swift law describes the yield stress evolution during plastic deformation as follows:

$$Y = K \left[ \left( \frac{Y_0}{K} \right)^n + \bar{\epsilon}^p \right]^n \quad (3)$$

where  $\bar{\epsilon}^p$  is the equivalent plastic strain and  $Y_0$ ,  $K$  and  $n$  are material parameters.

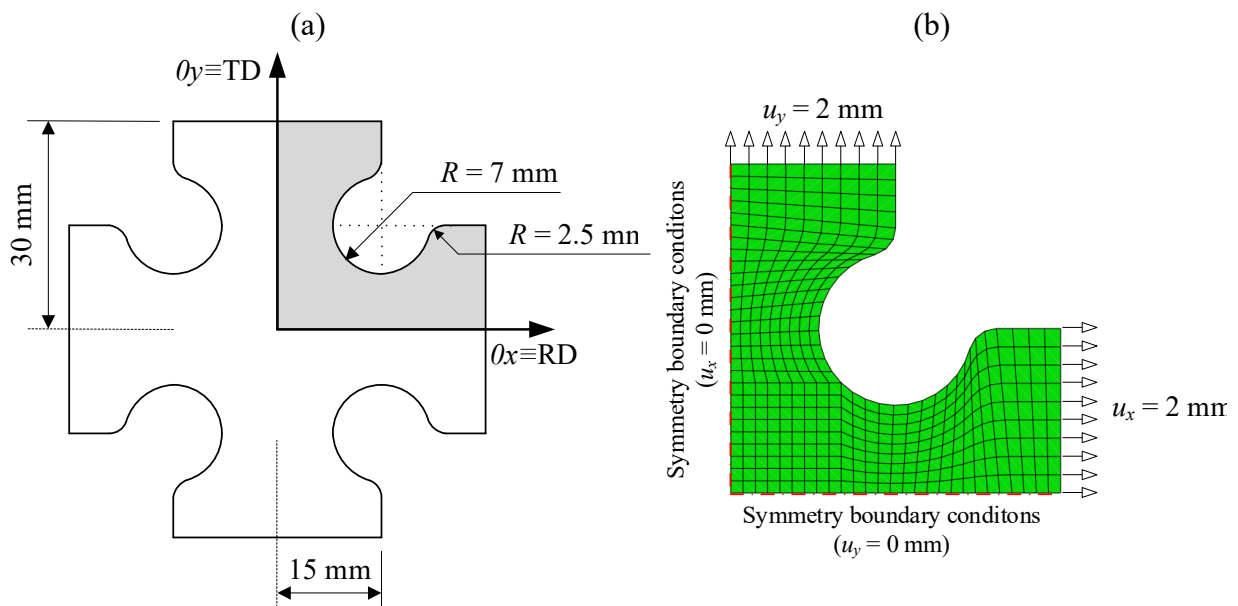


Fig. 2. Biaxial tensile test on a cruciform sample: (a) geometry and dimensions [9]; (b) boundary conditions and finite element mesh.

The database was populated with synthetic data generated by FEA simulations of the cruciform tensile test with different combinations of plasticity material parameters while maintaining the same geometry, boundary conditions and elastic properties. Table 1 shows the range of values of the plasticity material parameters considered as input space for the numerical simulations. Then, 2000 sets of parameters were generated using the Latin Hypercube Sample method, and numerical simulations of the cruciform tensile test were performed for each set. Fig. 3 presents an example of numerical results of the cruciform test that were used to build the database.

Table 1. Input space of plasticity material parameters.

Plasticity material parameters	Range
$Y_0$ [MPa]	80-300
$n$	0.1-0.3
$K$ [MPa]	280-700
$r_0$	1-2.5
$r_{45}$	1-2.5
$r_{90}$	1-2.5

