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Design of architected materials composed by periodic surfaces

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Abstract. Architected materials possess extraordinary mechanical properties, that cannot be met by natural materials, either in the static and in the dynamic range. For this reason, the study of metamaterials is a very active field of research. Although lattice architectures are commonly preferred, especially for the relative simplicity of manufacturing, geometries based on thin shells may present considerable advantages. Indeed it has been shown that shell metamaterials, still preserving lightness and versatility, can present excellent stiffness and strength properties, and ability to absorb strain energy by means of very large deformations, that make them useful for the design of shock absorbing elements. In the contribution will be examined a class of shell metamaterials, composed of triple periodic minimal surfaces TPMS, that having zero mean curvature can be in equilibrium with an isotropic state of stress. Two particular surfaces, the Schwarz P and the Schwarz G minimal surfaces, the first centro-symmetric, the second chiral, are examined.

Introduction

Metamaterials represent an innovative approach to the problem of obtaining unusual or extreme physical responses for advanced applications. Since their extreme macroscopic responses depend primarily on the internal low scale pattern, the understanding of how the microstructure topology influences the macroscopic properties is the key-point in the design of new advanced metamaterials.

Metamaterials are usually classified into three-dimensional and surface (planar) materials and are frequently based on elementary geometric patterns.

Periodic cellular materials are characterised by a unit cell that can be translated through them. If the cell is translated in two dimensions, they are designated prismatic cellular materials (e.g., honeycomb), while if the cell is translated in three dimensional periodicity, then they give rise to cellular structures.

A common typology of periodic material is constituted by truss lattices, in which case the unit periodic cell can assume several forms, that have been widely investigated. Hutchinson and Fleck (2006), Thomsen et al. (2018) performed a topological optimization of 2D periodic materials undergoing buckling type instabilities. Moreover, the non-linear response of planar periodic materials has been analyzed by Vigliotti et al. (2014) [6].

Shell lattice structures

AM has enabled the design and manufacturing of cellular structures whose unit cells are composed of plates or shells rather than struts. These lattice structures are commonly described as Triply Periodic Minimal Surfaces (TPMS)-like (though their surfaces do not necessarily have zero mean curvature) and are referred to as “shell lattices”.

In this way cellular materials are obtained, that can be closed or open as in natural foams. Manufacture of closed cell plate lattices remains problematic for powder-based AM systems due to the requirement of powder removal (see figure 1, from [1]). Open cells can be obtained exploiting the properties of the geometry of minimal surfaces.

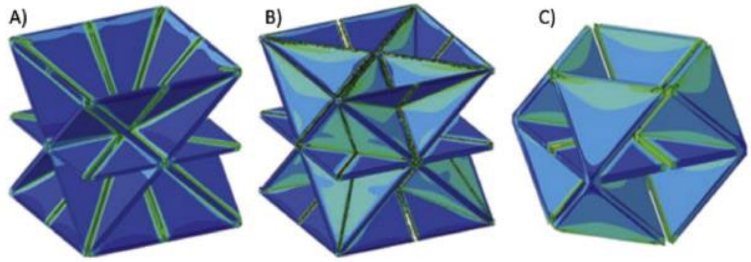

Figure 11. shell lattice cells generated by placing plates on the closest-packed crystals combining simple cubic (SC), BBS and SC-BCC-FCC (B) and SC FCC (C)[1]

Triply periodic minimal surface (TPMS)

A minimal surface is a surface that has a mean curvature equal to zero at every point. When the minimal surface is infinite and periodic in the three independent directions it is called a triply periodic minimal surface (TPMS). TPMS are surfaces that locally minimize surface area for a given boundary [5].

These surfaces have distinctive geometrical characteristics: a minimal surface is smooth everywhere, has no sharp edges or corners, and splits the space into two or more non-intersecting, intertwined and infinite domains that can be repeated periodically in three perpendicular directions.

The first examples of TPMSs were discovered by Schwarz in 1865, followed by his student Neovius in 1883. They described five TPMSs, namely, Schwarz primitive (P), Schwarz diamond (D), Schwarz hexagonal (H), Schwarz crossed layers of parallels, and Neovius (N). The gyroid (G) surface was described by Schoen in 1970, along with another eleven newly discovered TPMSs. In many ways, the G, D, and P surfaces are the most important most commonly observed TPMS structures.

