Materials Research Proceedings **27** (2023) 8-16 https://doi.org/10.21741/9781644902455-2

Long-term cable vibration monitoring and cable tension estimation of a cable-stayed bridge

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Keywords: Ambient Vibration, Cable-Stayed Bridge, Cable Tension Identification, Environmental and Operational Variations, Long-Term Monitoring

Abstract. This study aims to investigate the applicability of ambient cable vibrations for cable tension estimation and the identification uncertainty and effect of EOVs in the long-term SHM of cable tensions. An advantage of long-term ambient vibration monitoring is that there is no need to close roads for the monitoring campaign once a monitoring system is installed. A disadvantage of long-term environmental vibration monitoring is the difficulty in dealing with uncertainties caused by environmental and operational variations (EOVs). A Bayesian approach to quantify uncertainties in monitoring is thus proposed for the identification of cable tension. Variations of the identified cable tension in the short- and long-term monitoring are examined to discuss the need for normalization of EOVs in damage detection. Long-term monitoring of the cable-stayed bridge showed that it is possible to estimate cable tension using ambient vibration measurements, but that the seasonal variation is greater for longer cables than for shorter cables, making it clear that a trend component of the seasonal variation needs to be taken into account.

Introduction

The cable-stayed bridge, with excellent performance for long-span crossing, has been widely constructed around the world. As a crucial component in this structure, the stayed cable is always faced with long-term deterioration caused by corrosion, fatigue, etc. For the management and maintenance of the cable-stayed bridge, it is of great meaning to conduct the real-time long-term SHM in the stayed cables, among which the dynamic characteristics and cable tension are acknowledged as two informative features reflecting the condition of cables and bridge.

Without the request of artificial excitation, ambient-vibration-based long-term SHM offers a promising way for realizing remote and economical monitoring of bridges. There have been many types of research such as frequency domain decomposition (FDD), stochastic subspace identification (SSI), a series of Bayesian operational modal analysis methods (e.g. Fast Bayesian FFT), etc., which make the ambient-vibration-based modal analysis efficient and flexible. Further, cable tension, as a more intuitional physical feature, has also been investigated in the relation to the dynamics of cables [1–3].

By examining the estimated cable tension in long-term SHM, it is believed that potential damage effects in cables can be traced timely. However, there are still many issues remaining in the ambient-vibration-based long-term SHM. One is the low signal-to-noise ratio (SNR) with weak excitation which makes the identification uncertainty prominent. Another one is the effect of environmental and operational variations (EOVs), which raises the variability of long-term records in SHM. Therefore, to make a deep perception of these issues, this study investigates ambient-vibration-based cable tension estimation and the identification uncertainty and EOVs-induced variability in the long-term monitoring of cable tensions with Bayesian approaches.

Theoretical background

Fast Bayesian FFT

As one of the ambient-vibration-based operational modal analysis methods, a fast Bayesian FFT approach [4,5] is introduced here. By associating Bayesian inference with the FFTs of vibration response, Bayesian FFT gives a basic form as

$$p(\theta|\hat{\mathcal{F}}_k) = p(\hat{\mathcal{F}}_k)^{-1} p(\hat{\mathcal{F}}_k|\theta) p(\theta) \tag{1}$$

where θ denotes the system parameters of the structure to be identified, and $\hat{F}_k$ are the estimated FFTs data at different frequencies f_k.

Assuming that the power spectral density (PSD) is complex Gaussian distribution and independent at different frequencies, the posterior distribution in Eq. 1 has a linear relationship with the likelihood function as follows.

$$p(\theta|\hat{\mathcal{F}}_k) \propto p(\hat{\mathcal{F}}_k|\theta) = \frac{\pi^{-nN_f}}{\prod_k |E_k(\theta)|} exp\left[-\sum_k \hat{\mathcal{F}}_k^* E_k(\theta)^{-1} \hat{\mathcal{F}}_k\right] = e^{-L(\theta)} \tag{2}$$

The theoretical PSD matrix of data at the kth FFT for given θ is shown in Eq. 3.

