

## Friction-induced parametric oscillations in automotive drivelines: experimental analysis and modelling

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**Abstract.** Clutch judder is a friction-induced self-excited vibration occurring in automotive drivelines, an NVH issue studied for more than forty years and attributed by the scientific community to three possible causes: stick-slip, negative gradient of the coefficient of friction and geometric disturbances. However, these explanations fail to describe the kind of judder studied in this contribution, arising in presence of an oscillating component (dither) in the clutch actuation pressure. The analysis of experimental data collected on a dual-clutch transmission mounted on a specific test bench suggested the presence of a parametric resonance, generated by the dither. A specific 4 degrees of freedom model was then developed, able to predict with good accuracy the unstable parametric region in which judder occurs and useful in the design stage.

### Introduction

Among various NVH issues affecting automotive drivelines, the friction-induced self-excited vibration called clutch judder holds particular importance.

To the best of the authors' knowledge, the current scientific literature on the subject identifies three main factors responsible for clutch judder. In several cases, it is attributed to a negative gradient of the coefficient of friction between the clutch plates (in this case referred to as cold judder) [1-7]. In fact, if the friction coefficient decreases with increasing slip speed, the system becomes unstable in most operating conditions. The second identified factor is stick-slip [7-11], associated with the alternate sticking and slipping motion which occurs due to the discontinuity of the friction curve at very low relative speeds. Finally, the third factor is given by geometric disturbances in terms of both asperity on the disc surfaces and misalignments between the components of the transmission. In the first case, the asperities of the discs can give rise to hot spots and thermoelastic instability [12,13]. In the second case, the misalignments between some components of the transmission give rise to the periodic fluctuation of the clamp load and to the possible excitation of a frequency of vibration of the system [7,14].

Although it has been shown that clutch judder can have different characteristics and causes, the phenomenon here under analysis has not been investigated yet in the scientific literature. It occurs in dual-clutch transmissions of heavy vehicles (tractors), in which the clutch actuation pressure is made up of a constant main component plus a small oscillating component due to the vibration of the circuit valve (dither). The experimental data collected on a specific test bench, designed in partnership with CNH Industrial, show the onset of a cold judder (total absence of hot spots) at a frequency approximately equal to the dither frequency, as displayed in fig. 1. Furthermore, judder's magnitude varies depending on whether odd or even gears are engaged.

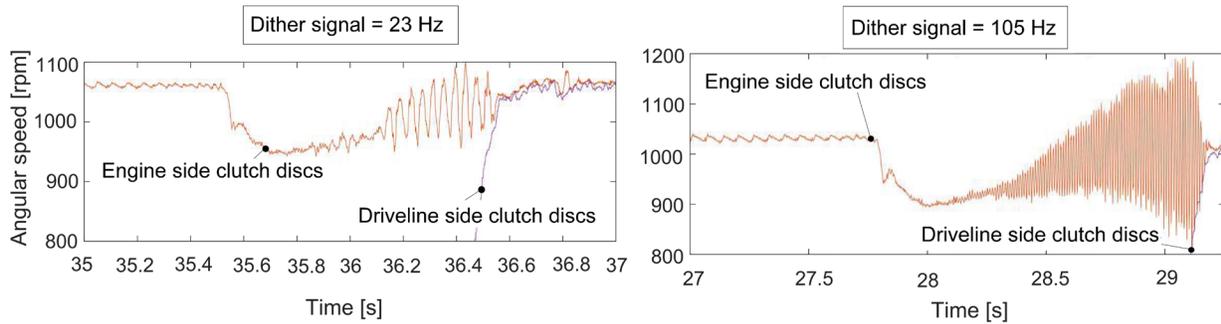


Fig. 1 - Experimental data: clutch judder arises at a high slip speed.

For a first qualitative investigation of this phenomenon, the authors developed a specific 3 degrees-of-freedom (dofs) minimal model, which was studied by means of Floquet theory [15], identifying parametric excitation due to dither as a new and unexplored source of clutch judder. For fitting experimental data and predicting the occurrence of the phenomenon, models with increasing complexity are here proposed, considering a larger number of dofs.