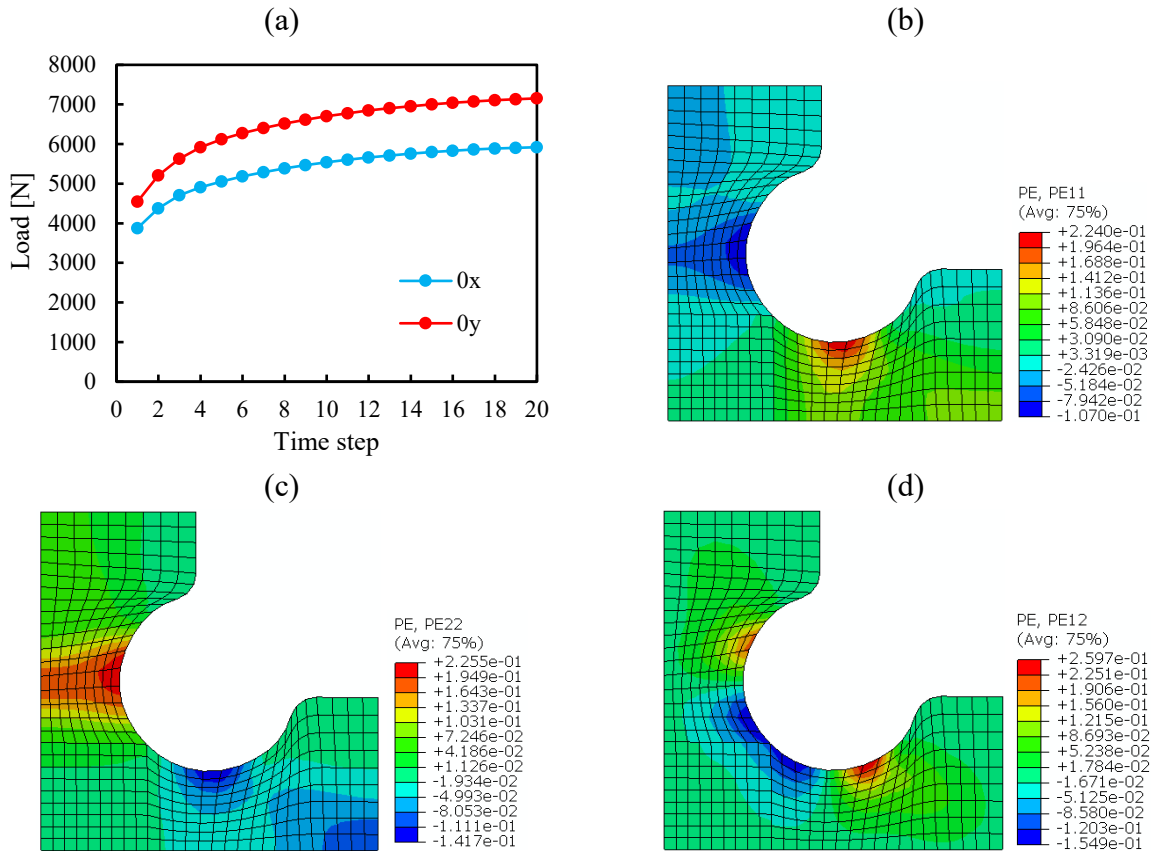


Fig. 3. FEA results of the cruciform test ( $Y_0=172\text{MPa}$ ,  $n=0.16$ ;  $K=486\text{MPa}$ ;  $r_0=2.38$ ;  $r_{45}=1.8$ ;  $r_{90}=1.06$ ): (a) load vs. displacement along the  $0x$  and  $0y$  axes; strain fields (b)  $\epsilon_{xx}$ , (c)  $\epsilon_{yy}$ , and (d)  $\epsilon_{xy}$ . The strain fields ( $\epsilon_{xx}$ ,  $\epsilon_{yy}$ , and  $\epsilon_{xy}$ ) were obtained for  $u_x = u_y = 2\text{ mm}$ .

### Feature importance.

Many ML systems are essentially considered black boxes, as it is hard to understand and explain how they work after training. Shapley Additive exPlanations (SHAP) are considered state of the art in Machine Learning explainability [11]. SHAP analysis explain how each feature (i.e. loads, strain fields) affects the outputs (i.e. material parameters) of the ML model. As an example, Fig. 4 presents the 20 most important features for predicting the hardening parameters  $Y_0$ ,  $n$  and  $K$ . In Fig. 4a, the feature “Force\_y\_1” (i.e. force  $0y$  at time step 1) is the most relevant feature to predict the initial yield stress,  $Y_0$ ; moreover, higher values of “Force\_y\_1” (towards red color) have a positive contribution on the prediction of  $Y_0$  (i.e. higher SHAP values), leading to higher values of  $Y_0$ . SHAP analysis confirmed that it is possible to substantially reduce the number of features in

the dataset, by excluding the less important ones and reducing the computational cost without compromising predictive performance.

