

## Graded meta-waveguides for elastic energy splitting

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**Abstract.** Engineered metamaterials for precise wave manipulation and control is a new challenge for many applications in the field of mechanical vibration control and energy harvesting. Inspired by the advancement in wave manipulation and localization performance of elastic waveguides thanks to the introduction of resonators, we try to assess if these systems can be applied also into the junctions between beams that make up lattices. The goal is to achieve control over elastic wave propagation and define whether the local resonators are able to enhance the redirection of the wave into a perpendicular beam. Furthermore, evaluations concerning elastic energy localization in the resonators are reported to assess whether these systems are also suitable for energy localization and energy harvesting.

### Introduction

Control over wave propagation has attracted growing interest across different realms of physics, with multiple realizations in electromagnetic [1] acoustic [2,3], and elastic systems [4]. Different physical effects have been developed in photonics and researchers have been borrowing those ideas to focus or confine elastic wave energy, such as the creation of elastic lenses [5,6,7,8], cavities [9,10], mirrors [11] or topological modes [12]. Metamaterials devised through the use of local lateral resonators have been recently employed to confine elastic energy for energy harvesting applications or vibration isolation. These resonators filter elastic waves and store elastic energy inside them, efficiently protecting the underlying guide from harmful vibrations [13,14]. More recently, it has been shown how broadband vibration isolation properties and wave redirection effects can be achieved leveraging graded arrays of resonators with spatially varying resonance frequency [15,16]. In the context of wave localization and wave redirection, the idea of implementing novel metalattices is now taking place. The underlying idea is to create metalattices that are able, thanks to their microstructure, to guide specific elastic waves, depending on the desired frequency and polarization, into certain predetermined paths in the lattice. The modification of the microstructure of the lattice is able to induce a change in the homogeneous properties at a macro level [17]. As a result, if the variation is properly engineered, lensing effects or in general wave manipulation effects are obtained. The idea now is the opposite: modifying the microstructure of a classic lattice so that the waves are redirected at the micro-structural level. The necessary step to achieve such structures would be to design junctions with peculiar properties that are able to influence the energy redirection between the single beams that compose the junction. To do so, this paper reports some numerical experiments conducted using local resonance systems to partially achieve such junctions. The idea is to engineer lateral resonators that localize and convert waves to achieve control over preferential wave paths.

### Analyses

A study of wave manipulation and control over the connection of two infinite waveguides is reported. The aim is to develop new knowledge over wave redirection in metalattices that employ

local resonance effects in specific junction points of frame structures to control the flow of elastic energy through space.

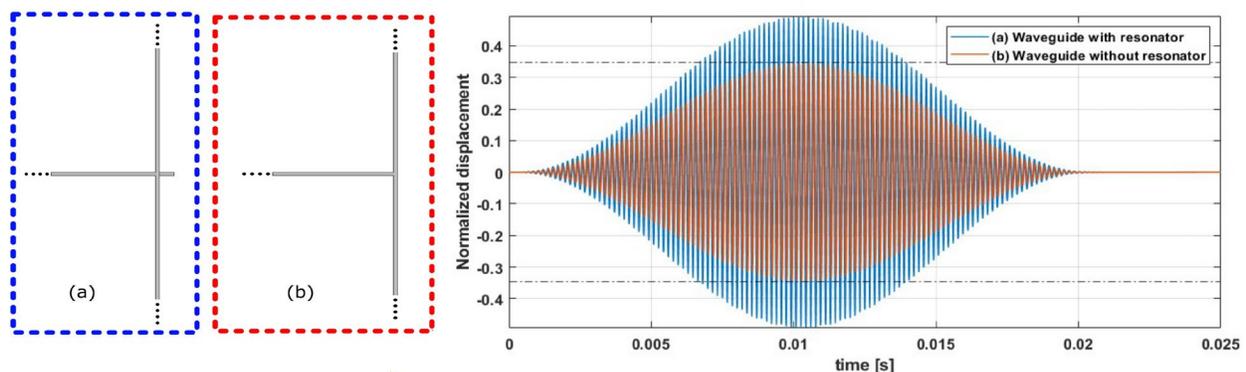
Starting from the analysis of a simple T-junction between infinitely long waveguides, the idea is to see if the presence of a resonator is able to single handedly improve the percentage of wave redirected into the perpendicular waveguide. The hypothesis is that, thanks to the localization of energy inside the resonator, the wave emitted by it through anchor losses or other coupling phenomena will be redirected also into the perpendicular waveguide. Then, from these results, rainbow reflection arrays will be implemented to see whether these structures are beneficial for the redirection of the waves.