### Methods

The system under analysis consists of four electro-hydraulically actuated wet clutches, eight gear ratios and three reduction ranges, as represented in fig. 2. Two clutches engage odd and even gears, respectively, while the other two engage forward and reverse. To improve the response of the hydraulic valves, they are actuated by dithering the main signal, consisting of a constant main pressure component  $p_m$  and an oscillating component  $p_f$ .

The hypothesis of parametric resonance as the cause of this kind of clutch judder was first assessed by the authors in [15], with the aid of a 3 dofs minimal model. Alongside the theoretical study of the phenomenon, an experimental campaign was conducted on a test bench. The input parameters selected for each test were the angular speed of the electric motor (replacing the engine of the tractor), the main actuation pressure, the dither (fluctuating) pressure ( $p_f$ ) and frequency ( $\omega_p$ ) for each of the four clutches. Considering an adequate range of variation for  $\omega_p$  and  $p_f$ , clutch engagements were performed for each pair ( $\omega_p$ ,  $p_f$ ). The output parameter selected as an indicator for the occurrence of instability was the angular velocity upstream of the clutch (see fig. 1). After computing the relative spectrum of each experimental time history, the maximum value (peak) of each spectrum was reported on ( $\omega_p$ ,  $p_f$ ) contour plots, as displayed in fig. 4b, which can be considered as potential and experimentally estimated stability charts. These charts showed that the essential model used in [15] cannot reproduce the identified unstable region. Therefore, models with higher complexity have to be considered.

### Results and discussion

A 10 dofs model of the transmission is developed, as shown in Fig. 2. This model is described by a system of second order differential equations with time-periodic coefficients within the damping matrix.

Stability can be studied by either the Monodromy Matrix Method (MMM), or the Harmonic Balance Method (HBM). Both methods are based on Floquet theory [16], with the difference that the computational cost of MMM lies in the integration of the equations of the system, while the computational cost of HBM lies in the solution of a quadratic eigenvalue problem [17,18,19].

As a result, stability maps are drawn, characterized by the presence of unstable tongue-shaped regions (Arnold tongues). The stability map of the 10 dofs model is quite complex, with Arnold tongues of different kinds, as displayed in fig. 3. Tongues 1, 2 and 3 are regions of ‘single-period instability, in which the system oscillates with a period equal to that of the parametric excitation (dither). On the other hand, in tongues 4 and 5 the unstable oscillation of the system has a double

period with respect to that of the parametric excitation, yielding a double-period instability. Finally, tongues 6, 7 and 8 are due to complex conjugate Floquet multipliers, coming out of the unit circle in the Argand plane [16], thus in general yielding non-periodic unstable solutions.