$$\mathrm{E}_k(\theta) = E(\hat{\mathcal{F}}_k \hat{\mathcal{F}}_k^{\ *}|\theta) + E[\varepsilon_k \varepsilon_k^* | \theta] = \sum_{i=1}^m \sum_{j=1}^m h_{ik} h_{jk}^{\ *} S_{ijk} \phi_i \phi_j^{\ T} + S_e \mathrm{I}_n = \Phi\ \mathrm{H}_k \Phi^T + S_e \mathrm{I}_n \tag{3}$$

The 'negative log-likelihood function' (NLLF) of Eq. 2 can be written as Eq. 4. The most probable value (MPV) of θ can be estimated as, $\hat{\theta} = \arg\min_\theta L(\theta)$.

$$L(\theta) = nN_f ln\pi + \sum_k ln\,|E_k(\theta)| + \sum_k \hat{\mathcal{F}}_k^* E_k(\theta)^{-1} \hat{\mathcal{F}}_k \tag{4}$$

where, the system parameter θ comprises modal frequencies $f_{i_{i=1}}^{r}$ and modal damping ratios $\zeta_{i_{i=1}}^{r}$ denoted in transfer functions $h_{ik_{i=1}}^{r}$ corresponding to each mode, partial mode shapes $\varphi_{i_{i=1}}^{r}$, PSD matrix of modal forces $S = [S_{ij}]_{r\times r}$, and the PSD matrix of prediction errors $S_e I_n$. In addition, r represents the number of dominant modes in a specified frequency band where the estimation is conducted. n is the number of sensors to collect the ambient vibration response. N_f is the number of FFT points in the specified frequency band.

Bayesian cable tension estimation

The relation between the modal frequency of cable and cable tension can be derived from the free vibration differential equation of cable as follows.

$$m\frac{\partial^2 v(x,t)}{\partial t^2} + EI\frac{\partial^4 v(x,t)}{\partial x^4} - T\frac{\partial^2 v(x,t)}{\partial x^2} - h(t)\frac{\partial^2 v(x,t)}{\partial x^2} = 0 \tag{5}$$

where $v(x,t)$ denotes the vertical vibration deflection, x is the longitudinal coordinate of the cable and t denotes time. The symbol m is the mass of the cable per unit length, EI denotes the flexural rigidity of the cable and T is the cable tension force. The notation $h(t)$ is the dynamic tension.

According to [1–3], the influence of ambient vibration-induced dynamic cable tension $h(t)$ and the cable sag is generally small and ignorable for simplicity. Assuming that the boundary condition is simply supported, the solution of Eq. 5 can be presented as follows.

$$\left(\frac{f_i}{i}\right)^2 = \frac{\pi^2 i^2}{4ml^4} EI + \frac{1}{4ml^2} T \tag{6}$$

where i is mode order and f_i denotes the ith modal frequency of the cable; l is the length of cable.

Then, when the cable vibration is more similar to a string (the contribution of *EI* on modal frequency is rather small), the equation can be further simplified as follows.

$$\left(\frac{f_i}{i}\right)^2 = \frac{1}{4ml^2}T \tag{7}$$

With identified modal frequencies of the cable, the estimation of cable tension from Eq. 6 and Eq. 7 can be treated as a regression problem. A basic form of the Bayesian linear regression (BLR) model can be written as follows.

$$y = X\beta + \varepsilon, \varepsilon \sim N(0, \sigma^2) \tag{8}$$

where y is an $n \times 1$ vector of response variable; X is an $n \times d$ matrix of predictor variables; β is a $d \times 1$ vector of coefficients; ε denotes the *iid* error term which obeys a normal distribution with zero mean and variance σ^2 for each observation; n is the number of observations, and d is the number of predictor variables. The Bayesian inference can then be used to obtain the posterior distribution of (β, σ^2) as follows.

$$p(\beta, \sigma^2 | y, X) = p(y|X)^{-1} \cdot p(y|X, \beta, \sigma^2) \cdot p(\beta, \sigma^2) \tag{9}$$

