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A reduced hysteretic model of stockbridge dampers

Francesco Bogani[1, a], Alex Sosio[1,b], Francesco Foti[1,c *]and Luca Martinelli[1,d]

[1]Department of Civil and Environmental Engineering (DICA), Politecnico di Milano, Milano, Italy

[a]francesco.bogani@mail.polimi.it, [b]alex.sosio@mail.polimi.it, [c]francesco.foti@polimi.it, [d]luca.martinelli@polimi.it

* corresponding author

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Abstract. This paper presents a reduced model to describe the nonlinear dynamic behavior of Stockbridge dampers. The proposed model is based on the classic Bouc-Wen hysteretic law and requires the identification of a small number of model parameters. The proposed formulation is used within an extended version of the classic Energy Balance Method to assess the role of the damper nonlinearities in mitigating aeolian vibrations of a reference overhead transmission line.

Introduction

Conductors and guard wires in overhead transmission line (OHL) are prone to vortex-induced-vibrations, also known as aeolian vibrations in the electrical engineering community. Whenever not properly controlled, aeolian vibrations can induce wear damage and fatigue failures of the cables. Stockbridge dampers are often used to mitigate the severity of such vibrations. Stockbridge dampers are characterized by a markedly nonlinear dynamic response, which is related to the intrinsic properties of their components, namely the hysteretic bending behavior of messenger cables.

This paper presents a reduced model to describe the non-linear dynamic behavior of one of such dampers. The proposed model, which is based on the classic Bouc-Wen hysteretic law, is then used within an extended version of the classic Energy Balance Method (EBM) [1] to assess the role of the damper nonlinearities in mitigating aeolian vibrations of a transmission line.

The Stockbridge damper model

As it is schematically depicted in Fig. 1, Stockbridge dampers are made of a metallic clamp, two inertial bodies and a short metallic strand, also known as “messenger cable”. The messenger cable is typically made of a straight core wire, which is surrounded by one or two concentric layers of helical wires. The clamp allows to rigidly connect the damper to OHL conductors and guard wires. Vibrations of the clamp set in motion the two branches of the messenger cable, which behave as two uncoupled cantilevers with lumped translational and rotational masses attached to their end sections.

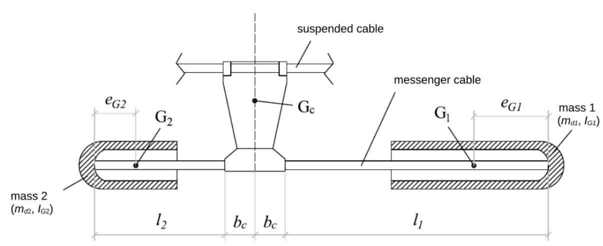

Figure 1. Geometry and inertial properties of a Stockbridge damper. Masses and centroidal mass moments of inertia are denoted respectively as M_{di} and I_{Gi} (i = 1, 2).

The dynamic response of Stockbridge dampers is markedly non-linear and hysteretic, due to frictional dissipation of energy localized on the contact surfaces between adjacent wires of the messenger cable. Non-linearity may be in principle exploited to increase the range of frequencies over which the damper can effectively mitigate aeolian vibrations. Only few models, however, can be found in the literature that specifically address the issue of the characterization of the nonlinear dynamic response of Stockbridge dampers [2-7]. Most of these formulations are based on computationally expensive strategies to model the hysteretic behavior of the messenger cable and /or require the identification of a significant number of parameters from experimental tests. A remarkable exception, within this context, is the model initially proposed by Pivovarov and Vinogradov [2]. The messenger cable is herein modelled as an equivalent single-degree-of-freedom (SDOF) system, with restoring forces defined through the phenomenological hysteretic model proposed by Bouc [8]. The model of Pivovarov and Vinogradov has been recently re-considered by Foti et al. [7], which adopted a formulation of the Bouc-Wen model based on a minimal set of parameters [9].