### Materials and methods

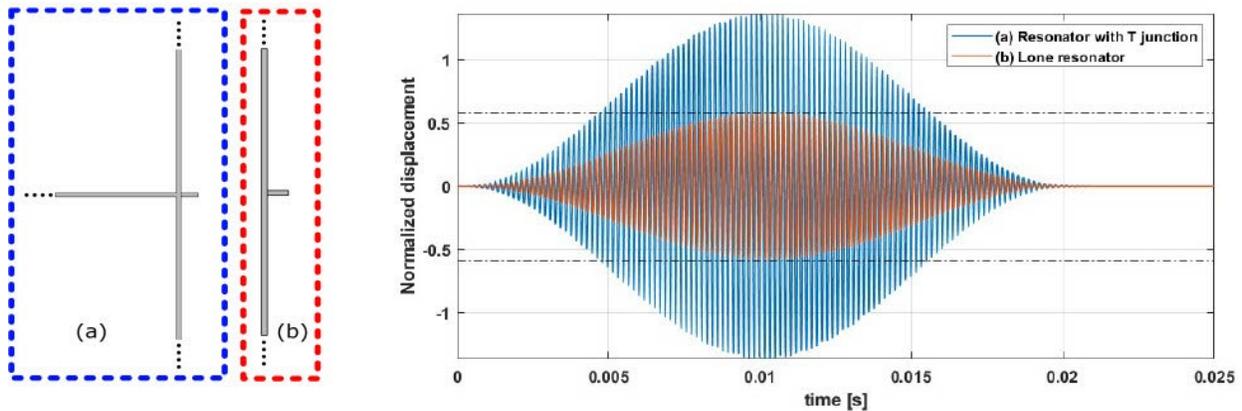
First, we evaluate the resonance frequency of the resonator implemented at the junction of the two waveguides with the specific boundary conditions (BCs). In the case of a 13 mm long, 2.5 mm wide, 1.5 mm thick resonator, made of aluminum and attached to the junction point of two infinite waveguides made of aluminum, the resulting resonance frequency is 5800 Hz. At the waveguides boundaries, ALID boundary conditions are implemented to model an infinite domain [18]. Further damping mechanisms are not contemplated. The analyses reported below are all conducted evaluating the displacement field generated by a travelling flexural wave. The numerical results reported are obtained through time domain implicit analyses in COMSOL Multiphysics.

### Single resonator at the junction

Fig. (1) reports the geometries analyzed and the results obtained through the comparison between the case with the resonator positioned at the junction point between the waveguides and the junction without the resonator. The displacement field has been normalized with respect to the maximum displacement generated at the input. It shows how the presence of the resonator influences the redirection of the wave at the frequency associated to its resonance. The maximum displacement field in the perpendicular waveguide is enhanced by 43% when the resonator is positioned on the junction. Furthermore, another analysis was conducted to see whether the energy traversing the waveguide is more localized on the resonator in the case of infinite waveguide or the T junction case described above. Fig. (2) shows the setup and the results. It is clear that, for a narrow band signal centered at the resonance frequency of the resonator, the maximum displacement field of the cantilever is enhanced by 134%.



