

Seismic damage assessment of building with strong non-linearity based on particle filter

Takenori Hida^{1, a*} and Ryoji Ishizaka^{1, b}

¹Ibarkai University, Hitachi-shi, Ibaraki, Japan

^atakenori.hida.mn75@vc.ibaraki.ac.jp, ^b21nm805l@vc.ibaraki.ac.jp

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Abstract. Various structural health monitoring (SHM) methods based on system identification using strong motion records of buildings were proposed in previous studies. Methodologies based on linear state-space models such as subspace state-space system identification (4SID) [1][2] were often used to assess the structural damage in past studies. However, it is inappropriate to apply methods based on linear state-space representations when evaluating damage to buildings that exhibit strong nonlinearity, such as wooden structures that were severely damaged due to an earthquake. To evaluate the seismic damage of a building, it is desirable to observe the strong motion on all floors of the building. However, such cases are rare in real buildings due to various restrictions such as cost and/or spatial limitation. Based on the background mentioned above, this paper investigates the applicability of the particle filter [3][4] to assess the structural integrity of highly nonlinear buildings using the strong motion records observed on a limited number of floors.

Introduction

Structural Health Monitoring (SHM) methods could play important role to evaluate the seismic damages to buildings during an earthquake. Various SHM approaches based on the system identification using strong motion records observed at a building were proposed in the past studies. In the several past studies, a methodology based on the linear state space model such as the subspace state-space system identification (4SID) [1][2] was used. However, there are limited knowledge on the validity of the methodology for the building with a strong non-linearity, such as wooden structure suffered severe damage due to an earthquake. Moreover, it is inappropriate to apply the method based on linear state-space representations when evaluating damage to buildings that exhibit strong nonlinearity.

Considering the background mentioned above, this paper investigates the applicability of the particle filter [3][4], which is one of methodologies of the data assimilation, to the structural health monitoring of buildings with a strong non-linearity.

First, we performed a non-linear seismic response analysis of 2DOF system consisting of two springs of the Bouc-Wen model, which can reproduce the various shapes of restoring force characteristics. Then the parameters of the model were identified by the particle filter, and the evaluated hysteresis loops were compared with those of the analytical results to examine the accuracy of this approach.

In order to evaluate the seismic damage to buildings in detail, it is desirable to observe the strong motion on all floors of the building, but such cases are rare in the real buildings due to various restrictions such as cost and/or spatial limitation. To cope with these difficulties, we also investigate the identification accuracy of the above-mentioned approach by using the strong motion records observed on the limited number of stories, e.g., 1st floor and roof floor of the 2-story building.



Overview of particle filters

Figure 1 shows an overview of the particle filter. The particle filter uses a state-space representation shown below.

$$\begin{cases} \mathbf{x}_t = f(\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{v}_t), & \mathbf{v}_t \sim N(\mathbf{0}, \mathbf{V}_t) \\ \mathbf{y}_t = h(\mathbf{x}_t, \mathbf{u}_t, \boldsymbol{\varepsilon}_t), & \boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \mathbf{E}_t) \end{cases} \quad (1)$$

Where \mathbf{x}_t is the state vector at time t , \mathbf{u}_t is the input vector, \mathbf{y}_t is the output vector, \mathbf{v}_t is the system noise vector, $\boldsymbol{\varepsilon}_t$ is the observation noise vector, \mathbf{V}_t is the system noise covariance matrix, \mathbf{E}_t is the observation noise covariance matrix. The parameters to be identified are incorporated into the state, and many random numbers (particles) are generated. Then the particles are duplicated according to the likelihood evaluated from the analytical results calculated by the seismic analysis models with each parameter (particle) and observation data. The particles with low likelihood are vanished. The duplicated particles are taken as the state of the next step. Finally, the appropriate parameters can be identified by repeating this procedure at each step. In this study, we used a merging particle filter [5] that can avoid reducing particle diversity. The number of margined particles n is set to three.