As a major drawback, SDOF phenomenological models only allow to reproduce the dynamic response associated to the translation of the masses attached at the end sections of the messenger cable. In the present work, the formulation proposed by Foti et al. [7] is generalized to account for the rotational motion of the damper masses.

To keep the presentation of the model as straight as possible, let us herein focus on the special case of symmetric Stockbridge dampers (i.e. $m_{d1}=m_{d2}=m_d$, $I_{G1}=I_{G2}=I_G$, $l_1=l_2=l_d$, $e_{G1}=e_{G2}=e_G$ - see Fig. 1) subject to a pure vertical translation of the clamp. Details on the general case of non-symmetric dampers subject to a combined rotational and translational motion of the clamp can be found in [10].

The force F_d exerted by the damper on the cable (cable-damper interaction force) can be expressed as:

$$F_d = (2m_d + m_c)\,\dot{v}_c + 2m_d\,\ddot{u}_1 - 2\,m_d\,e_G\,\ddot{u}_2 \tag{1}$$

where m_c is the mass of the clamp, v_c is the velocity of the clamp, u_1 and u_2 are, respectively, the relative displacement of the centroid with respect to the clamp and the rotation of the rigid body attached at each end of the messenger cable (see also the formulation presented in [3]).

The dynamics of the two degrees-of-freedom u_1 and u_2 is assumed to be approximately described by the following decoupled equations of motion:

$$M_i\,\ddot{u}_i + F_i = Q_i, i = 1,2 \tag{2}$$

where the generalized masses M_i read: $M_1 = m_d$ and $M_2 = I_G + m_d\,e_G^2$; the generalized external forces Q_i read: $Q_1 = -\,m_d\,\dot{v}_c$ and $Q_2 = m_d\,e_G\,\dot{v}_c$; and the generalized restoring forces F_i are described through the following equations:

$$\begin{cases} F_i = k_{min,i}\,u_i + \left(k_{max,i} - k_{min,i}\right)u_{0,i}\,z_i \\ \qquad u_{0,i}\,\dot{z}_i = \dot{u}_i - |\dot{u}_i| z_i \end{cases} \tag{3}$$

where z_i is a non-dimensional hysteretic variable with values in the range [-1, 1], while $k_{min,i}$ $k_{max,i}$ and u_{0i} are the parameters of the model. On the overall, hence, the total number of parameters of the proposed model that need to be identified from experimental tests is equal to six (three for each degree-of-freedom of the masses).

The Energy Balance Method

The technical approach currently adopted to assess the severity of aeolian vibrations relies on an application of the Energy Balance Method (EBM), e.g. [1]. Without loss of generality, the EBM will be presented in this section with reference to a taut cable of length l, suspended to rigid horizontal supports and equipped with a single Stockbridge damper. The damper is assumed to be attached at a distance x_d from one of the supports.

The EBM is based on the assumption of steady-state mono-modal vibrations of the cable. For each vibration frequency f, the single-peak antinode vibration amplitude y_{max} is obtained by imposing the balance between the average power per vibration cycle imparted by the wind to the cable (P_w) and the one dissipated in the coupled cable-damper system, that can be expressed as: $P_c + P_d$, where the term P_c accounts for the internal dissipation within the cable (also known as cable "self-damping"), while P_d is the average power per vibration cycle dissipated by the damper. The equation, hence, reads:

$$P_w\,(y_{max}, f) - P_c(y_{max}, f) - P_d(y_{max}, f) \;=\; 0, \text{for any } f \tag{4}$$

The wind input power P_w is typically obtained from wind-tunnel tests. Empirical results for laminar wind conditions can be expressed in the following general form:

$$\frac{P_w}{l} \;=\; D^4 f^3 \mathrm{fnc}\left(\frac{y_{max}}{D}\right) \tag{5}$$

where D is the diameter of the cable and "fnc" is a nonlinear function of the non-dimensional vibration amplitude y_{max}/D. Different expressions have been proposed in the literature for the function "fnc" (see e.g. [1, 11]) along with correction coefficients that allows to modify Eq. (5) to account for the effect of turbulence (see e.g. [12]). Without loss of generality, the present work will focus only on laminar wind conditions and will adopt the definition of "fnc" recommended in [11].