Further, the marginal posterior of β can be given as,

$$p(\beta | y, X) = \int p(\beta, \sigma^2 | y, X)\, d\sigma^2 \tag{10}$$

When the Jeffreys non-informative prior is given as Eq. 11,

$$p(\beta, \sigma^2) \propto \frac{1}{\sigma^2} \tag{11}$$

the marginal posterior of β is analytically tractable and follows a d dimensional *t-location-scale* distribution shown in Eq. 12.

$$p(\beta | y, X) \sim t_d\left(\hat{\beta}, \frac{(y - X\hat{\beta})'(y - X\hat{\beta})}{n - d}(X'X)^{-1}, n - d\right) \tag{12}$$

where the three parts in the right hand are the location parameter, scale parameter, degree of freedom, in sequence. $(\cdot)'$ represents a transposition of $(\cdot)$. The notations (y, X, β, n, d) are the same as Eq. 8, while $\hat{\beta}$ is the least-squares estimate of β with a form as follows.

$$\hat{\beta} = (X'X)^{-1}X'y \tag{13}$$

Without loss of generality, taking Eq. 6 into the form as Eq. 8, the Bayesian cable tension estimation framework can be established as shown in Eq. 14.

$$y = \left\{\left(\frac{f_i}{i}\right)^2\right\}_{n\times 1}, X = \begin{bmatrix} \frac{\pi^2 i^2}{4ml^4} & \frac{1}{4ml^2} \end{bmatrix}_{n\times 2}, \beta = \begin{Bmatrix} EI \\ T \end{Bmatrix}_{2\times 1} \tag{14}$$

The posterior distribution of β contributes to a simultaneous estimation of cable tension and flexural rigidity, along with the estimation uncertainty.

Predictive probabilistic model considering ensemble variability

The Bayesian estimates of cable tensions in long-term SHM is a sequence involving identification uncertainty within each estimate, and EOVs-induced variability among different estimates. Under the framework of the Bayesian cable tension estimation, the identification uncertainty can be clarified by posterior variance. Furthermore, to represent the ensemble variability by integrating the identification uncertainty and EOVs-induced variability in long-term SHM, a predictive probabilistic model is proposed with a mixture model [6].

Assuming that the long-term SHM is conducted over a rather long period with sufficient data sets covering almost all the environmental and operational situations (EOSs) of a bridge in general state, the predictive probabilistic model of cable tension at a certain future time (without any other information which implies corresponding EOS) is given by a mixture model under the theorem of total probability as follows.

$$p(T_f|D) = \sum_{i=1}^{N_s} p(EOS_i|D)p(T_f|D, EOS_i) = \frac{1}{N_s}\sum_{i=1}^{N_s} p(T|D_i) \quad \textbf{(15)}$$

where $p(T_f|D)$ denotes the predictive probabilistic model of cable tension T_f at a certain future time, given the past long-term SHM data sets $D = \{D_i\}_{i=1}^{N_s}$. The probability of the case that unknown EOS in a certain future time corresponds to the EOS in either segment D_i in the past long-term SHM is assumed to be equal as $p(EOS_i|D) = 1/N_s$, without any additional information indicating corresponding EOS. $p(T_f|D, EOS_i)$ represents the predictive distribution of cable tension T_f at a certain future time with a definite EOS corresponding to that of D_i, and is equal to the posterior distribution $p(T|D_i)$ acquired by Bayesian cable tension estimation at corresponding data segment D_i.

The predictive probability model can be regarded as an integration of the identification uncertainty in the long-term SHM and the variability due to EOVs, representing the ensemble variability of cable tension over a long period of similar length under general bridge condition. Then, the damaging effect may be compared with the ensemble variability that offers information for the management of cable-stayed bridges.

Ambient vibration monitoring on a cable-stayed bridge

Target bridge and monitoring system

The target bridge is a single-tower cable-stayed bridge shown in Fig. 1. The span length of the bridge is about 124 m and the height of the pylon is about 48 m. A short-term ambient vibration test was carried out in November 2020. The corresponding sensor setup and structural layout are shown in Fig. 2. Ambient-vibration signals from cables at the anchor, cables at the bridge deck, bridge deck, and the pylon were collected during the short-term test. Further, the long-term SHM of two cables (the longest one (C1) and the shortest one (C5)) at the bridge deck was conducted from December 2020 to January 2022, with the ambient vibration signals recorded remotely.