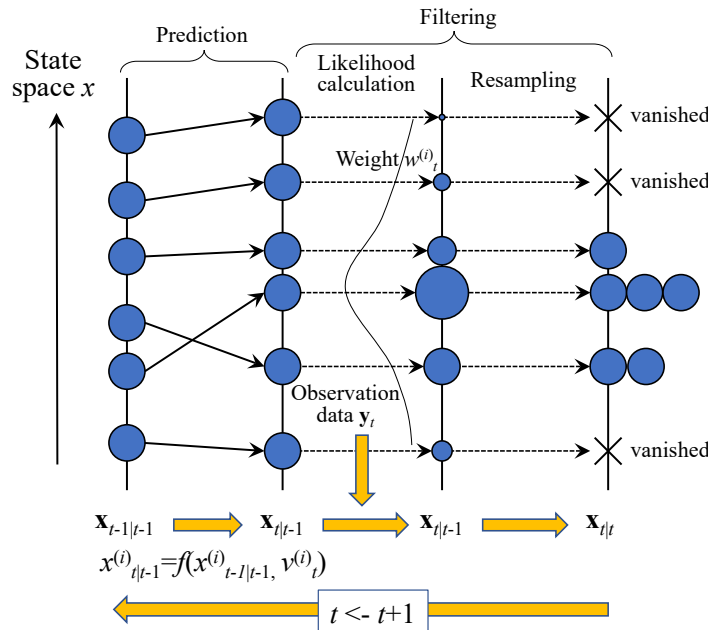


Figure 1 Outline of particle filter

Estimation of Response on Unobserved Floor of 2DOF System

In this study, the seismic response of an unobserved floor is predicted using the data obtained from the seismic response analysis of the shear 2DOF system. Figure 2 shows the analysis model. In this study, we assume that strong motion observations are performed only on the bottom floor and the roof floor. These data are used to predict the seismic response of the non-observation floor (second floor).

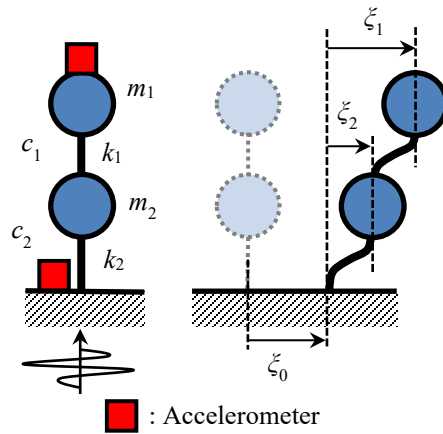


Fig. 2 Analysis model

When the restoring force characteristics of each layer are modeled using the Bouc-Wen model [6][7], the equation of motion of the 2DOF system is expressed by the following equations.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \ddot{\xi}(t) + \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{bmatrix} \dot{\xi}(t) + \begin{bmatrix} \alpha_1 k_1 & -\alpha_1 k_1 \\ -\alpha_1 k_1 & \alpha_1 k_1 + \alpha_2 k_2 \end{bmatrix} \xi(t) + \begin{bmatrix} (1-\alpha_1)k_1 & 0 \\ -(1-\alpha_1)k_1 & (1-\alpha_2)k_2 \end{bmatrix} \begin{bmatrix} z_1(\xi_1 - \xi_2, t) \\ z_2(\xi_2, t) \end{bmatrix} = - \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \ddot{\xi}_0(t) \quad (2)$$

$$\dot{z}_i(r, t) = \frac{1}{x_{y,i}} \left\{ A_i \dot{r}(t) - \beta_i \dot{r}(t) |z_i^n(r, t)| - \gamma_i |\dot{r}(t)| |z_i^n(r, t)| \right\}, n = 1, 3, 5, \dots \quad (3)$$

Where m_i are the mass of each floor, c_i is the damping coefficient, and k_i is the initial stiffness of floor i . $\xi(t)$ and $\xi_0(t)$ is the displacement vector of the masses and ground, respectively. z_i is a non-observable hysteretic parameter, A , β , γ , $x_{y,i}$ and n are dimensionless quantities controlling the behavior of the model. Table 1 shows the set parameters.