*Fig.1: On the left the analyzed geometries. On the right the computed normalized displacement on the perpendicular waveguide for the case of junction with lateral resonator (a), and junction without lateral resonator (b).*



*Fig.2: On the left the analyzed geometries. On the right the computed normalized displacement at the resonator's tip for the case of T junction (a), and lone lateral resonator on the waveguide (b).*

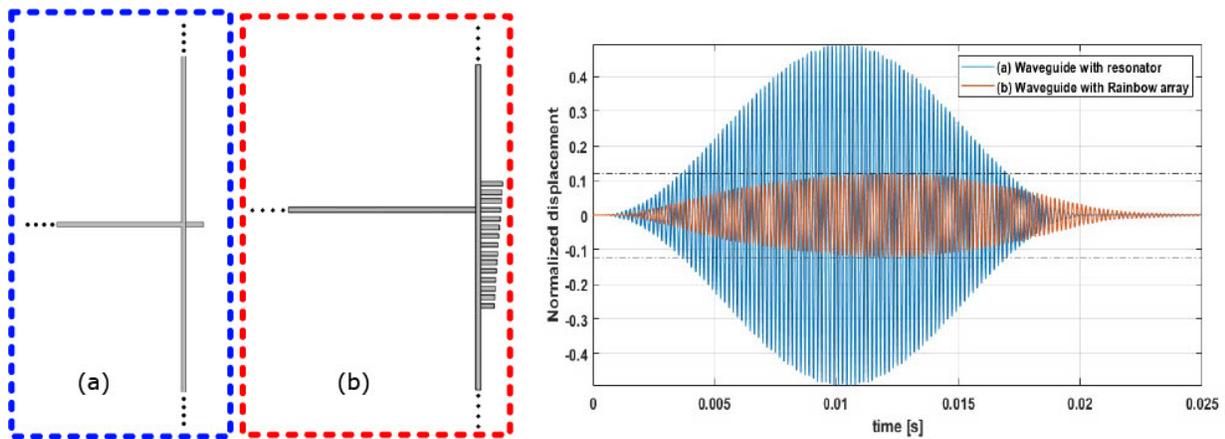
This is an interesting result for energy localization and energy harvesting application, given that the presence of the junction enhances the energy stored in the lateral resonator.

### **Asymmetric rainbow at the junction**

Having seen how the presence of a single resonator is able to improve wave redirection and energy localization, the next step is to evaluate the effect of placing the resonator inside a rainbow reflection array. This has been done by considering that a rainbow reflection array is able to increase the interaction time between the wave and the target resonator [19]. The goal is to assess whether this system can increase both localization of the wave at the target resonator positioned at the junction point and also redirect more efficiently the energy along the perpendicular waveguide. Before stating the results, it is to be noticed that the resonance frequency of the target resonator is now shifted with respect to the previous analyzed case: this is due to the interaction with the neighbor resonators that pushes down the resonance by 300 Hz. Furthermore, the rainbow reflection configuration is composed of 15 linearly increasing in length and equally spaced resonators so that the target resonator (the 12<sup>th</sup>) is positioned exactly at the junction point between the two waveguides.

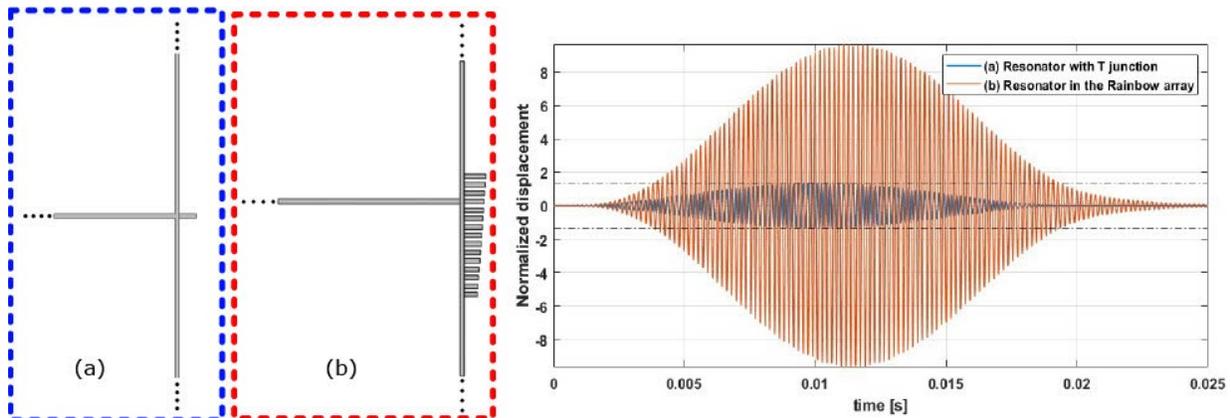
Now the idea is to compare the redirection efficiency of the rainbow structure at the resonance frequency of the target resonator. The analyses are all performed with a narrowband signal generated at the resonance frequency of the target resonator.

