

Continuous particle swarm optimization for model updating of structures from experimental modal analysis

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Abstract. The aim of this work is to present a structural model updating procedure based on a recent variant of Particle Swarm Optimization, called Continuous Particle Swarm Optimization. In order to validate the proposed model, the modal parameters of a real scale large electrical device have been identified by measuring the dynamical response of the system subjected to a measured input. Numerical results confirm the robustness and reliability of the proposed method.

Introduction

The discrepancy between the design of an artifact and its actual construction generates uncertainty on the mechanical behavior of the structures which depends on a large number of variables such as geometrical and mechanical properties of materials and structural parts, boundary conditions and so on.

In the past decades, the scientific literature proposed several model updating methods [1] mostly based on heuristic optimization techniques and on the experimental characterization of the structures in order to fill up the aforementioned gap.

Among the classical optimization techniques, the Continuous Particle Swarm Optimization (CPSO) [2, 3] has been recently developed showing good performance in terms of accuracy and computational time reduction in comparison with the others.

Moreover, the dynamical behavior of the artifact needs to be identified through experimental campaigns performed by means of two different approaches. The first one is the so-called Operational Modal Analysis (OMA) in which the modal parameters are obtained only from the measured data using environmental vibrations as unknown input (i.e., wind load, micro-tremors, traffic), without any artificial excitations applied on the structure. The second approach is the Experimental Modal Analysis (EMA) in which the identification of modal parameter is evaluated by applying a measured input on the system and measuring its response.

In the present paper, the CPSO has been adopted with the aim to develop a robust and fast structural model updating procedure. The method has been validated on a real scale large electrical device whose dynamical properties have been characterized by using EMA approach.

The experimental seismic response of the structure under investigation has been acquired by taking advantage of the shaking table system of the Laboratory of Earthquake engineering and Dynamic Analysis (L.E.D.A.) [4].

The paper is organized in three sections: in the first one the experimental model of a high voltage Current Transformer is presented as well as its numerical twin model defined by means of Finite Element Method (F.E.M.); the second section deals in detail with an original model updating procedure achieved through CPSO. Last, the comparison in terms of displacement between acquired data and dynamical response reproduced by the optimized numerical model shows a good agreement validating the proposed method.

Experimental Model

It is common practice that the design and construction process of industrial artifacts need to be verified by means of experimental test to guarantee the goodness of the actual objects to be placed in the market and their reliability during the operative lifetime. For such a reason, a great number of national and international standard have been delivered for example to qualify the artifacts under seismic events [5-6].

In order to foresee the behavior of these objects and their components under several input motions without the necessity to run many expensive experimental campaigns, it is often convenient to develop a reliable twin numerical model starting from the mechanical and dynamical data acquired during the mandatory qualification tests.

In this paper a model updating procedure is discussed to develop a numerical model able to reproduce the dynamic response of a high voltage Current Transformer which have been tested at the Laboratory of Experimental Dynamic of the L.E.D.A. Research Institute.

The Transformer has been mounted on a support structure by means of a six-spring damper and instrumented by several accelerometers and strain gauges to acquire the response in terms of accelerations and strains, respectively. Moreover, an infrared optoelectronic system has been deployed to measure the absolute 3D displacements of the structure under test. In Fig. 1a a picture of the experimental setup is reported and, for the sake of shortness, only the sensors A1 (tri-axial accelerometer) and D1 (markers for displacement measure) at the top of the transformer are highlighted.

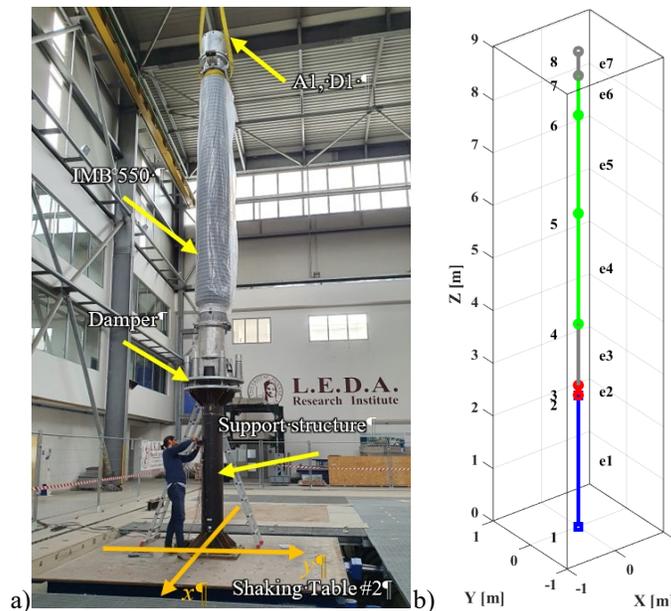


Figure 1. a) Experimental setup for shaking table tests; b) FEM model: nodes and element geometry.