Table 1 Parameter of Bouc-Wen

m_1	m_2	k_1	k_2	c_1	c_2	A_1	A_2	n_1	n_2
8	10	700	800	0	0	1	1	1	1
α_1	α_2	β_1	β_2	γ_1	γ_2	x_{y1}	x_{y2}		
0.95	0.9	0.3	0.5	0.9	0.7	0.01	0.01		

The north-south component of the strong motion record observed at JR Takatori station during the 1995 Hyogo-ken Nanbu Earthquake occurred in Japan was used as the input motion. The time step of seismic analysis was set to 0.001 second. Assuming strong motion observation, the likelihood is calculated using the absolute acceleration obtained every 0.01 second (Fig. 3). The standard deviation of the observation noise was set to 1 m/s², and the prior distribution in the particle filter was set to uniform distribution. The prior distribution and system noise settings are shown in Table 2.

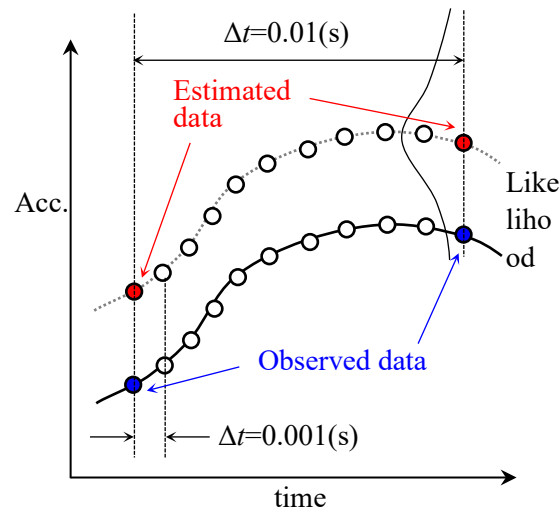


Figure 3 Likelihood calculation method

Table 2 Parameter setting of particle filter

Parameter	Min. and max. value of prior distribution (Uniform distribution)	Standard deviation of system noise
$\xi_1(t), \xi_2(t)$	-0.001 ~ 0.001	0.00001
$\dot{\xi}_1(t), \dot{\xi}_2(t)$	-0.005 ~ 0.005	0.00001
$\ddot{\xi}_1(t), \ddot{\xi}_2(t)$	-0.01 ~ 0.01	0.00001
$z_1(t), z_2(t)$	-0.1 ~ 0.1	0.0000001
k_1, k_2 (kN/m)	500 ~ 1000	0.1
c_1, c_2 (kNs/m)	0	0
α_1, α_2	0.5 ~ 1.0	0.0001
β_1, β_2	0.05 ~ 0.5	0.0001
γ_1, γ_2	0.2 ~ 0.6	0.0001
x_{y1}, x_{y2} (m)	0.005 ~ 0.02	0.00001

The number of particles was set to 100, 200, 500, and 1000. 100 cases were calculated for each to investigate the variation due to the random number of particles. Figure 4 shows the relationship between the number of particles and the root-mean-square error (RMSE) of the observed and estimated acceleration for each floor. The larger the number of particles, the smaller the RMSE. Particularly, In the case of 1000 particles, the RMSE is extremely small.

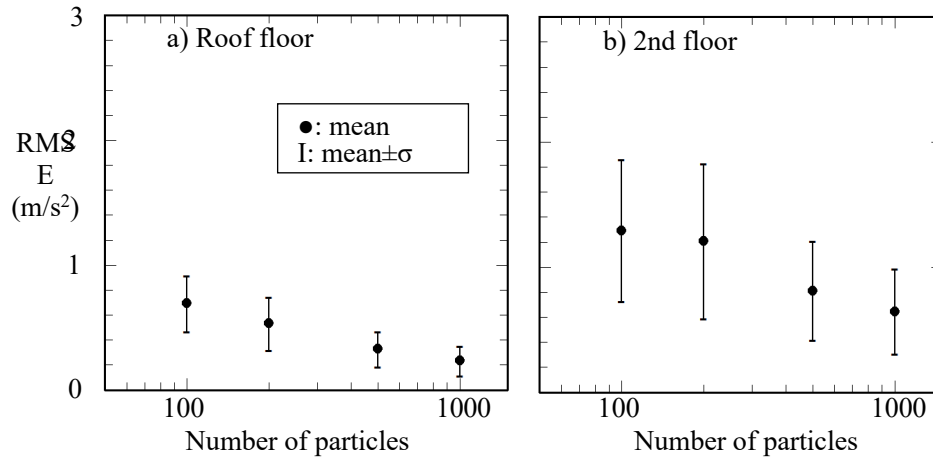


Figure 4 Relationship between number of particles and RMSE of absolute acceleration on each floor

Figure 5 shows an example of identification results of k_1 , k_2 and the absolute acceleration of each floor when the number of particles is 1000. It can be seen that k_1 and k_2 converge to the true values as time progresses, and the acceleration of the unobserved floor (2nd floor) is predicted with good accuracy. Based on this, the number of particles is assumed to be 1000 in the following discussion.

