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# Numerical modeling of dynamic crack propagation mechanisms using a moving mesh technique based on the ALE formulation

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**Abstract.** This work proposes a new FE model to predict dynamic crack propagation mechanisms in quasi-brittle materials. The numerical strategy uses a Moving Mesh (MM) technique consistent with the Arbitrary Lagrangian-Eulerian (ALE) formulation to reproduce the variation of the geometry of the computational domain caused by dynamically growing cracks. Specifically, the motion of the mesh nodes takes place consistently with conditions dictated by classic Fracture Mechanics, which provide conditions concerning the direction of propagation and the velocity of advancing cracks. As a remarkable key novelty, the proposed method introduces the ALE formulation of the M-integral for extracting Dynamic Stress Intensity Factors (DSIFs) at a moving crack front. This formulation allows extracting fracture variables on deforming elements without losing accuracy. Comparisons with analytical and numerical data are proposed to assess the validity and efficiency of the proposed strategy.

## Introduction

Recent developments in computational fracture mechanics have led to a proliferation of numerical methodologies for analyzing the failure behavior of most materials typically used in civil engineering and mechanical applications. Many numerical methodologies have been developed in the context of the Finite Element method because of its widespread diffusion and flexibility in modeling complex structures (see, for instance, [1-10]). Unfortunately, each method presents advantages and disadvantages, so it is relatively complicated to identify the best one.

Currently, there is an increasing request for numerical methods that combine high accuracy and computational efficiency [11, 12].

Consistently with such a necessity, this work presents an effective numerical model for reproducing dynamic fracture phenomena in quasi-brittle materials. The proposed model consists of an FE code enhanced by a Moving Mesh (MM) technique based on the Arbitrary Lagrangian-Eulerian (ALE) formulation [13, 14]. Specifically, the MM is used to reproduce the geometric variation of the computational domain because of the dynamically advancing cracks. Besides, it offers a reliable evaluation of key fracture variables (*i.e.*, Dynamic Stress Intensity Factors (DSIFs)) owing to the use of the Interaction Integral method (*i.e.*, *M*-integral). In particular, this work proposes as a key novelty the ALE formulation of the dynamic *M*-integral, which permits using the Interaction integral in the context of the ALE formulation.

The validity of the present approach is assessed by comparing the proposed method's previsions with experimental data and numerical results available in the literature.

## Theoretical background

*Arbitrary Lagrangian-Eulerian formulation*

Traditional problems of continuum mechanics are formulated either in a spatial or a material system of coordinates, denoted as $R_x$ and $R_X$, respectively. The ALE formulation uses a third

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coordinate system, that is the mesh (or referential) coordinates system $R_\chi$, in which the coordinate $\boldsymbol{\chi}$ identifies the nodes of the computational mesh. A proper set of mapping functions serve to link $R_x$, $R_X$, and $R_\chi$ (see Fig. 1). The use of the ALE formulation requires that the governing equations of the problem must be expressed in $R_\chi$ instead of $R_X$. According to the ALE, the gradient of a vectorial field $\mathbf{v}^M(\mathbf{X},t)$ in the material domain can be stated in the referential one using the following expression:

$$\nabla^M \mathbf{v}^M = \frac{\partial \mathbf{v}^M}{\partial \mathbf{X}} = \frac{\partial \mathbf{v}^R}{\partial \boldsymbol{\chi}} \frac{\partial \boldsymbol{\chi}}{\partial \boldsymbol{\Psi}} = \nabla^R \mathbf{v}^R \mathbf{J}_{\boldsymbol{\Psi}}^{-1} \tag{1}$$

where $\nabla^M(\cdot) = \partial(\cdot)/\partial\mathbf{X}$, $\nabla^R(\cdot) = \partial(\cdot)/\partial\boldsymbol{\chi}$, and $\mathbf{J}_{\boldsymbol{\Psi}}$ is the Jacobian of the mapping function $\boldsymbol{\Psi}(\boldsymbol{\chi},t)$.