With the aim to define a numerical twin model of the unit under test, a simplified Finite Element Model has been built by means of a Matlab® self-developed FEM software (Fig. 1b). The FEM model to be updated consists of n. 8 nodes (node n. 1 is fully restrained while nodes 2 to 8 are not constrained) and n. 7 elements among which elements 1, 3, 4, 5, 6, 7 have been discretized as 3D lumped-mass frame elements whose bending properties depend on the stiffness coefficients $EI_{x,i}$ and $EI_{y,i}$ [7], while element n. 2 has been modeled as a 3D spring with axial stiffness k_z and rotational springs $k_{\phi,x}$ and $k_{\phi,y}$ (lateral k_x , k_y and torsional $k_{\phi,z}$ are herein considered 10 times bigger than the others in order to simulate the actual behavior of the spring). Elements n. 3 and 7

(corresponding to the massive oil tanks of the transformer) are much stiffer than the other elements and are not considered in the optimization process.

On each frame element with parameters to be identified (element n. 1, 4, 5 and 6) the damping has been modeled according to Raileigh formulation in such a way that their internal damping matrix is

$$C_i = \alpha_{0,i}M_i + \alpha_{1,i}K_i, \quad i = 1, 4, 5, 6 \tag{1}$$

while for the spring element n. 2 the damping coefficients to be optimized are those corresponding to the axial and rotational degrees of freedom, namely c_z , $c_{\varphi,x}$ and $c_{\varphi,y}$, respectively.

Nominal values of the materials and geometrical properties of each element have been obtained by the design information available from the constructor and from the static tests performed before the seismic qualification tests.

Modal Updating by means of Continuous Particle Swarm Optimization

In this paper, a model updating strategy is developed by assuming that the simplified FEM model, previously described, is linear and the difficulties in modeling joints, flanges, the presence of non-structural components and other complicated boundary conditions could be compensated for by adjusting the stiffness and damping coefficients of some relevant elements of the structure.

As customary [1], a Finite Element Model Updating procedure can be defined as an iterative method where some physical parameters are updated until the FEM model reproduces the measured data to a sufficient degree of accuracy. This can be achieved by minimizing a so-called objective function with respect to the variation of the selected physical parameters:

$$\mathbf{x}_{opt} = \min_{\mathbf{x}} [J(\mathbf{x})], \quad \underline{\mathbf{x}} < \mathbf{x} < \overline{\mathbf{x}} \tag{2}$$

where $J(\mathbf{x})$ is the objective function, \mathbf{x} is the array of the physical parameters to be updated that can assume values in the interval $(\underline{\mathbf{x}}, \overline{\mathbf{x}})$.

In the following, the experimental data acquired during seismic tests will be assumed correct and considered as the refence signals to use for the computation of the objective function. Since the aim is to obtain a numerical model able to reproduce the dynamic response of the structure, the objective function adopted in this work is computed as:

$$J(\mathbf{x}) = w_1 \left[\frac{\text{rms}(u_{8,x}^{\text{num}}(\mathbf{x}, t_i, t_f) - u_{8,x}^{\text{exp}}(t_i, t_f))}{\text{rms}(u_{8,x}^{\text{exp}}(t_i, t_f))} \right] + w_2 \left[\frac{\text{rms}(u_{8,y}^{\text{num}}(\mathbf{x}, t_i, t_f) - u_{8,y}^{\text{exp}}(t_i, t_f))}{\text{rms}(u_{8,y}^{\text{exp}}(t_i, t_f))} \right] \tag{3}$$

where $w_1 = w_2 = 0.5$ are weighting parameters, rms is the root mean square function, $u_{8,x}$, $u_{8,y}$ are the displacements with respect to ground at the top of the transformer (node n. 8) in x and y global direction, respectively, for the numerical (num) and the experimental (exp) system. The time window $[t_i, t_f]$ have been chosen to be the time interval between the 2% and 98% contribution of the experimental shaking table input acceleration time histories $\ddot{\mathbf{u}}_g^{\text{exp}}(t)$ to the Arias intensity defined as:

$$A = \int_0^{T_f} \ddot{\mathbf{u}}_g^{\text{exp}}(t) dt \tag{4}$$

where $[0, T_f]$ is the whole signal duration.