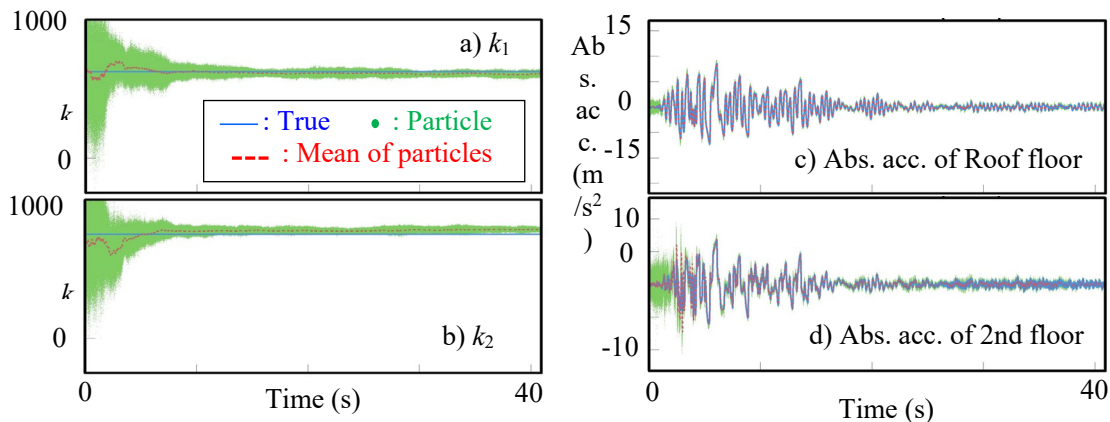


Figure 5 Example of result of identification (Particle number = 1000)

Verification by Seismic Response Analysis of RC Two-Story Building

Next, a particle filter is applied to the data obtained from the seismic response analysis assuming a two-story RC building. The analysis model was the same as in fig. 2, and the mass of each floor was set to 400 tons. The restoring force characteristic is a degrading trilinear model, and the damping characteristic was proportional to the tangent stiffness. The north-south component of the strong motion record observed at the JR Takatori station was used as the input motion. It is assumed that strong motions were observed on the bottom floor and the roof floor. The response of the second floor is predicted using these observation records. The parameters of the prior distribution and system noise were set as shown in Table 3.

Table 3 Parameter setting of particle filter

Parameter	Min. and max. value of prior distribution (Uniform distribution)	Standard deviation of system noise
$\zeta_1(t), \zeta_2(t)$	-0.0005 ~ 0.0005	0
$\dot{\zeta}_1(t), \dot{\zeta}_2(t)$	-0.01 ~ 0.01	0
$\ddot{\zeta}_1(t), \ddot{\zeta}_2(t)$	-0.1 ~ 0.1	0
$z_1(t), z_2(t)$	-0.01 ~ 0.01	0
k_1, k_2 (kN/m)	5000 ~ 20000	10
c_1, c_2 (kNs/m)	1000 ~ 6000	1
α_1, α_2	0.1 ~ 0.5	0.0002
β_1, β_2	0.05 ~ 0.5	0.0002
γ_1, γ_2	0.2 ~ 0.8	0.0002
x_{y1}, x_{y2} (m)	0.002 ~ 0.01	0.000002

Fig. 6 shows the time histories of the relative displacement and absolute acceleration response estimated by the particle filter with the Bouc-Wen model with $A = 1, n = 11$. The relative displacement, relative velocity, and absolute acceleration of each floor including non-observed floors are predicted with good accuracy.