TPMS are widespread in nature. One outstanding example are the cubic membranes, biological membranes formed from lipid bilayer sheets, highly curved. They not only separate the inside and outside of an organism but are also involved in many biological processes, such as selective permeability and energy production.

Single gyroid geometries have been widely discovered in nature, such as alveolar surface of mammalian lung , the prolamellar bodies in plant cells , intracellular cubic membrane in cell organelles , wing scales of green butterflies and exoskeletons of beetles. The SG structure found in the green butterflies is of special interest, exhibiting a complete photonics bandage and polarization dependent optical properties and negative refractions due to inherent chirality.

TPMS can be divided into balanced and unbalanced surfaces. Examples of balanced surfaces are the Gyroid and Primitive surfaces that divide the volume into congruent or interchangeable regions that may (Gyroid) or may not (Primitive) be mirror images of the one another. Unbalanced surfaces such as the I-WP divide the space into labyrinths of unequal morphologies [f14]-[5].

Minimal surfaces can be found by means of form-finding algorithms [cite Bletzinger], However, useful approximations can be obtained using level-set equations derived from a sum defined in terms of the Fourier series [5].

$$\sum_{i=1}^{N} F_i(\boldsymbol{k_i}) \cos(2\pi \boldsymbol{k}_i \cdot \boldsymbol{r} - \boldsymbol{\alpha_{ki}}) = t$$

where F is the amplitude, $\boldsymbol{k}_i$ a parameter that defines the relative dimensions of the periodic cell in the three directions, and α a phase parameter. Particularly simple expressions can describe P D and G surfaces. In this paper we focus on gyroid and Schwarz primitive geometries, whose approximated expressions are given by

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Schwarz primitive:

$$g_P = \cos\frac{2\pi x}{a} + \cos\frac{2\pi y}{a} + \cos\frac{2\pi z}{a} = t$$

Gyroid

$$g_G = \sin\frac{2\pi x}{a}\cos\frac{2\pi y}{a} + \sin\frac{2\pi y}{a}\cos\frac{2\pi z}{a} + \sin\frac{2\pi z}{a}\cos\frac{2\pi x}{a} = t$$

The constant *t* is denoted as the level parameter. The best approximation to minimal surfaces is obtained setting *t*=0.

Varying the value of the level parameter, two families of surfaces are obtained. In the case of P surface, $t \in (-1,1)$, while in the case of the G surface $t \in (-\pi, \pi)$. Figures 2,3 show the surfaces that can be obtained for some values of the level parameter. While for the P surface the sign is critical, it is inessential for the G surface, given its rotational symmetries.

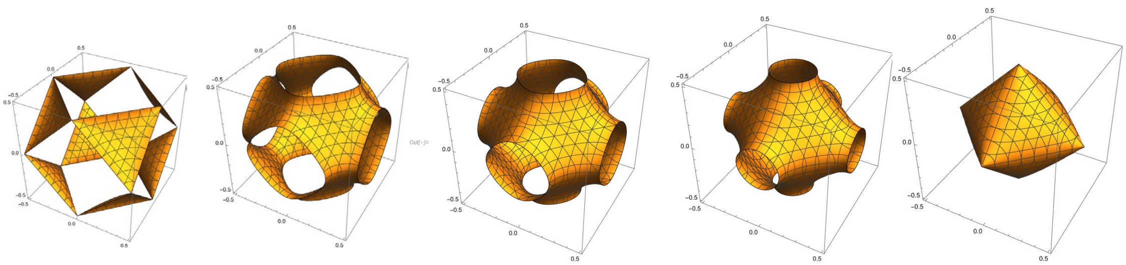


Figure 2. Isolevel surfaces obtained from expression (2), for t=-1, -0.5, 0, 0.5 1. The Schwarz P surface is the one corresponding to t=0.