Fig. 1. A side view of the cable-stayed bridge.

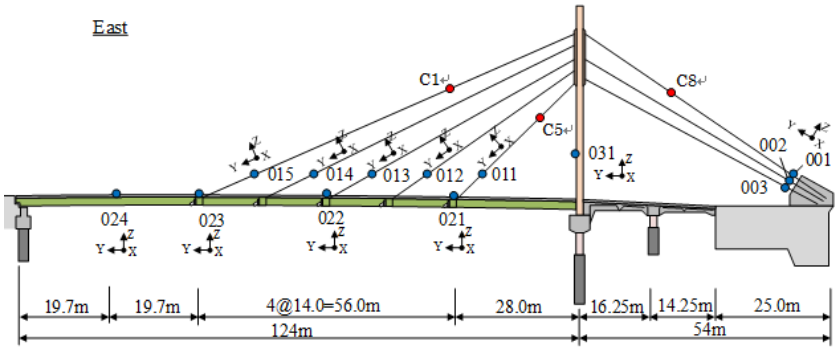


Fig. 2. Layout and sensor setup of the short-term SHM system.

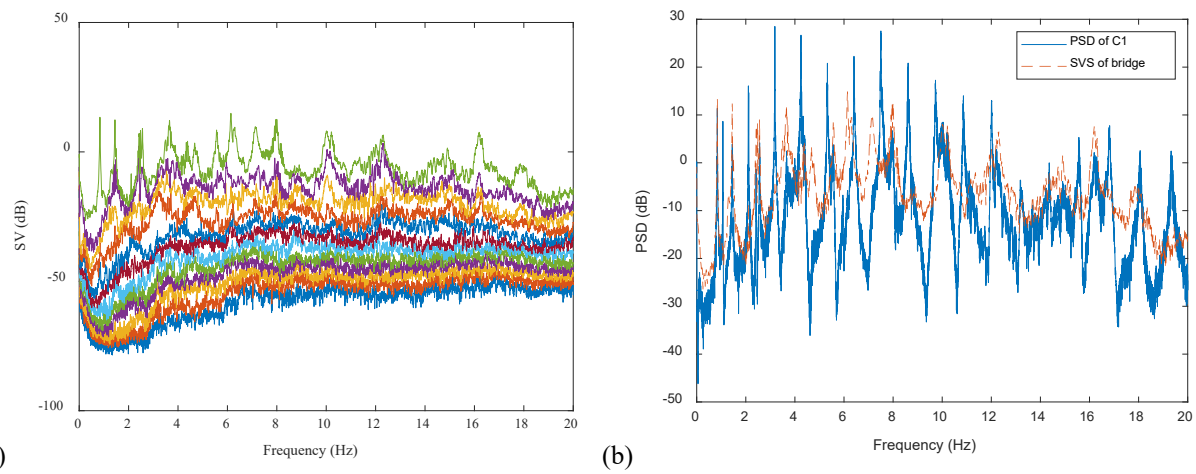


Fig. 3. (a) SVS of bridge and (b) PSD of cable C1.

Table 1. Results of operational modal analysis of cables by Bayesian FFT.