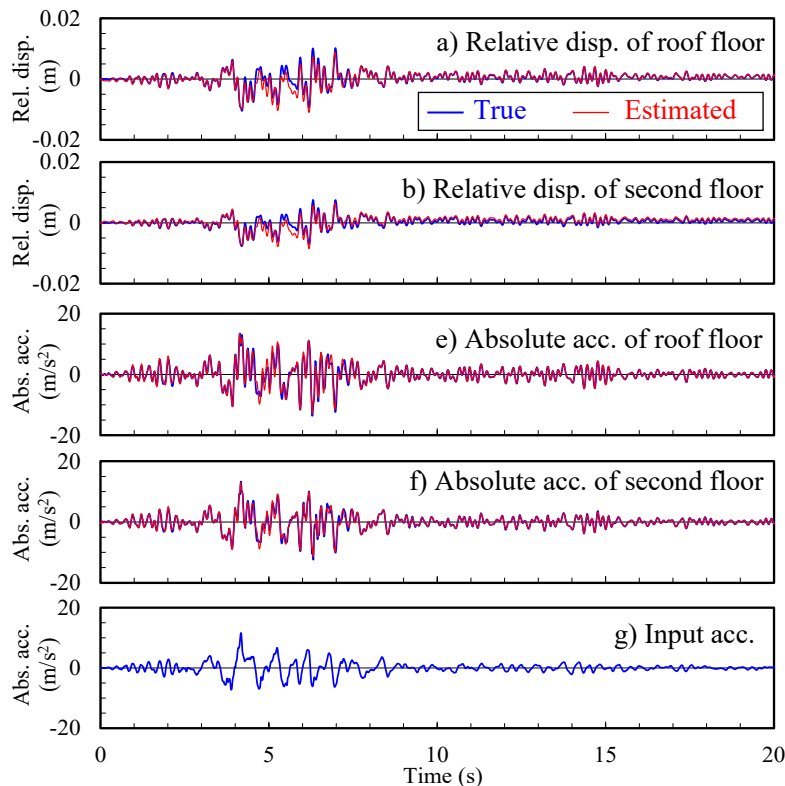


Figure 6 Time history of seismic response of

Fig. 7 compares the restoring force characteristics in the analysis and the estimated results. Although the inter story drift was slightly different, the estimated restoring force characteristics of 1st and 2nd floor generally corresponded to those of the true ones. In particular, the 2nd floor has not yet yielded, and the yield load and yield displacement of the 1st floor were well identified. The maximum inter-story displacement on the 1st floor also corresponds roughly. These results show

that the method proposed in this study enables us to predict the position and degree of damage of the building based on the strong motion records at fewer points.

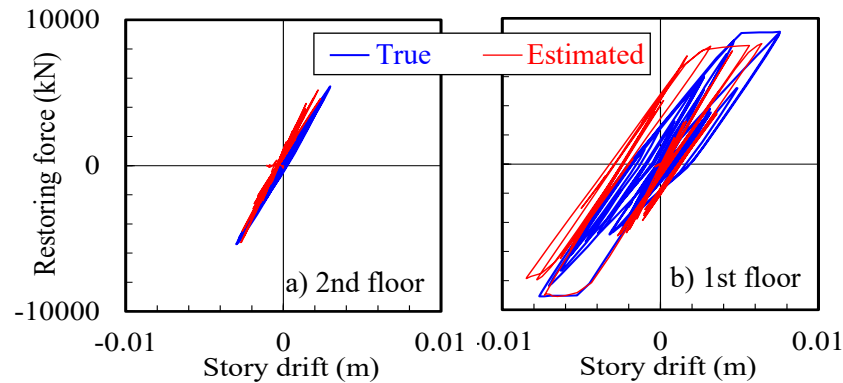


Figure 7 Restoring force characteristics

Conclusion

In this study, based on the nonlinear seismic response analysis of the 2DOF system, we investigated the method for estimating the seismic response of the unobserved floor using the Bouc-Wen model and the particle filter. The following conclusions were obtained.

- 1) In this study, when the number of particles was 1000, the seismic response of the unobserved floor of the building can be predicted with good accuracy.
- 2) It was possible to predict with good accuracy even for buildings whose restoring force characteristics differed from that of the Bouc-Wen model.
- 3) By identifying the restoring force characteristics exhibiting strong nonlinearity, the seismic damage such as yielding of a building can be evaluated.
- 4) It is also possible to predict the building response on non-observed floors. This result suggests that the damaged parts of the building can be identified based on the strong motion records at fewer points.

Acknowledgments

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