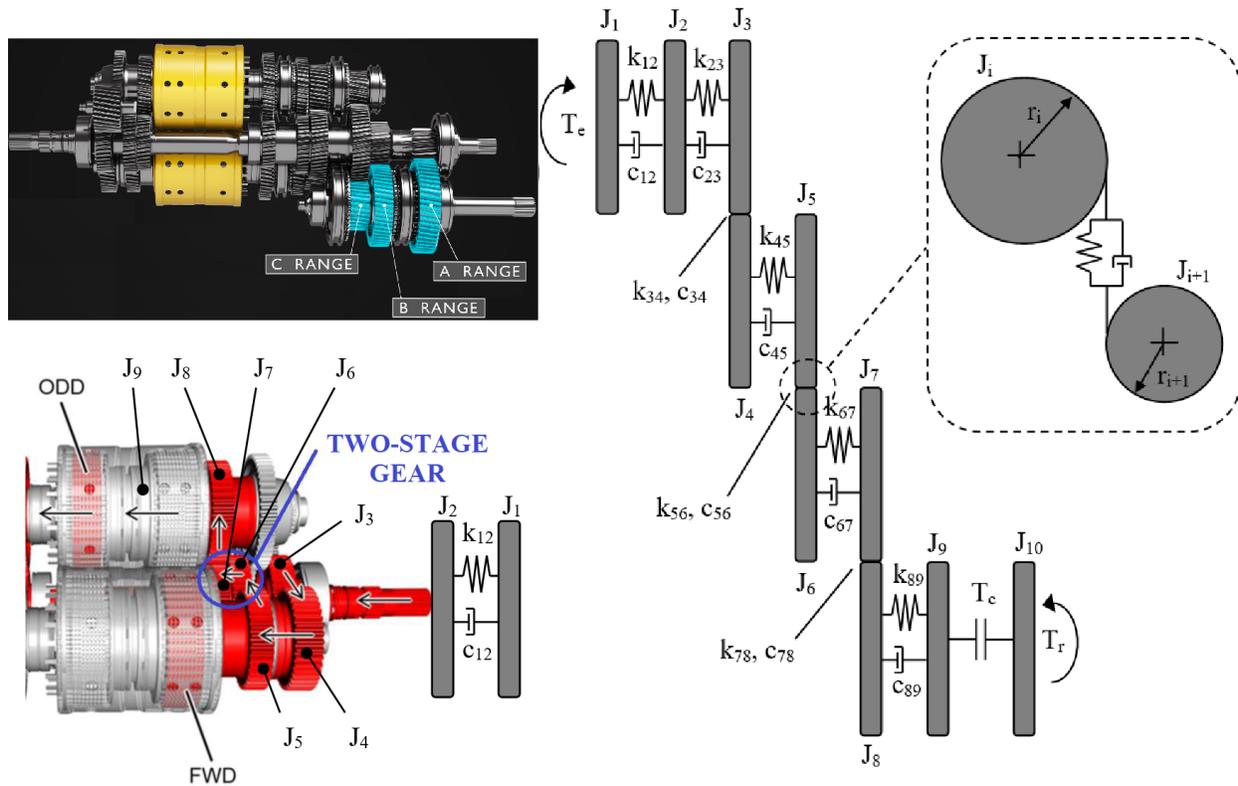


Fig. 2 – Dual-clutch transmission under analysis and 10 dofs model.

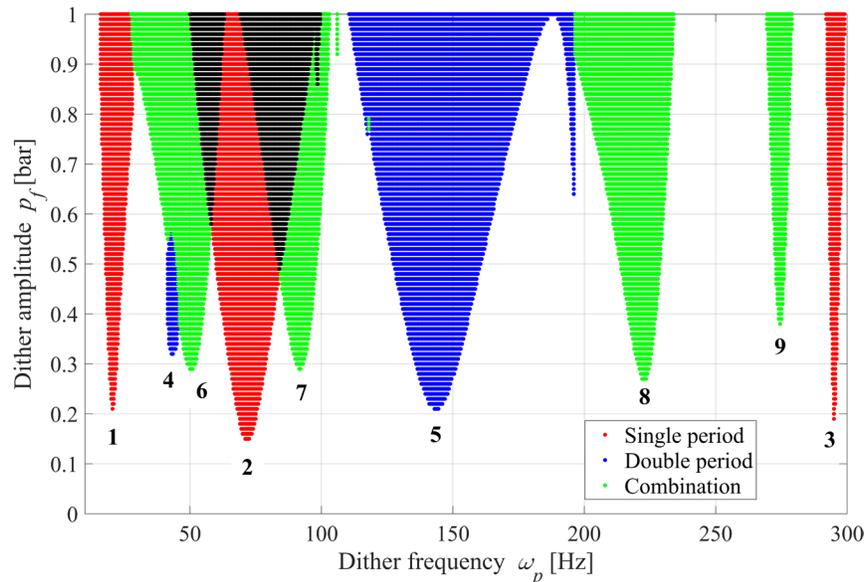


Fig. 3 – Stability map obtained with the 10 dofs model.

