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Anisotropic behaviours and strain concentration in lattice material evaluated by means of discrete homogenization

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Abstract. Lattice fibre materials are challenging the standard modelling approaches due to their specific nature that results in peculiar effective behaviours such as extremely anisotropic materials or generalized continuum media. In this context, the aim of this paper is to the determine qualitatively and quantitatively the role of the morphological and mechanical parameters by investigating simple archetypical microstructures. The study is conducted through an up-scaling approach making use of the Homogenization method of discrete periodic media in the framework of a variational approach. The results of the homogenization have been validated comparing the response of the continuum with the response of discrete models.

Introduction

Lattice fibre materials are becoming widespread in engineering applications, thanks to their versatility and their efficient mechanical and physical properties. In addition to conventional fibre materials, widely used in aero-spatial technology and in civil engineering, innovative materials have been introduced by the new manufacturing technologies like additive printing.

The aim of the contribution is to investigate the elastic mechanical behaviour of plane networks (fibre sheets), that present strong directional properties, which can be tuned by suitable design of the network and selection of the fibre mechanical characteristics.

In order to describe the network behaviour, the effective mechanical properties of plane fibre networks have been derived using discrete homogenization [1,2,3]. The resulting equivalent continuum presents strong directional properties, with anisotropy ratios much larger than those that can be found in natural materials.

The results of the homogenization have been validated comparing the response of the continuum with the response of ‘‘discrete’’ models, for which each element is modelled as a slender beam, and boundary conditions are accounted for in an exact way.

Brief description of Discrete homogenization technique and some applications on lattice structural types

Discrete homogenization applies to lattices constituted by a finite number of nodes and micro-beam elements. It derives from classical two-scales homogenization, in which the fast (microscopic) variables are substituted by the node numbering. Scale separation hypothesis is assumed, that is, the ratio between the dimension of the unit cell ℓ_c and the dimensions of the network, L is a very small parameter $\varepsilon = \ell_c/L << 1$. Homogeneous Euler–Bernoulli beams are considered for the lattice, for more detaileds on the homogenization method see [3,4].

Two types of networks are examined in this paper, both composed by straight members with rigid connection, representing both biaxial and quadriaxial repetitive cells. The biaxial cell is composed by two families of fibres, as shown in Fig. 1(a); in this case will be considered either orthogonal rectangular cells (so that the fibres form an angle $\alpha = \pi/2$) and skew cell (with an angle

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between the fibres $0 < \alpha < \pi/2$). The quadriaxial cell is always considered with orthogonal fibres as shown in Fig. 1(b). The elements of the networks are considered as having rectangular cross-sections, with unit depth normal to the plane of the lattice. Plane stress are therefore considered for the homogenized model. Referring to the elements indicated in Fig. 1, the following symbols will be used: l_i is the length of member b_i; I_i is the in-plane moment of inertia of member b_i; A_i is the area of member b_i; h, s will denote the height and width of the members; E_i is the Young Modulus of member b_i; α is the opening angle of the biaxial cell.

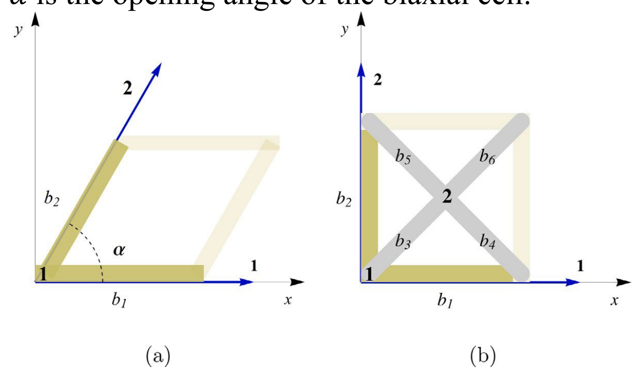

Fig. 1 Case studies: (a) Skew cell with rigid connection, (b) Rectangular cell braced with rigid connection.

The homogenized constitutive equations of the network constituted by skew repetitive cells with straight fibres forming an angle α between themselves and for the rectangular cell braced with rigid connection can be found in [4,5].