Fig. (3) reports the results of the comparison between the case of one single resonator at the junction point (the same one shown before) with the asymmetric rainbow case. The result shows that the rainbow structure is not able to efficiently redirect the wave into the perpendicular waveguide.



*Fig.3: On the left the geometries analyzed. On the right the computed displacement on the perpendicular waveguide for the case of junction with lateral resonator (a), and junction with rainbow reflection array (b).*

As for the efficiency of energy localization inside the resonator, Fig. (4) reports that, as stated in previous works [19,20] the enhancement of energy localization is confirmed. However, even if the displacement field of the resonator is enhanced 8 times in the rainbow array, the redirection on the perpendicular waveguide is halved. The energy is simply back scattered towards the input, as it is in the case of simple one dimensional systems.



*Fig.4: Normalized displacement field of resonators' tip for the case of junction with lateral resonator (a), and junction with rainbow reflection array (b).*

The rainbow configuration can be modified both by changing the length of the lateral resonators and the spacing between them. Changing the latter, results in a drastic change in redirection efficiency. For an array with a distance between the resonators in the order of one eighth of the wavelength (in the previous case it was 12 times) the redirection is 3% less than that for the single resonator case, while the energy displacement field of the resonator is still maintained at approximately 8 times the one of the single resonator. These results are reported in Fig. (5) and Fig. (6). The reason why the change in distance of the resonator is so important is thought to be linked to the conversion of the travelling wave from flexural to torsional. A torsional wave is better able to be redirected in the perpendicular waveguide. On this topic it has already been shown how a staggered configuration of resonators is able to achieve efficient mode conversion when the distance and the length of the resonators is properly set [21].

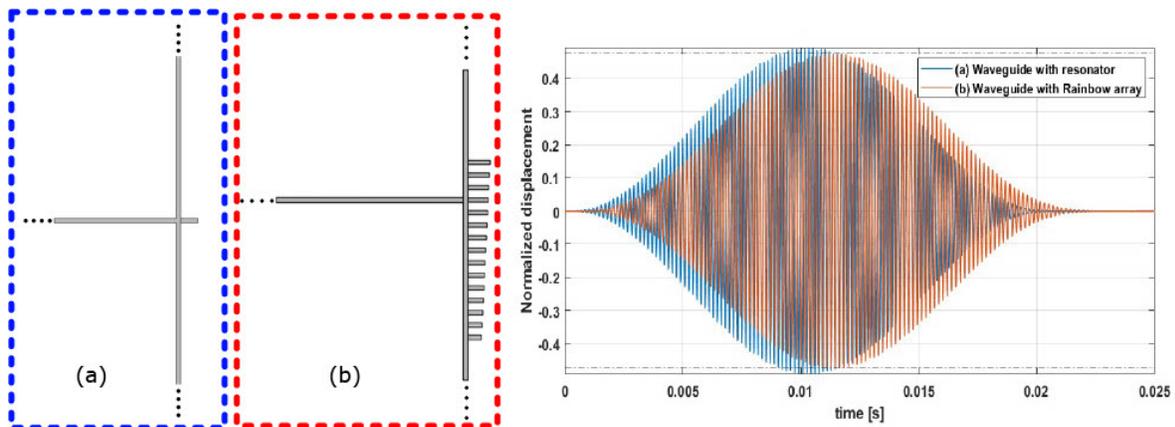


Fig.5: On the left the geometries analyzed. On the right the computed displacement on the perpendicular waveguide for the case of junction with lateral resonator (a), and junction with rainbow reflection array (b).