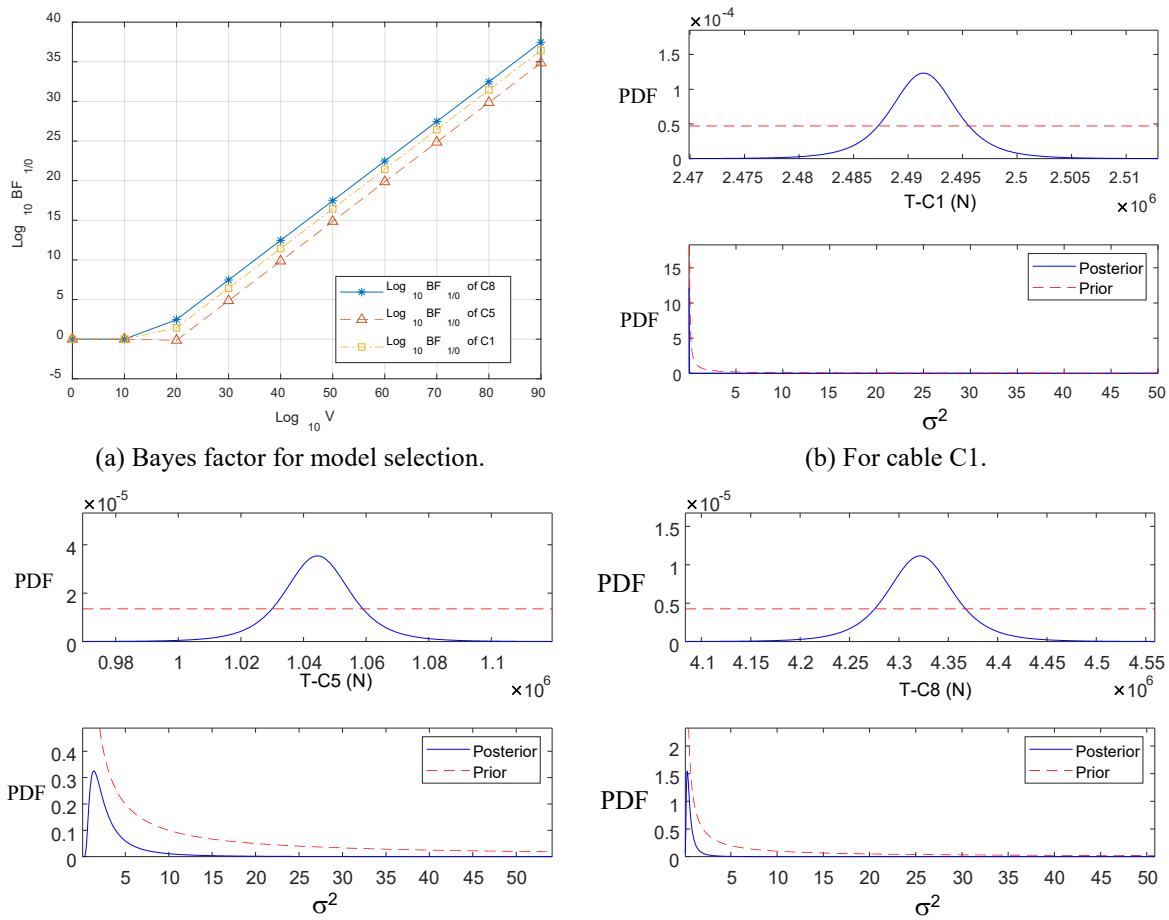
Cable	[Modal frequency (Hz); standard deviation (Hz)] for each modal order				
	1	2	3	4	5
C1	1.07; 0.0010	2.11; 0.0012	3.18; 0.0009	4.24; 0.0020	5.31; 0.0013
C5	2.07; 0.0013	4.14; 0.0014	6.26; 0.0031	8.45; 0.0058	10.63; 0.0035
C8	1.64; 0.0026	3.32; 0.0088	4.96; 0.0040	6.65; 0.0057	8.14; 0.0033

Operational modal analysis and cable tension estimation

To get the dynamic characteristics of the bridge and cables, the fast Bayesian FFT was first applied for operational modal analysis of the bridge and cables. In this paper, three typical cables (C1, C5, and C8 cables as marked in Fig. 2) are selected to simplify the discussion even though all the cables were measured during the short-term test. As a pre-step for specifying initial values and searching band of optimization algorithm, the singular value spectrum (SVS) of the bridge (see Fig. 3(a)) and the power spectral density (PSD) plots of cables (see Fig. 3(b)) were investigated. Details about the fast Bayesian FFT with SVS can be found in [4, 5]. In the PSD plot of cables shown in Fig. 3(b), the first SVS line of the bridge is overlapped to help eliminate frequencies originating from the global modes of the bridge. It can be noted from Fig. 3(b) that some global modes of the bridge appearing in the SVS also appear in the PSD of cables, which indicated the ambient vibration of stayed cable was mixed with interference from the global vibration of the bridge. The mean and standard deviation of the identified frequencies from the ambient-vibration-based operational modal analysis are summarized in Table 1.