Analysis of the properties of the equivalent materials

The design of lattice structures, like woven fabrics and networks, with controlled anisotropy and mechanical properties is of critical importance for various applications. In order to evaluate the characteristics of anisotropy of networks having as elementary cell one of those examined in the previous sections, we examine the uniaxial stiffness of the network defined as the ratio between an uniaxial stress applied along an axis ξ rotated by an angle θ respect to the x axis (always taken coincident with the direction of fibre 1 and the corresponding strain, see Fig. 2. The ratio $D_{\xi\xi} = \sigma_{\xi\xi}/\epsilon_{\xi}$ denotes the uniaxial stiffness in the direction ξ.

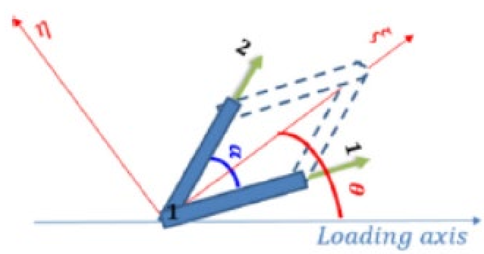

Fig. 2 Definition of the rotated axis ξ.

Extension tests of networks modelled by square biaxial or quadriaxial cells arranged at various angles with respect to the extension axis are considered. The following geometrical and mechanical properties of the fibres have been considered: $l_1 = l_2 = 5$ mm; $h_1 = h_2 = 1$ mm, $s_1 = s_2 = 1$ mm, $E_1 = E_2 = 1600$ Mpa.

Biaxial network

The uniaxial stiffness, normalized with respect to the modulus E_1 of the beams lying in the 1-direction, are represented in polar plots in Fig. 3(a) and 3(b).

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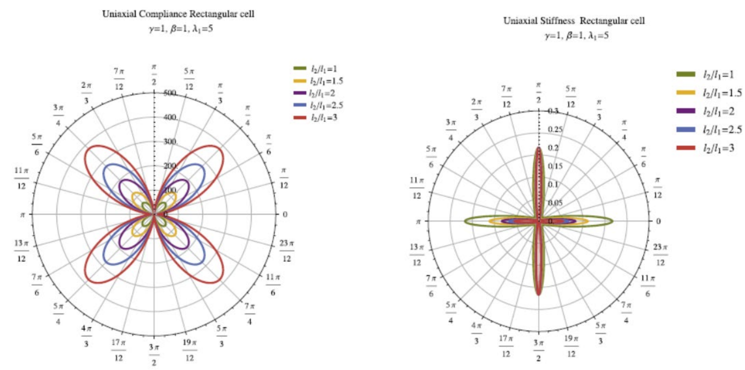
Defining the dimensionless parameters $\rho = l_2/l_1$ $\beta = h_2/h_1$ $\gamma = E_2/E_1$ $\mu_1 = l_1/h_1$, the different curves in the plots refer to cells having beams with different length with ρ ranging from 1 to 3, equal section $\beta = 1$, equal modulus $\gamma = 1$, and slenderness of the first set of beams $\mu_1 = 5$.

The material obtained presents a sharp contrast of anisotropy, reaching two order of magnitudes between the directions of the fibres and a direction at 45° with it. For large ratios l_2/l_1 there is also a strong difference between the stiffness along the directions of the fibres.



(a) Polar plot of the dimensionless uniaxial compliance $C_{\xi\xi}E_1$

(b) Polar plot of the dimensionless uniaxial stiffness $\frac{D_{\xi\xi}}{E_1}$

Fig. 3 Uniaxial stiffness of a material formed by rectangular reference cells with rigid connections as function of the load directions for different aspect ratio of the cell.

Extension tests are usually performed to get an estimate of the elastic properties of fibre networks. According to ISO standard 13934-1, the sample length-to-width ratio used in laboratory tests for assessing the uniaxial elastic modulus should be equal to 2.

In order to evaluate whether this aspect ratio is appropriate for obtaining stable values of the elastic parameters, the apparent elastic modulus D_{yy} in the axial direction was numerically evaluated as a function of the slenderness of the sample for various inclinations θ of the material axis 1 with respect to the loading direction.

The aspect ratio was varied in the range 0.1 to 10.0. The results are summarized in the plot of Fig. 4(a). All the results are contained within two boundaries, represented by the upper and lower dashed curves, which represent the uniaxial stiffness evaluated analytically under the hypothesis of uniaxial strain state ($\epsilon_{zz} = \gamma_{yz} = 0$, upper curve) and of uniaxial stress state ($\sigma_{zz} = \tau_{yz}$, lower curve).