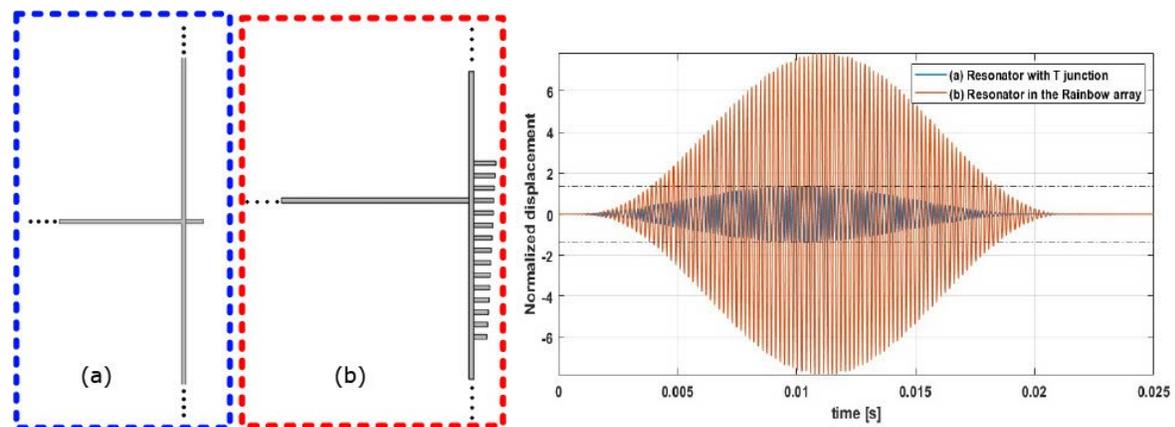


Fig.6: Normalized displacement field of resonators' tip for the case of junction with lateral resonator (a), and junction with rainbow reflection array (b).

In the end, we observe that the resonator and the resonators array are able to influence the wave redirection mainly through conversion of the flexural input wave mode into a propagating torsional mode. Further studies on this topic are necessary to assess whether this wave conversion effect is the key component in wave redirection.

## Conclusions

The paper contains some studies aimed at evaluating the effectiveness of implementing different configurations of lateral resonators at 90 degrees junctions of infinite beams. It was showed how the presence of the resonators can be an advantage or disadvantage in terms of wave redirection. It was seen how a single resonator positioned at the junction point is able to partially influence the energy redirection towards the perpendicular waveguide and how its own motion is enhanced by the junction. For what concerns the rainbow reflection system, the results indicate that the single resonator is more effective with respect to the array if the resonators are closely packed together. But for the case of more spaced resonators the two configurations are comparable in wave redirection efficiency. Furthermore, the rainbow array allows for a great enhancement of the displacement of the target resonator. Further numerical and analytical studies must be performed to correctly understand the phenomena that define wave redirection.

## References

- [1] Pendry, J. B., Holden, A. J., Robbins, D. J. & Stewart, W. Magnetism from conductors and enhanced nonlinear phenomena. *IEEE transactions on microwave theory and techniques* 47, 2075–2084 (1999). <https://doi.org/10.1109/22.798002>