Given that the experimental campaign identified only one region of single-period instability, as shown in Fig. 4b, this suggests that the adopted 10 dofs model can actually be simplified into a more convenient minimal model. The stability map in fig. 3 shows only two single-period tongues

at low frequencies (1 and 2), while the others are located at much higher frequencies (other single-period tongues appear at frequencies outside the upper limit of the  $x$ -axis in fig. 3).

Analyzing the model, it was found that the part of the transmission between the engine and the first gear wheel is the one that has a preponderant influence on unstable tongues 1 and 2.

Therefore, an excellent compromise has been found with a 4 dofs minimal model in which all the components placed between the clutch damper ( $J_2$ ) and the clutch pack have been incorporated into a single rotating mass and a single torsional stiffness (fig. 4a).

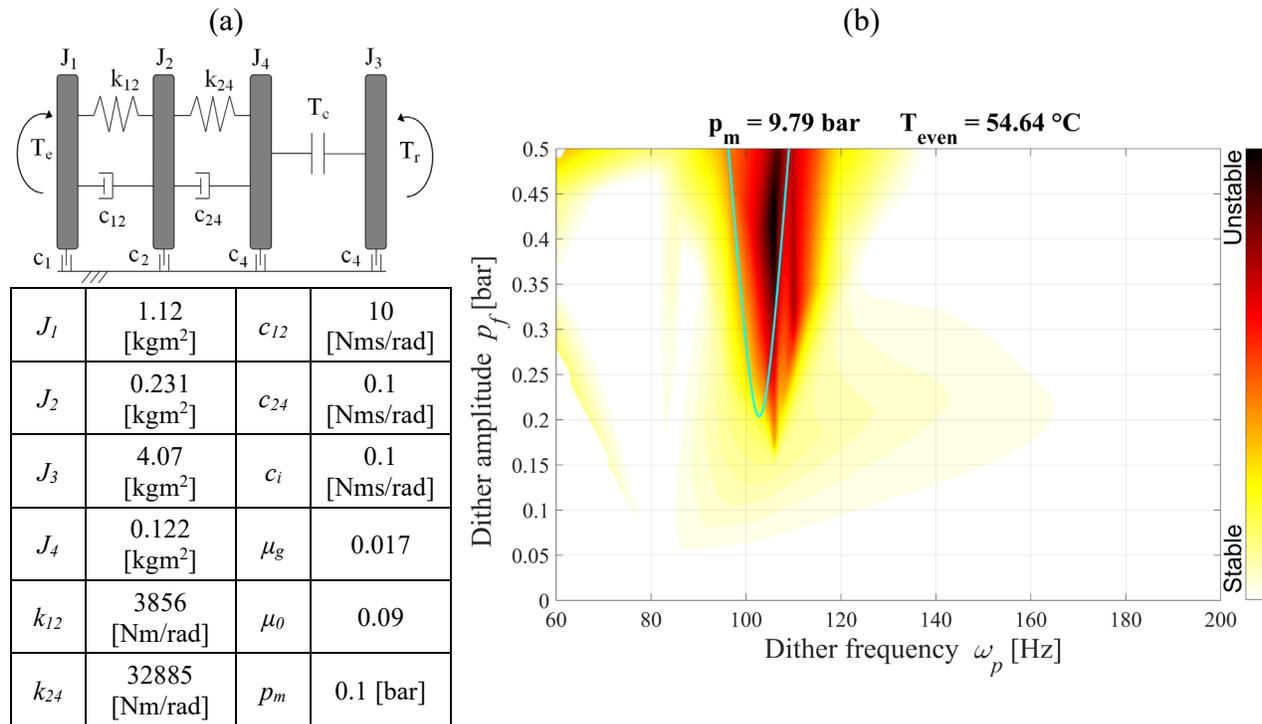


Fig. 4 – Four dofs model (a) and comparison with the experimental stability chart (b).