Quadriaxial network

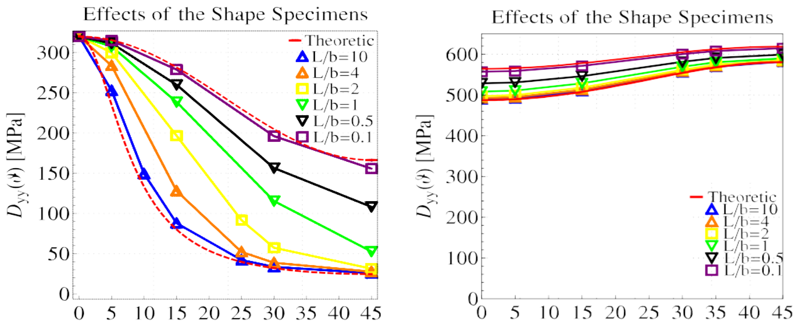
The results of this section refer to a quadriaxial lattice with orthogonal fibres and rigid connections, as the one examined, in the special case that the internal fibres along the diagonals (usually less stiff) are identical among themselves. The internal node is located at the centre of the cell. The following dimensionless parameters are used: $\gamma_1 = E_2/E_1$; $\gamma_2 = E_{int}/E_1$; $\beta_1 = h_2/h_1$; $\beta_2 = h_{int}/h_1$; $\rho = l_2/l_1$ where E_{int}, h_{int} are the Young modulus and the thickness of the internal fibres.

The length of the internal fibres is $l_{in} = l_1/2\sqrt{1+\rho^2}$. The slenderness of the external fibre 1 is denoted by $\mu_1 = l_1/h_1$. Notice that the slenderness of the beams in the 2-direction and of the internal beams are respectively $\mu_2 = \mu_1\,\rho/\beta$ and $\mu_{int} = \mu_1\,/\beta_2\sqrt{0.25+0.25\rho^2}$.

The case of a square braced cell, $\rho = 1$, with two equal external fibres and $\beta_1 = 1$, $\gamma_1 = \gamma_2 = 1$, is examined first. The slenderness ratio of the external beams has been fixed to $\mu_1 = 5$.

The uniaxial stiffness and compliance are reported in Fig. 5, for different values of β_2 varying from 0.3 to 2. According to whether the internal fibres have a larger thickness than the external ($\beta_2 > 1$), or the opposite, a greater stiffness is obtained at the braces direction (45°), or at 0°. However, in any case, a more isotropic behaviour is obtained with respect to the biaxial lattice.

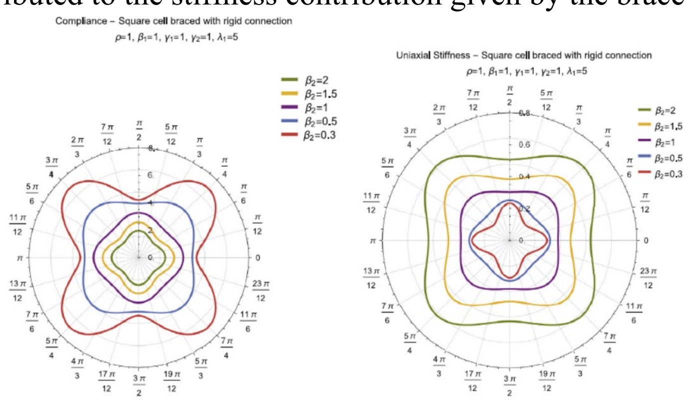
The effect of the internal fibres is to reduce the contrast of anisotropy. The insertion of diagonal fibres makes the transition of the stiffness from direction 1 to direction 2 more smooth.



(a) (b)

Fig. 4 Uniaxial stiffness of a material formed by rectangular reference cells with rigid connections as function of the load directions for different aspect ratio of the cell.

The apparent longitudinal elastic modulus as a function of the fibre for aspect ratios of the specimen in the range 0.1 to 10 and for orientations ranging from $\theta = 0^\circ$ to $\theta = 45^\circ$ is presented in Fig. 4(b). Comparing this plot with the one of Fig. 4(a) it is possible to observe that in the case of quadriaxial networks the behaviour of the material is almost independent on the mesh inclination. This effect can be attributed to the stiffness contribution given by the braces.