- [2] Craster, R. V. & Guenneau, S. Acoustic metamaterials: Negative refraction, imaging, lensing and cloaking, vol. 166 (Springer Science & Business Media, 2012). <https://doi.org/10.1007/978-94-007-4813-2>
- [3] Liu, Z. et al. Locally resonant sonic materials. *Science* 289, 1734–1736 (2000). <https://doi.org/10.1126/science.289.5485.1734>
- [4] Craster, R. & Guenneau, S. World scientific handbook of metamaterials and plasmonics (2017). <https://doi.org/10.1142/10642-vol2>
- [5] Tol, S., Degertekin, F. L. & Erturk, A. Gradient-index phononic crystal lens-based enhancement of elastic wave energy harvesting. *Applied Physics Letters* 109, 063902 (2016). <https://doi.org/10.1063/1.4960792>
- [6] Tol, S., Degertekin, F. L. & Erturk, A. Phononic crystal luneburg lens for omnidirectional elastic wave focusing and energy harvesting. *Applied Physics Letters* 111, 013503 (2017). <https://doi.org/10.1063/1.4991684>
- [7] Zareei, A., Darabi, A., Leamy, M. J. & Alam, M.-R. Continuous profile flexural GRIN lens: Focusing and harvesting flexural waves. *Applied Physics Letters* 112, 023901 (2018). <https://doi.org/10.1063/1.5008576>
- [8] Allam, A., Sabra, K. & Erturk, A. Sound energy harvesting by leveraging a 3D-printed phononic crystal lens. *Applied Physics Letters* 118, 103504 (2021). <https://doi.org/10.1063/5.0030698>
- [9] Wu, L.-Y., Chen, L.-W. & Liu, C.-M. Acoustic energy harvesting using resonant cavity of a sonic crystal. *Applied Physics Letters* 95, 013506 (2009). <https://doi.org/10.1063/1.3176019>
- [10] Qi, S., Oudich, M., Li, Y. & Assouar, B. Acoustic energy harvesting based on a planar acoustic metamaterial. *Applied Physics Letters* 108, 263501 (2016). <https://doi.org/10.1063/1.4954987>
- [11] Carrara, M. et al. Metamaterial-inspired structures and concepts for elastoacoustic wave energy harvesting. *Smart Materials and Structures* 22, 065004 (2013). <https://doi.org/10.1088/0964-1726/22/6/065004>
- [12] Darabi, A., Ni, X., Leamy, M. & Al`u, A. Reconfigurable Floquet elastodynamic topological insulator based on synthetic angular momentum bias. *Science advances* 6, (2020). <https://doi.org/10.1126/sciadv.aba8656>
- [13] Gonella, S., To, A. C. & Liu, W. K. Interplay between phononic bandgaps and piezoelectric microstructures for energy harvesting. *Journal of the Mechanics and Physics of Solids* 57, 621–633 (2009). <https://doi.org/10.1016/j.jmps.2008.11.002>
- [14] Sugino, C. & Erturk, A. Analysis of multifunctional piezoelectric metastructures for low-frequency bandgap formation and energy harvesting. *Journal of Physics D: Applied Physics* 51, 215103 (2018). <https://doi.org/10.1088/1361-6463/aab97e>
- [15] Colombi, A., Colquitt, D., Roux, P., Guenneau, S. & Craster, R. V. A seismic metamaterial: The resonant metawedge. *Scientific reports* 6, 1–6 (2016). <https://doi.org/10.1038/srep27717>
- [16] Colombi, A. et al. Elastic wave control beyond band- gaps: shaping the flow of waves in plates and half-spaces with subwavelength resonant rods. *Frontiers in Mechanical Engineering* 3, 10 (2017). <https://doi.org/10.3389/fmech.2017.00010>
- [17] Aguzzi, G. et al. Octet lattice-based plate for elastic wave control. *Scientific reports* 12, 1–14 (2022). <https://doi.org/10.1038/s41598-022-04900-0>
- [18] Rajagopal, P., Drozd, M., Skelton, E. A., Lowe, M. J. & Craster, R. V. On the use of absorbing layers to simulate the propagation of elastic waves in unbounded isotropic media using commercially available finite element packages. *NDT & E international* 51, 30–40 (2012). <https://doi.org/10.1016/j.ndteint.2012.04.001>
- [19] De Ponti, J. M. et al. Graded elastic metasurface for enhanced energy harvesting. *New Journal of Physics* 22, 013013 (2020). <https://doi.org/10.1088/1367-2630/ab6062>
- [20] De Ponti, J. M. et al. Experimental investigation of amplification, via a mechanical delay-line, in a rainbow - based metamaterial for energy harvesting. *Applied Physics Letters* 117, 143902 (2020). <https://doi.org/10.1063/5.0023544>
- [21] De Ponti, J. M. et al. Selective mode conversion and rainbow trapping via graded elastic waveguides. *Physical Review Applied* 16, 034028 (2021). <https://doi.org/10.1103/PhysRevApplied.16.034028>