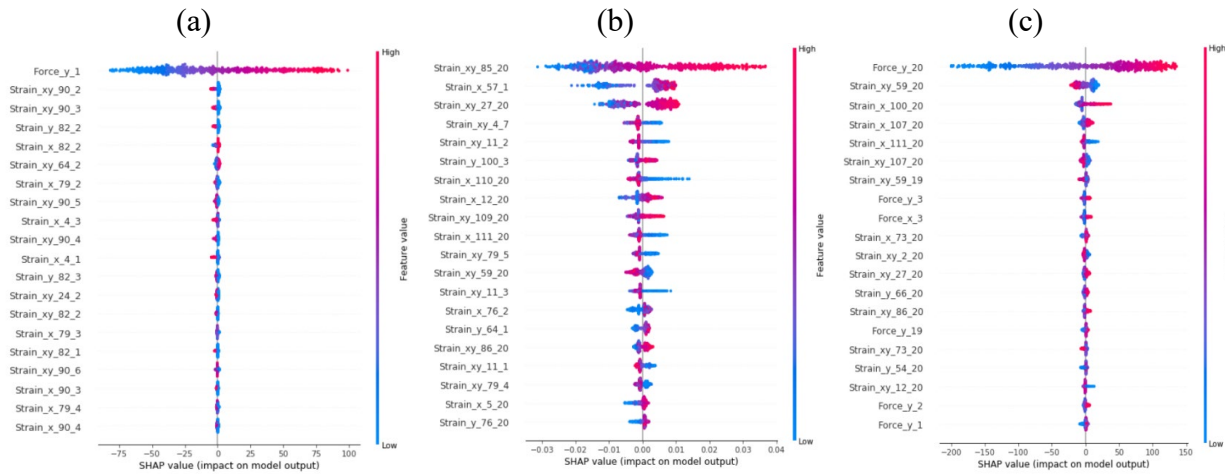


Fig. 4. SHAP analysis of the 20 most important features for predicting the hardening parameters (a)  $Y_0$ ; (b)  $n$  and (c)  $K$ .

### Virtual experiments.

Virtual experiments enable a proper comparison between simulations and experiments by overcoming issues that arise from the direct numerical-experimental comparison (e.g. alignment, data locations, discretization, filtering effects, experimental uncertainties, etc.) and making the database features one step closer to measurements collected from real experiments. After finishing the feature importance step, the numerical results of the cruciform test were used to generate synthetic images that were then processed with DIC. The DIC setting were selected using the Performance Analysis module from MatchID [12], representing a good compromise between spatial and measurement resolution, as shown in Fig. 5. Each sample was divided into 6455 subsets. In each subset, the location in pixels and the strains  $\epsilon_{xx}$ ,  $\epsilon_{yy}$ , and  $\epsilon_{xy}$  were recorded. 2000 samples were generated, 1800 for training and 200 for testing.

### XGBoost algorithm.

Extreme Gradient Boosting (XGBoost) is an efficient open-source implementation of the gradient boosted trees algorithm [13]. Gradient boosting is an algorithm in which new models are created from previous models' residuals (weak models) and then combined to make the final prediction (strong models). When adding new models, it uses a gradient descent algorithm to minimize the loss. XGBoost attempts to minimize the regularized objective as follows:

$$obj(\theta) = \sum_i L(\hat{y}_i(\theta), y_i) + \sum_k \Omega(\theta) \quad (4)$$

where  $L$  is the training loss function that measures the deviation between the value  $\hat{y}_i$  predicted by the model and the actual value  $y_i$  of sample  $i$ ;  $\Omega$  is the regularization function (i.e. a penalty term) that measures the complexity of the model, which tends to prevent overfitting;  $\theta$  represents the set of parameters to be calibrated during training.

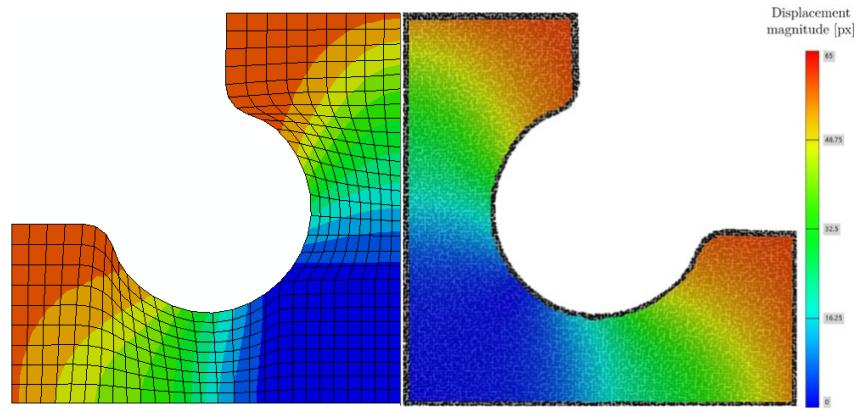


Fig. 5. Comparison of the displacement magnitude in the cruciform test, between FEA (left) and virtual experiments (right).