(a) Polar plot of the dimensionless uniaxial compliance $C_{\xi\xi}E_1$

(b) Polar plot of the dimensionless uniaxial stiffness $\frac{D_{\xi\xi}}{E_1}$

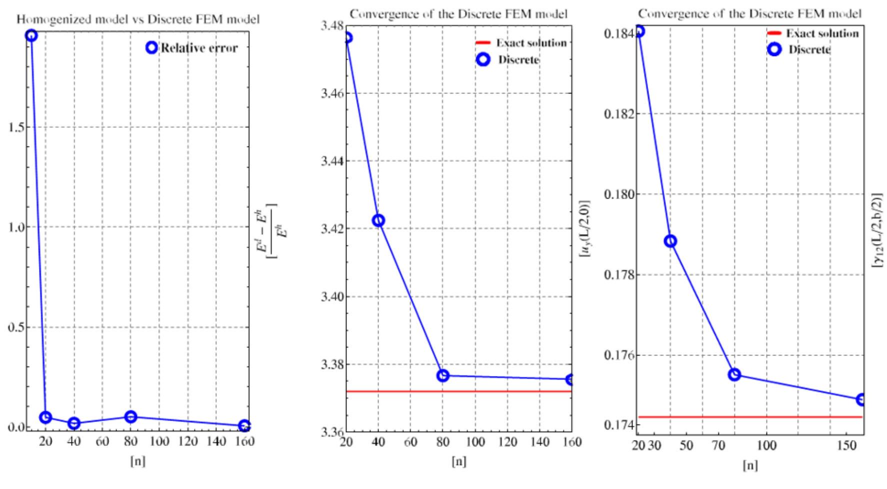
Fig. 5 Uniaxial stiffness of a material formed by square braced cell with rigid connections as function of the load directions for different thickness ratios β_2.

Validation

The numerical results obtained using the homogenization model in the simulation of the bias test of the biaxial network with aspect ratio $\rho = 2$ have been compared to the response of a meso-model of the network, realized by means of discrete linear beams.

Four discrete models with a variable number of cells have been employed, corresponding to 20, 40, 80, 160, cells along the longest side of the specimen. For both families of equal fibres the Young modulus E = 1600 MPa and slenderness ratio l_i/h_i = 10 have been used.

The elastic properties of the homogenized material are E_{xxxx} = 160 MPa, Γ = 0.8 MPa. As the number of elements in the discrete model grows, so as to satisfy the scale separation hypothesis, the results of the discrete and of the homogenized models tend to converge. Fig. 6(a) shows the relative error for the strain energy between the discrete model and a reference value evaluated with the homogenized continuum model, using a very fine mesh. Similar convergence plots refer to the lateral contraction of the specimen, measured at its centre (Fig. 6(b)) and to the shear strain γ_{12} between the fibres directions in the centre of the specimen (Fig. 6(c)).



(a) (b) (c)

Fig. 6 Convergence of the results of the discrete models and of the homogenized model as the number of discrete cells increases.

Cylindrical structure

In this section a cylinder whose wall is made by a periodic lattice of biaxial orthogonal fibres with circular cross-section is examined. The discrete and homogenized continuous model will be compared. The homogenised model was defined taking into account both the flexural and the membrane properties simultaneously. The cylindrical structure is shown in the figure 7.

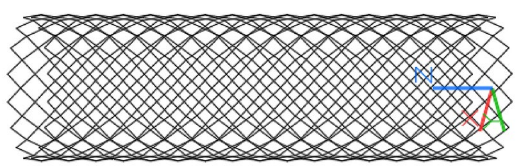


Fig. 7 Cylindrical structure

The geometrical parameters used for the model are: diameter D = 100 mm and height H = 300 mm of the cylindrical structure; dimensions of the repetitive cell l_1 = 4.0 mm e l_2 = 4.0 mm, with rotations of load at 45° of the cell with respect the axis z; diameter of the beams of the cell is d = 0.5 mm. The mechanical properties of the material constituting the lattice are the elastic module E = 1600 N/mm^2 and poisson's ratio ν= 0.15. A line load that tends to uniformly stretch the cylinder has been applied.