At each step of the iterative procedure adopted, the FEM software computes the global Mass **M**, Damping **D** and Stiffness **K** matrices of the system and solves the numerical time integration problem

$$\begin{cases} \mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{M}\boldsymbol{\tau}\ddot{\mathbf{u}}_g^{\text{exp}}(t) \\ \mathbf{u}_0(0) = \mathbf{0}; \dot{\mathbf{u}}_0(0) = \mathbf{0} \end{cases} \quad (5)$$

where $\mathbf{u}(t)$ is the displacement vector of the nodes of the structure for all the 6 degrees of freedom in space with respect to ground, the upper dots indicate time derivative, $\boldsymbol{\tau}$ is the load incidence matrix and $\ddot{\mathbf{u}}_g^{\text{exp}}(t)$ is the array of the x, y and z direction experimental acceleration time histories recorded at the base of the structure during the shaking table seismic tests.

To compute the optimal minimum value of the objective function $J(\mathbf{x})$ and the corresponding optimal parameter set \mathbf{x}_{opt} , the latest variant of PSO, the so called CPSO, introduced in [2,3], has been applied.

The discrete formulation of PSO has been extended in the continuous time space by means of an integration over the time interval, $[0, T]$, or rather, the bird flying motion evolution is defined by a continuous time function that is the integral of a given Cauchy problem associated with a second order ordinary differential equation describing the dynamics of a damped harmonic oscillator. In details, let μ denote the inertia weight, c_c and c_s the cognitive and social constant, p_{ib} the best position of the i -th bird history, p_{gb} the best position in the swarm history, r_1 and $r_2 \in [0, 1]$ random values.

Eventually, let $\omega(t), \zeta(t): [0, T] \rightarrow [0, +\infty)$ be the angular frequency and the damping ratio of the oscillator, respectively, such that, $\forall t \in [0, T]$,

$$\begin{cases} 2\zeta(t)\omega(t) = 1 - \mu(t) \\ \omega^2(t) = c_c r_1(t) + c_s r_2(t) \end{cases} \quad (6)$$

thus, if the time interval is split in N sub-interval of range $dt = T/N$, the particle position updates

$$p(t) = \cup_{k=0}^{N-1} p_{[(k-1)dt, kdt]}(t) \quad (7)$$

where $p_{[(k-1)dt, kdt]}(t)$ is the particle movement computed as follows.

Let be $\omega(t) = \omega_k, \zeta(t) = \zeta_k, f_k = c_c p_{ib}(kdt) + c_s p_{gb}(kdt) \forall k \in [kdt, (k+1)dt], k = 1, \dots, N$ and

$$\lambda_1^k = -\omega_k \left[\zeta_k + \sqrt{(\zeta_k^2 - 1)} \right] \quad \lambda_2^k = -\omega_k \left[\zeta_k - \sqrt{(\zeta_k^2 - 1)} \right]. \quad (8)$$

If $\zeta \neq 1$ then

$$p_{[(k-1)dt, kdt]}(t) = c_1^k \exp(\lambda_1^k t) + c_2^k \exp(\lambda_2^k t) + \frac{f_k}{\omega_k^2} \quad (9)$$

with

$$c_1^k = \frac{\left(p_k - \frac{f_k}{\omega_k^2} \right) \lambda_2^k - \dot{p}_k}{(\lambda_2^k - \lambda_1^k) \exp(\lambda_1^k kdt)} \quad \text{and} \quad c_2^k = \frac{\dot{p}_k - \left(p_k - \frac{f_k}{\omega_k^2} \right) \lambda_1^k}{(\lambda_2^k - \lambda_1^k) \exp(\lambda_2^k kdt)}. \quad (10)$$

Instead, if $\zeta = 1$ that is $\lambda_1^k = \lambda_2^k = \overline{\lambda^k}$, it results

$$p_{[(k-1)dt, kdt]}(t) = (c_1^k + c_2^k t) \exp(\overline{\lambda^k} t) + \frac{f_k}{\omega_k^2} \tag{11}$$

where

$$c_1^k = \left[\left(p_k - \frac{f_k}{\omega_k^2} \right) \left(1 + \overline{\lambda^k} kdt \right) - \dot{p}_k kdt \right] \exp - \left(\overline{\lambda^k} kdt \right) \tag{12}$$

$$c_2^k = \dot{p}_k - \overline{\lambda^k} \left(p_k - \frac{f_k}{\omega_k^2} \right). \tag{13}$$

Numerical results

The procedure described in the previous section has been applied for the computation of the optimal parameters able to reproduce the dynamical response of the numerical twin model.