The cable self-damping P_c can be modeled by means of both empirical and theoretical expressions (see e.g. [13] for a detailed discussion on this topic). Without loss of generality, in the present work the theoretical expression derived by Foti and Martinelli [13] under the assumption of micro-slip conditions on the contact surfaces between adjacent wires of the vibrating cable will be adopted:

$$\frac{P_c}{l} \;=\; \frac{128\pi^5\gamma^3 RTS\, EI_{max,ef}}{3\, c_0\mu}\;\frac{y_{max}^3 f^7}{T^4} \tag{6}$$

where T is the tensile force (approximately assumed as constant along the cable span); γ, RTS and $EI_{max,ef}$ are, respectively, the mass per unit of length, the Rated Tensile Strength (RTS) and the maximum bending stiffness of the cable; the coefficient c_0 is a construction parameter that only depends on the geometry of the cable cross section; and μ is the friction coefficient adopted to describe the contact conditions between the wires of the cable (see [13] for further details).

The power dissipated by the damper, P_d, reads:

$$P_d \;=\; \tfrac{1}{2}\cos(\zeta)|Z_d| v_c{}^2 \tag{7}$$

where $|Z_d|$ and ζ are, respectively, the module and phase of the mechanical impedance function (Z_d) of the damper. By adopting the classic undamped taut string model to describe the cable vibrations, the clamp velocity v_c can be related to the vibration frequency f and antinode vibration amplitude y_{max} through the equation:

$$v_c \;=\; f\, y_{max} \left|\sin\left(\frac{2\pi f}{\Omega_1}\;(1-\alpha)\right)\right| \tag{8}$$

where $\alpha = x_d/l$ and Ω_1 is the fundamental circular vibration frequency of the bare cable, i.e. $\Omega_1 \;=\; \frac{1}{l}\sqrt{\frac{T}{\gamma}}$. Substitution of Eqs. (5)-(8) in the balance equation (4) yields a non-linear algebraic equation that can be solved to get the maximum expected nondimensional aeolian vibration amplitude y_{max}/D as a function of the vibration frequency f.

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It is worth noting that the procedure described up to this point applies as presented strictly to linear dampers i.e., in other terms, to dampers whose dynamic behavior can be characterized by a single impedance function. Whenever dealing with nonlinear damper models, the impedance function of the damper will depend on the motion amplitude in addition to the frequency, hence an iterative approach is required.

Example of application

The proposed modeling strategy is applied to investigate the aeolian vibrations of a benchmark OHL span already studied elsewhere (see e.g. [7, 14]). The length of the span is 450 m and the cable is a ACSR Bersfort 48/7 conductor (diameter D=35.6 mm, mass per unit of length γ=2.375 kg/m, Rated Tensile Strength RTS=180 kN, c_0 =0.139, μ =0.3). The cable is strung at T=0.4RTS.

A single symmetric Stockbridge damper that was experimentally tested by Sauter [15] is assumed to be present at the 1% of the span (i.e. at the non-dimensional arc-length coordinate $\alpha=x_d/l$=0.01). Geometrical and inertial parameters of the damper are fully reported in [10, 15]. Sauter [15] performed sweep tests on a shaker at two different constant values of the clamp velocity: v_c=50mm/s and v_c=200 mm/s. From these tests the parameters of the proposed non-linear model of the damper were identified to match the experimental data obtained for the clamp velocity v_c=200 mm/s. The identified model parameters are: $k_{min,1}$ = 2000 N/m, $k_{max,1}$ = 8000 N/m, $u_{0,1}$ = 0.002 m, $k_{min,2}$ = 68.5 N/m, $k_{max,2}$ = 275 N/m, $u_{0,2}$ = 0.0075 rad.