Performance metrics.

The performance of the ML model was evaluated by comparing the predicted and simulated parameters using the Coefficient of Determination ( $R^2$ ), given by:

$$R^2 = 1 - \frac{\sum_{i=1}^j (y_i - y_i^*)^2}{\sum_{i=1}^j (y_i - \bar{y})^2} \quad (5)$$

where  $y_i$  and  $y_i^*$  are respectively the real and predicted values for the constitutive parameters of simulation  $i$  from the test set,  $\bar{y}$  is the average of the real values of the constitutive parameters, and  $j$  is the total number of simulations of the test set.

## Results and Discussion

Performance evaluation.

Fig. 6 compares the  $Y_0$ ,  $n$  and  $K$ ,  $r_0$ ,  $r_{45}$  and  $r_{90}$  values predicted by the ML model with the real values considered in the FEA simulations. In general, the ML model has superior predictive performance, which is confirmed by the values of  $R^2$  presented in the figures.

Robustness Analysis.

The robustness of the ML model was analyzed in relation to the boundary conditions of the cruciform test. In fact, the ML model was trained keeping the boundary conditions unchanged; however, during the performance of experimental tests there may be, to a greater or lesser extent, misalignments in the placement/test of the specimen, which may influence the measurements and consequently the calibration of the constitutive models. In this context, one of the samples from the testing set was chosen ( $Y_0=172$  MPa,  $n=0.16$ ;  $K=486$  MPa;  $r_0=2.38$ ;  $r_{45}=1.8$ ;  $r_{90}=1.06$ ) and the orientation of the material axes was varied w.r.t. the global axes system  $0xy$  (see Fig. 2) to simulate various cases of sample misalignments that may occur in material testing. Fig. 7 shows the relative error in the prediction of each material parameter, for rotations of the material axes ranging between  $0^\circ$  and  $10^\circ$  w.r.t. the global axes system (the  $0^\circ$  case is taken as reference for calculating the relative error in predictions); this figure also includes an extreme case of  $45^\circ$ . In general, the parameter predictions remain robust regarding possible sample misalignments in-between  $0^\circ$  and  $10^\circ$ ; in this interval, the maximum variation of the relative error occurs for parameter  $r_0$  (about 2.65%). This preliminary robustness analysis suggests that sample misalignments do not significantly influence the predictive performance of the ML model, regardless of the overall quality of predictions obtained for each parameter (poor quality for parameters  $n$ ,  $r_{45}$  and  $Y_0$ , which requires careful analysis to assess the cause).