At one end of the cylinder translations are prevented, at the other end only axial displacements are allowed. The same simulation using the homogenized continuum has been performed. The vertical displacements of both models, figures 8 and 9, are quite similar. The maximum displacement value is 27.982 mm 28.346 mm respectively for the discrete model and for the homogenized model. The relative error between the maximum displacement obtained for the discrete model and that obtained for the homogenised is:

$$e_r = \frac{|28.346 - 27.982|}{27.982} \times 100 = 1.3\%$$

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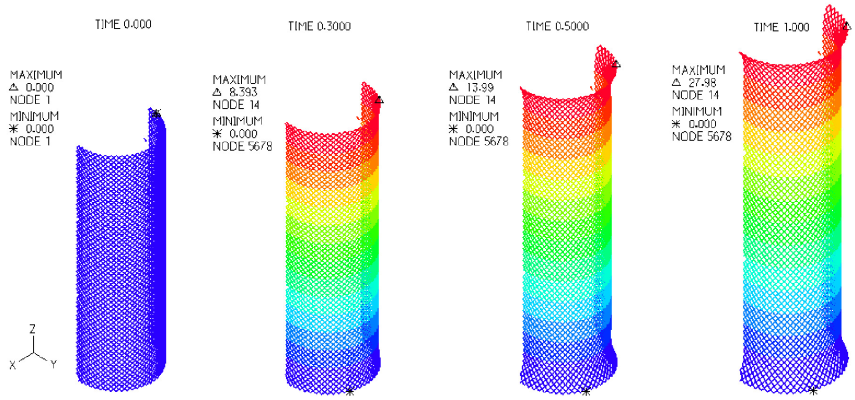

Fig. 8 Band plot of the vertical dislacement for the discrete FEM model for different increment of the load.

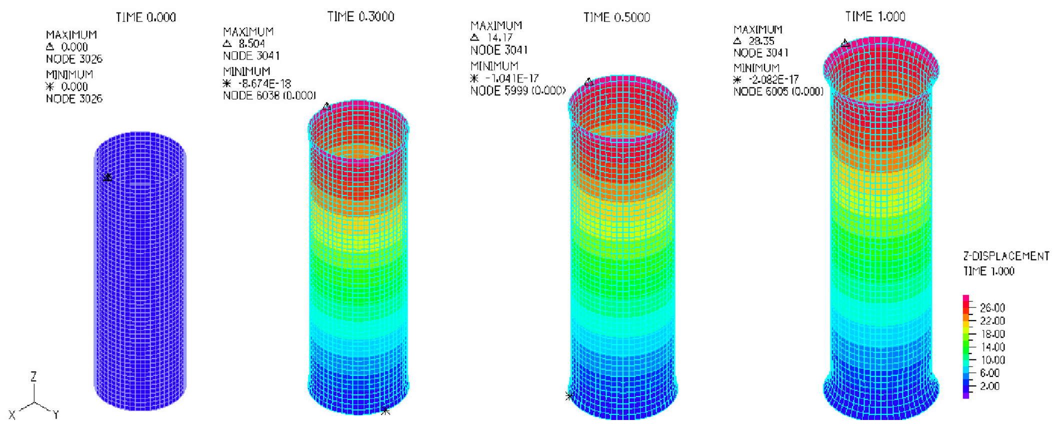

Fig. 9 Band plot of the vertical dislacement for the discrete FEM model for different increment of the load.

References

[1] Abdoul-Anziz, H., Seppecher, P., "Strain gradient and generalized continua obtained by homogenizing frame lattices", Math. Mech. Complex 6, 3, 213-250, (2018). https://doi.org/10.2140/memocs.2018.6.213

[2] dell'Isola, F., Giorgio, I., Pawlikowski, M., Rizzi, N.L., "Large deformations of planar extensible beams and pantographic lattices: heuristic homogenization, experimental and numerical examples of equilibrium", Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci. 472, 472, (2016). https://doi.org/10.1098/rspa.2015.0790

[3] Caillerie, D., Mourad, A., Raoult, A., "Discrete homogenization in graphene sheet modeling", J. Elasticity, 84, 33-68, (2006). https://doi.org/10.1007/s10659-006-9053-5

[4] Gazzo, S., Cuomo, M., Boutin, C., Contrafatto, L., "Directional properties fibre network materials evaluated by means of discrete homogenization", European Journal of Mechanics, A/Solids, 82 (2020). https://doi.org/10.1016/j.euromechsol.2020.104009

[5] Cuomo, M., Boutin, C., Contrafatto, L., Gazzo, S., "Effective anisotropic properties of fibre network sheets", European Journal of Mechanics, A/Solids, 93, 104492, (2022). https://doi.org/10.1016/j.euromechsol.2021.104492