The required simulations were performed by applying a sinusoidal motion of the clamp with a frequency sweep from 0 to 100 Hz during a total duration of 200 s. The non-linear equations of motion of the damper were then numerically integrated.

Fig. 2 shows the matching between experimental results and the damper model output in terms of the real part of the damper impedance function. The predictions of the proposed damper model match very well the experimental results, not only for the clamp velocity v_c=200 mm/s, used in the identification process, but also for v_c = 50mm/s.

Fig. 3 shows the comparison of the OHL aeolian vibration amplitude computed with a linear model of the damper (Fig. 3a) and with a nonlinear model of the same (Fig. 3b). The vibration amplitude of the OHL without the damper is reported as well for comparison purposes. As it can be appreciated, the damper (no matter the model) is effective in the range of frequency approximately 10-20 Hz. The curve for the nonlinear damper model tends to follow the curve associated to higher vibration velocites at the lower end of the cited frequency range, while it follows the lower velocity one at the higher ends.

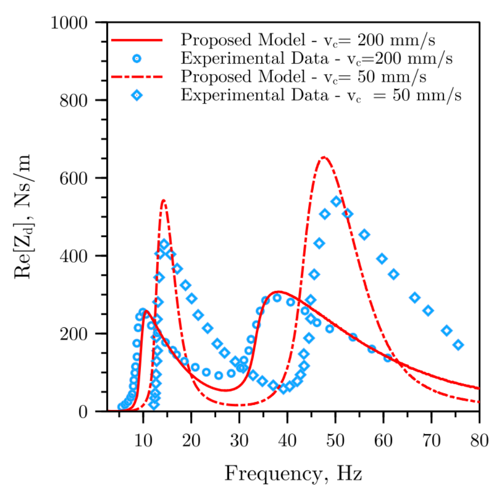

Figure 2. Absolute value of the real part of the impedance function of the identified model of the damper, compared to the experimental values in [15].

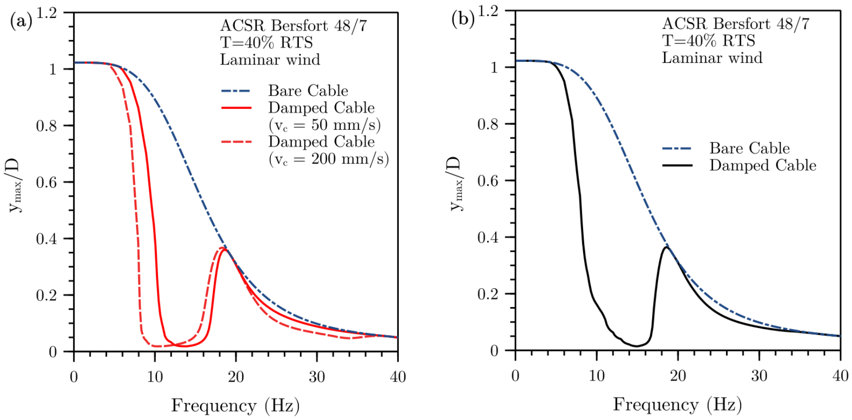

Figure 3. (a) Nondimensional vibration amplitudes of the line with and without linear damper strung at T= 0.4RTS. (b) Same for nonlinear damper.

Conclusions

A novel model for the nonlinear dynamic response of Stockbridge dampers has been presented. The model is based on a classic Bouc-Wen hysteretic law, and depends on a small number of physical parameters that can be identified from dynamic tests. A procedure to account for the Stockbridge damper nonlinear response on the assessment of aeolian vibrations of OHL has been briefly presented. Even though only one damper and one line has been considered in this work, numerical results suggest that an appropriate linear model is adequate to get meaningful estimates of vibrations amplitudes. Availability of a practical nonlinear model of the Stockbrigde damper paves the way to enhanced integrated design strategies of the dampers and the overhead lines that fully exploit the damper nonlinearities.

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