The proposed Bayesian cable tension estimation is applied to identify cable tensions from ambient vibrations. A model selection between Eq. 6 for the simply supported beam theory and

Eq. 7 for the string theory was carried out by the means of Bayes factor $B_{1/0}$ which is the ratio of two model evidence with two hypotheses H_0 and H_1. Therein, H_0 denotes the null hypothesis meaning that identified modal frequencies support Eq. 6 more, i.e. the effect of the flexural rigidity on cable tension identification cannot be ignored, while H_1 denotes the alternative hypothesis meaning that the data support Eq. 7 more, i.e. the effect of flexural rigidity can be ignored. The Bayes factor along with increasing uncertainty in the prior distribution is presented in Fig. 4(a), where V is a hyper-parameter of Gaussian-Inverse-Gamma conjugate prior which controls the uncertainty in the prior of parameters (*EI*, *T*). It indicates that with less prior information about cable tension, the identified modal frequencies support the string theory more, i.e., the effect of flexural rigidity can be ignored in the subsequent procedure of the Bayesian cable tension estimation. The estimated cable tension using the Bayesian linear regression is shown in Figs. 4(b-d). It is noted that the identification uncertainty in the three cables was different; i.e., the lowest identification uncertainty was observed at the longest cable at the bridge deck (C1 cable).



(a) Bayes factor for model selection. (b) For cable C1.

(c) For cable C5. (d) For cable C8.

Fig. 4. (a) Bayes factor along with increasing uncertainty in prior distribution of cable tension; (b)(c)(d) Prior and posterior estimates of cable tension T and error variance σ^2.

Finite element analysis (FEA)

As a verification of the ambient-vibration-based operational modal analysis and cable tension estimation by field test, a finite element model of the cable-stayed bridge was also created as shown in Fig. 5. The simulated cable tensions are shown in Table 2 compared with the ambient-vibration-based estimates and the design value. From Table 2, it can be noted that the FEA and design value match well as the design value was also decided by means of FEA with beam elements during the

design stage. The ambient-vibration-based estimates for the cable at the anchor were close to the FEA and design values, while the estimates for the cables at the bridge deck were lower than those of FEA and design values. A possible reason for this phenomenon may be the complexity of ambient vibration for the cable at the bridge deck end. On one hand, the ambient vibration of the cable is coupled with the bridge deck. On the other, the vibration model of cable at the bridge deck can be viewed as a string with vibrating support, which is kind of different from the above string model. These two aspects may further decrease the rigidity of cable at the bridge deck, which is not considered in Eqs. 5, 6, and 7.

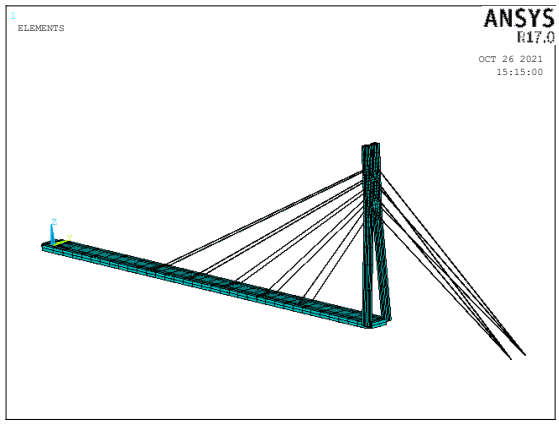


Fig. 5. Finite element model of the cable-stayed bridge.

Table 2. Comparison of Bayesian cable tension estimates, FEA and design value.