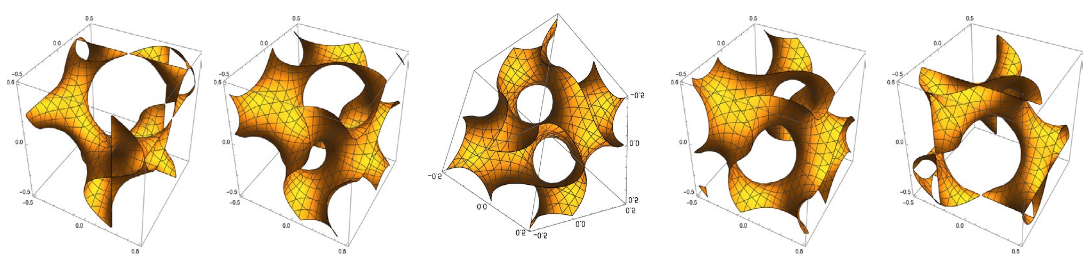
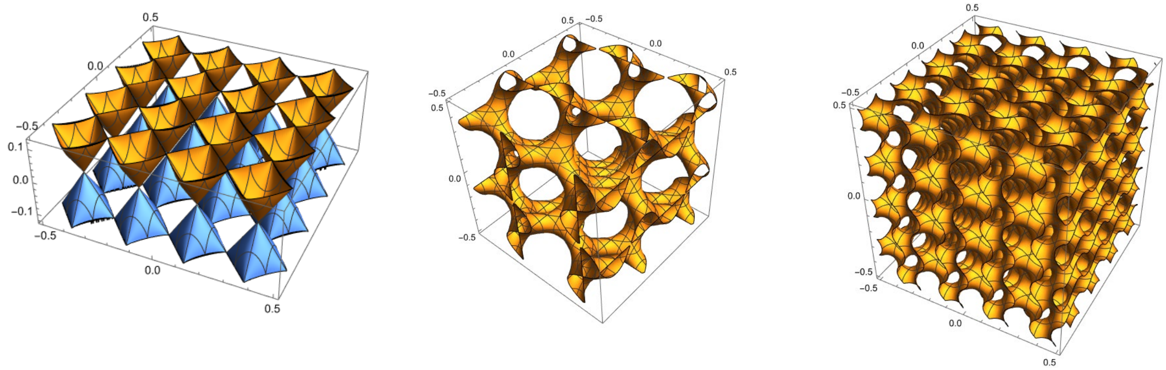


Figure 3. Isolevel surfaces obtained from expression (3)), for t=-1, -0.5, 0, 0.5 1. The Single Gyroid surface is the one corresponding to t=0.

Unit cell for metamaterials can be obtained assigning a thickness to one of the iso-level surfaces. However, this procedure is safe only for very thin surfaces, otherwise there is the risk of intersections. A consistent way for building solid microstructures is obtained using the analytical expressions (2), (3). Since minimal surfaces divide the space in two non connected regions, it is possible to build a so called solid network as the region of space for which $g_G \leq t_1$ (similarly for the P surface). A sheet solid can then be obtained from the intersection of two solid networks with close values of the level parameter. It is observed that replicating the procedure, it is possible to build hierarchical minimal surface unit cells. This aspect will be object of future investigations.

TPMSs are obtained replicating the unit cells in three orthogonal directions. Figure 4 illustrate the procedure. It is interesting to underline that choosing values of the level set parameter close to the extreme values one can approach plate lattices or rod lattices. This possibility makes the present approach very appealing for optimization procedures.



(a) (b) (c)

Figure 4. TPMS obtained as repetition of unit cells. (a) P surface with t=-1, (b) G surface with t=1, (c) G surface with t=-0.25

Among all possible lattices, the unique geometry of the Gyroid minimal surface and its related morphologies makes Gyroid structured materials a fascinating subject across many disciplines including biology, physics, optics and topological photonics. Gyroids, like most biomimetic designs, are mechanically robust as evolution of self-assembly leads to mechanically stable structures.

Gyroids present cubic symmetry and chirality. They bisect space into a pair of 3D labyrinths, one left-handed (LHD) and one right-handed (RHD). These channels resemble a spiral-like shape and are indeed chiral [F20]. With the left- and right-handed channels having opposing handedness, the infinitely periodic gyroid surface has zero net geometric chirality due to equal amounts of LHD and RHD curvature

Physical and Mechanical properties of TPMS-Gyroid structure in the linear range

From an engineering point of view, the main properties that need to be determined in order to investigate the linear response of the material are density and stiffness. Next some preliminary observations about them will be outlined. Similar results can be found in [..] for the density, and in [Maskery] for the elastic properties.