A very good correspondence was found between the theoretical stability map obtained with the 4 dofs model, as shown in fig. 4b (single-period tongue superimposed on the experimental chart), adopting the data reported below fig. 4a.

The equations of motion of the 4 dofs model shown in fig. 4a read:

$$\mathbf{M}\ddot{\boldsymbol{\theta}}(t) + \mathbf{C}(t)\dot{\boldsymbol{\theta}}(t) + \mathbf{K}\boldsymbol{\theta}(t) = \mathbf{g}(t) \tag{1}$$

with:

$$\mathbf{M} = \begin{bmatrix} J_1 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 \\ 0 & 0 & J_4 & 0 \\ 0 & 0 & 0 & J_3 \end{bmatrix}, \quad \mathbf{C}(t) = \begin{bmatrix} c_1 + c_{12} & -c_{12} & 0 & 0 \\ -c_{12} & c_2 + c_{12} + c_{24} & -c_{24} & 0 \\ 0 & -c_{24} & c_4 + c_{24} + \mu_g f(t) & -\mu_g f(t) \\ 0 & 0 & -\mu_g f(t) & c_3 + \mu_g f(t) \end{bmatrix} \tag{2}$$

$$\mathbf{K} = \begin{bmatrix} k_{12} & -k_{12} & 0 & 0 \\ -k_{12} & k_{12} + k_{24} & -k_{24} & 0 \\ 0 & -k_{24} & k_{24} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{\theta}(t) = \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ \theta_4(t) \\ \theta_3(t) \end{bmatrix}, \quad \mathbf{g}(t) = \begin{bmatrix} T_c \\ 0 \\ -\mu_0 f(t) \\ \mu_0 f(t) - T_r \end{bmatrix}$$

and:

$$f(t) = N_c R_m S p(t), \quad T_c = \mu(t) N_c R_m S p(t), \quad p(t) = p_m + p_f \cos(\omega_p t) \tag{3}$$

where  $J_i$  are moments of inertia,  $k_{ij}$  are torsional stiffness coefficients,  $c_{ij}$  are internal damping coefficients,  $c_i$  are external damping coefficients,  $N_c$  is the number of pairs of clutch plates,  $R_m$  is the mean radius of the clutch friction surfaces,  $p(t)$  is the actuation pressure,  $S$  is the area of action of  $p(t)$ ,  $T_c$  is the friction torque and  $\mu$  is the friction coefficient.

As a first approximation, the friction coefficient is considered linearly dependent on the relative speed of the clutch plates and is expressed as:

$$\mu(t) = \mu_0 + \mu_g(\dot{\theta}_4 - \dot{\theta}_3) \quad (4)$$

The values adopted for the parameters in the 4 dofs model are reported below fig 4a.

Sensitivity analysis showed that the experimentally identified single-period tongue is mainly influenced by the equivalent stiffness  $k_{24}$  and moment of inertia  $J_4$  of the components between the damper and the clutch. Furthermore, the damping coefficient of the clutch damper  $c_{12}$  has a great impact on the stability map because, if considering lower values, it gives rise to two combination regions which are symmetrical with respect to the identified single-period tongue, giving rise to a sort of tricuspid tongue. This tricuspid tongue is also visible in the map of the 10 dofs model, composed of unstable regions 6, 2 and 7: by reducing  $c_{12}$  the tips of tongues 6 and 7 are lowered and thus the unstable region broadens.

Finally, it should be pointed out that the adopted model does not take into account several aspects of the complexity of the real transmission, such as nonlinear effects and asymmetries, able to produce relevant modifications in the stability thresholds, as for instance ‘merging’ effects between adjacent instability tongues [18,19].

## Summary

The analysis of clutch judder on a dual-clutch gearbox for tractors has led to the identification of parametric resonance due to dither as a new possible cause for the onset of vibration.

Models with different levels of complexity have been developed, and a 4 dofs minimal model was identified, able to predict with good accuracy the unstable parametric region in which judder occurs and representing a useful tool at the design stage.

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