Cable	Sensor	Bayesian estimates	FEA	Design
		Tension (kN)	Tension (kN)	Tension (kN)
C1	015	2505	3127	3120
	115	2494	3140	
C5	011	1056	1200	1360
	111	1082	1209	
C8	001	4374	4256	4400
	101	4422	4257	

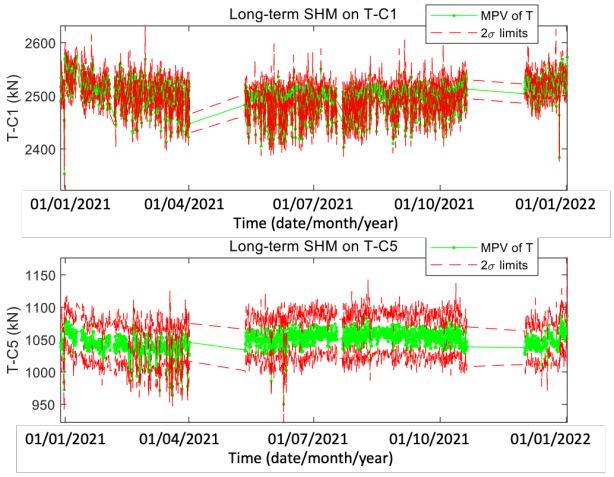


Fig. 6. Long-term sequence of Bayesian cable tension estimates (upper plot for C1; lower plot for C5).

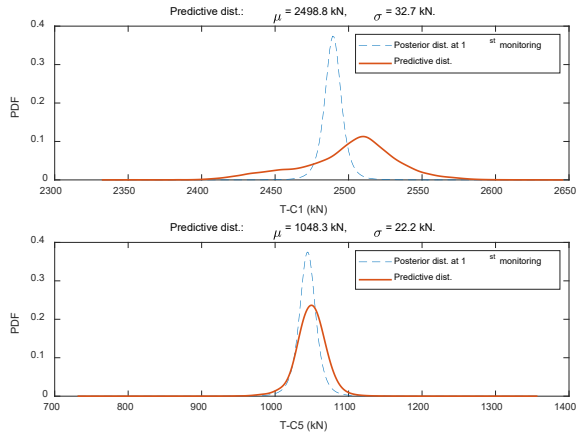


Fig. 7. Ensemble variability involving the identification uncertainty and EOVs (upper plot for cable C1, lower plot for cable C5).

Ambient-vibration-based long-term SHM of cables

Long-term ambient vibration monitoring was also conducted for the longest and the shortest cables at the bridge deck, i.e. C1 and C5 cables. Although an inconsistency between the ambient-vibration-based cable tension estimates and the FEA was observed for the cables at the bridge deck in the last section, the ambient-vibration-based results can still be used as a nominal damage-sensitive feature for anomaly indication in long-term SHM with its convenience. The long-term sequence of Bayesian cable tension estimates is shown in Fig. 6. Observing the sequence of cable tension estimates, it can be noted that the identification uncertainty and the effect of EOVs in two cables occupied different extents. Further, by using the predictive probabilistic model introduced before, the predictive distribution for an unknown future time point is given by Fig. 7 with a depiction of ensemble variability. From Fig. 7, it is clearly noted that the effect of EOVs is dominant in the longest cable at the bridge deck, while for the shortest cable, the identification uncertainty contributes more to the ensemble variability. This observation may offer some guidance for the research of seasonal effects in long-term SHM of cable tensions.

Conclusions

This paper investigates the Bayesian operational modal analysis and cable tension estimation of a cable-stayed bridge using ambient vibration and investigated the identification uncertainty and effect of EOVs in long-term SHM of cables.

For the operational modal analysis, the analysis showed that the interference from the bridge vibration to the cables should be noticed, which may further affect the accuracy of identified modal frequency and cable tension.

An ambient-vibration-based Bayesian cable tension estimation method was introduced and verified comparing with the design value and FEA. It is noted that the estimates for the cable at the bridge anchor are close to the design value as well as FEA, while the estimates for the cables at the bridge deck are overall lower than the design value and FEA, which is inferred as a result of the different physical models of cables at bridge deck and ground anchor side, respectively.

The identification uncertainty and effect of EOVs in cable tensions were investigated in the long-term SHM. It is noted that the effect of EOVs is more prominent in the longest cable than the shortest one, which indicated the longer cables in the cable-stayed bridge may be more sensitive to the EOVs and it may be worthwhile to conduct data normalization in these cables.

Acknowledgments

This study is supported by Research Committee on Technological Development for Infrastructure and Japanese Society for the Promotion of Science (JSPS) Grant-in-Aid for Scientific Research

(B) under project No.22H01576. Those technical and financial supports are gratefully acknowledged.

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