Figure 5a shows a plot of the relative density for a P sheet network as a function of the level parameter. The relative density is defined as the volume occupied by the sheet with respect to the volume of the bounding cube. Three different values of the thickness are considered, viz., h/L= 0.05, 0.1, 0.15, with L the size of the bounding cube. The density is relatively independent from the level parameter, and of its order of magnitude can be approximated as 1.5 h.

A similar result is found for the gyroid, also in this case the relative density is almost constant with the level parameter.

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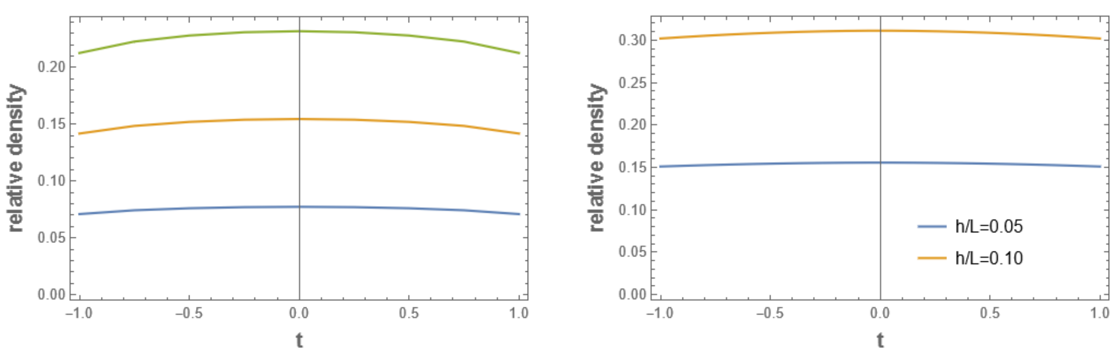

(a) (b)

Figure 5 Relative density of sheet networks (a) P surface (b) G surface

Many investigations have focused on the compressive behaviour of TPMS. Maskery et al.. Wang et al. Yan et al. A stable and progressive buckling collapse is observed during compression, and absorbed energy and crushing load are affected by relative density. Li et al. conducted a comparison of gyroid structures with different lattices at quasi-static loading Ref. Their results highlighted that the gyroid lattice structures with the relative density of 20% and 30% absorb more energy per unit mass than the other structures, indicating the potential application in protective structures. [Abueidda]

Maskery et al.[The deformation and elastic anisotropy of a new gyroid-based honeycomb made by laser sintering I. Maskery*, I.A. Ashcroft] and Abueidda et al. [Effective conductivities and elastic moduli of novel foams with triply periodic minimal surfaces Diab W. Abueidda a,b, Rashid K. Abu Al-Rub a,∗, Ahmed S. Dalaq a, Dong-Wook Lee a, Kamran A. Khan c, Iwona Jasiuk] among others studied the mechanical performance of three TPMS structures: Primitive, Gyroid, and Diamond. They showed that the Primitive structure has higher elastic modulus than the other two TPMS designs. The Primitive type also exhibits strut stretching and buckling, as opposed to the bending dominated deformation modes observed in other TPMS.

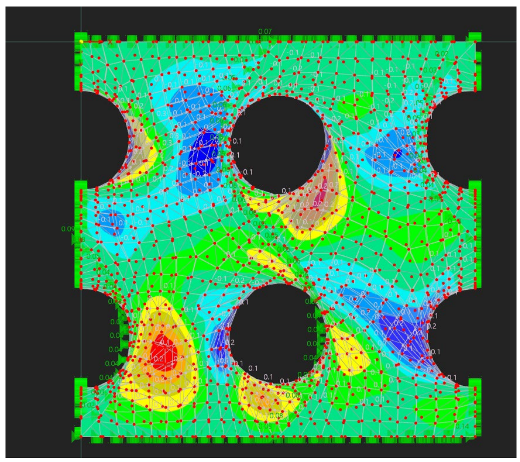

Figure 7 shows the horizontal displacements of a unit gyroid cell to uniaxial stretch. At odd with P cells, the gyroid tends to rotate in addition to deform. This aspect has been not observed in previous studies, indicating the need of using non standard material models for interpreting the behaviour of lattice gyroid metamaterials, opening the way to very interesting applications.

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