The parameters \mathbf{x} to be optimized are summarized in Table 1.

Table 1. Optimization parameters.

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
k_z	$k_{\varphi,x}$	$k_{\varphi,y}$	c_z	$c_{\varphi,x}$	$c_{\varphi,y}$	$EI_{x,1}$	$EI_{y,1}$	$\alpha_{0,1}$	$\alpha_{1,1}$	$EI_{x,(4,5,6)}$	$EI_{y,(4,5,6)}$	$\alpha_{0,(4,5,6)}$	$\alpha_{1,(4,5,6)}$

In order to achieve the optimum value of the parameters in Table 1, the CPSO has been executed by choosing: $c_c = 1.49$, $c_s = 1.49$, number of population $n_{pop} = 100$, $T = 200$, $N = 200$ and maximum number of iterations 200.

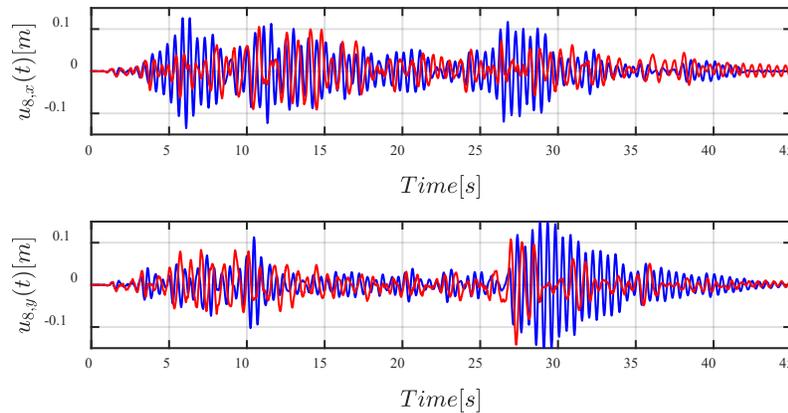


Figure 2. Displacements at the top of the structure (node 8) in x (upper) and y (lower) direction vs time: red line represents experimental data; blue line represents numerical simulation by using nominal parameters.

The Fig. 2 and Fig. 3 depict the displacement versus time of node 8 in the x and y direction considering the experimental data (red line) and numerical simulation (blue line) concerning the nominal parameters (Fig. 2) and the optimized parameters (Fig. 3). The comparison of the time signals in the latter figure shows a good match and it is possible to state that the proposed methodology allows to define a set of structural parameters able to correctly simulate the dynamic of the structure under investigation.

Several runs of the optimization procedure have been executed and a good agreement among them has been found in terms of both the optimum points and the corresponding objective function values.

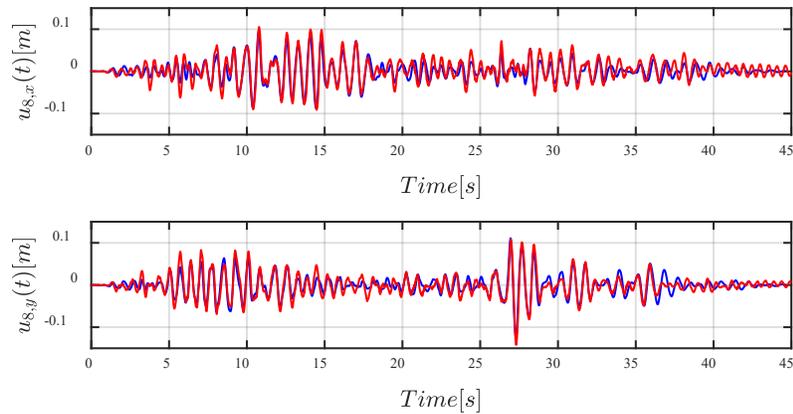


Figure 3. Displacements at the top of the structure (node 8) in x (upper) and y (lower) direction vs time: red line represents experimental data; blue line represents numerical simulation by using optimal parameters.

Summary

A model updating analysis based on CPSO has been presented and verified to numerically reproduce the dynamical response of a high voltage Current Transformer.

Future developments of this work will consider the comparison of the proposed model updating procedure with other classical optimization algorithms such as Genetic Algorithms and PSO, other objective functions depending on different components of the structural response and a modified version of the FEM model to investigate the role played by geometrical and mechanical nonlinearities.

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