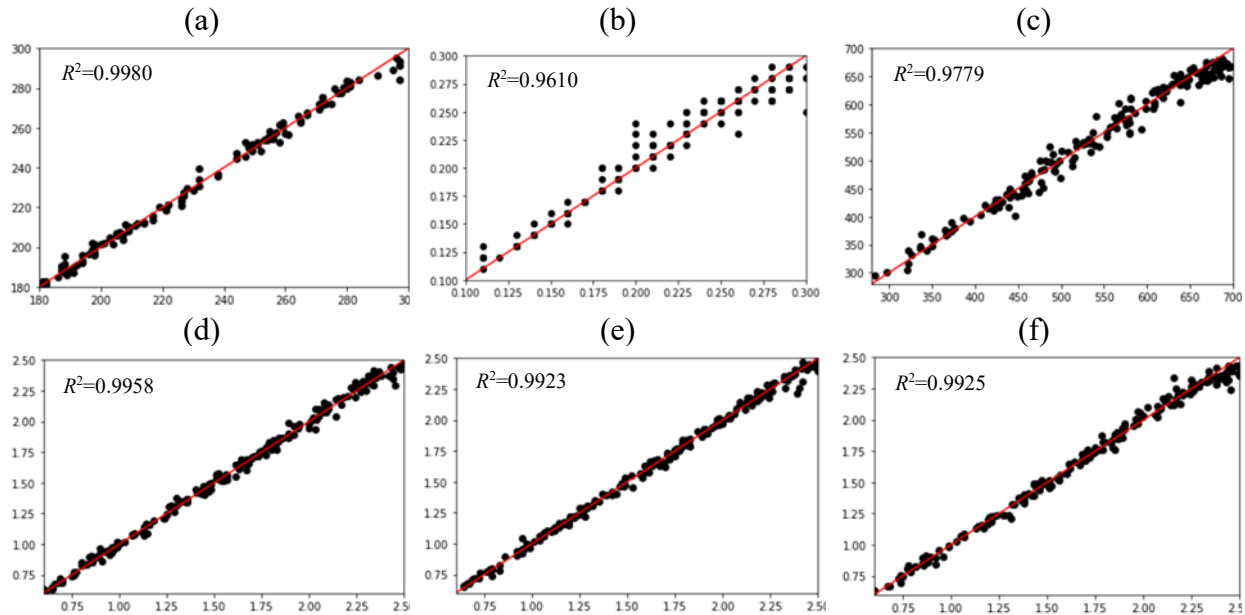


Fig. 6. Predicted (vertical axis) vs. real (horizontal axis) values of the constitutive parameters: (a)  $Y_0$ ; (b)  $n$ ; (c)  $K$ ; (d)  $r_0$ ; (e)  $r_{45}$ ; (f)  $r_{90}$ . The  $R^2$  values are also presented.

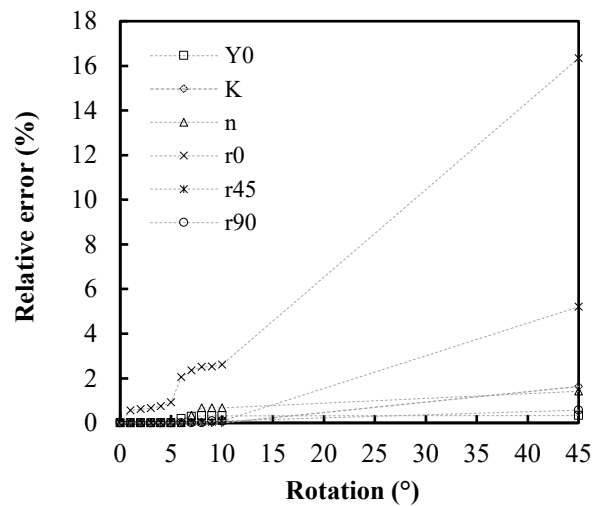


Fig. 7. Relative error in the prediction of the material parameters as a function of the rotation of the material axes w.r.t. the global axes system.

### Summary

A machine learning approach was developed for calibrating constitutive models that describe the plastic behaviour of metal sheets. The database was populated with numerically generated features (strain fields and loads) collected from FEA simulations of the biaxial tensile test on a cruciform sample. Feature importance analysis allowed to identify the most relevant features in the database for identifying the constitutive parameters of the Swift hardening law and the Hill'48 yield criterion. A virtual experiments database was established from the FEA-generated database using synthetic images and DIC processing, making the numerically generated features one step closer to measurements collected from real experiments. A robustness analysis on the ML approach was performed for the boundary conditions to simulate possible misalignments that may occur in



material testing, namely on the rotation of the material axes w.r.t. the global axes system. It was concluded:

- Feature importance enabled the identification of the most important features of the database for identifying constitutive parameters. ML training was greatly accelerated, without compromising predictive performance;
- Virtual experiments allow for accurate model predictions of the constitutive parameters, bridging the gap between simulation results and experimental measurements;
- Sample misalignments do not significantly influence the performance of the ML model.

The trained ML model enables an almost instantaneous parameter identification, without being significantly influenced by sample misalignments. On the other hand, it is limited to the identification of Hill'48+Swift law parameters, which can be unsuitable for materials whose plastic behaviour is best described using more advanced constitutive models; also, the trained ML model considers uniform displacements imposed at the end of both arms of the sample. Therefore, it is envisaged to extend the robustness analysis to the displacement boundary conditions (i.e. when it is not possible to achieve uniform displacement at both arms of the sample), and to the type of material constitutive model that describes the plastic behaviour of the material.

### Acknowledgments

This research has received funding from the Research Fund for Coal and Steel under grant agreement No 888153. This research was also sponsored by FEDER funds through the program COMPETE (Programa Operacional Factores de Competitividade), by national funds through FCT (Fundação para a Ciência e a Tecnologia) under the projects UIDB/00481/2020, UIDP/00481/2020, CENTRO-01-0145-FEDER-022083 and LA/P/0104/2020. J. Henriques was supported by a grant for scientific research from the Portuguese Foundation for Science and Technology (ref. 2021.05692.BD). All supports are gratefully acknowledged.

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