*Governing equations*

The governing equations at the base of the proposed method are twofold: the fundamental equations of solid mechanics and those concerned with the MM problem. In particular, the latter is consistent with the Laplace regularization approach, which smoothly moves mesh nodes, avoiding excessive distortions for the Finite Element. Besides, additional equations of Fracture Mechanics are included for (i) identifying crack onset conditions, (ii) evaluating the direction of propagation, and (iii) quantifying the velocity of the crack tip.

*The Interaction Integral method*

The Interaction Integral method, also known as *M*-integral, is a strategy used in numerical methods to extract DSIFs at the crack front. The M-integral expression derives from the J-integral applied to a superimposed state formed by two admissible fields. In particular, the first regards the problem under investigation (denoted as the actual state), while the second is an auxiliary state (aux) with DSIFs known. The expression of the *M*-integral assumes the following form:

$$\begin{aligned} M = \int_A \left[ \left( \sigma_{ij}^{act} u_{j,1}^{aux} + \sigma_{ij}^{aux} u_{j,1}^{act} \right) - \left( \sigma_{ij}^{act} \varepsilon_{ij}^{aux} + \rho \dot{u}_j^{act} \dot{u}_j^{aux} \right) \delta_{1i} \right] q_{,i} dA + \\ + \int_A \left[ \rho \left( \ddot{u}_j^{act} u_{j,1}^{aux} + \ddot{u}_j^{aux} u_{j,1}^{act} \right) - \rho \left( \dot{u}_{j,1}^{act} \dot{u}_j^{aux} + \dot{u}_{j,1}^{aux} \dot{u}_j^{act} \right) \right] q dA \end{aligned} \tag{2}$$

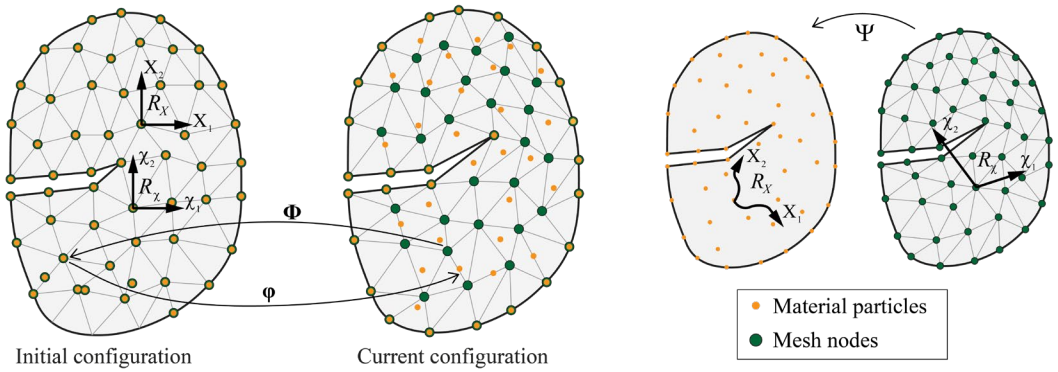


*Fig. 1 – A schematic representation of Spatial, Material, and Referential coordinate systems.*

## Numerical implementation

The proposed strategy has been implemented in COMSOL Multiphysics [15], a commercially available software. COMSOL offers a helpful tool that connect it with MATLAB software, thereby managing COMSOL functions through script codes [16, 17]. This feature has served to develop a

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user-made script that manages the propagation process automatically. The fundamental steps of the script are reported in Table 1. For further clarification, see [18-20].

*Table 1 – Main steps of the MATLAB script*

| | | |
|---|---|---|
| START | 1. | Set data of the problem: Geometry, material properties and boundary conditions |
| | 2. | Define stretching segment and initial mesh configuration |
| | 3. | Loading process: Evaluate DSIFs and assess crack initiation conditions |
| | 4. | Propagation process: Evaluate crack tip velocity and check mesh quality |
| STOP | 5. | Check tolerance to the angle variation, crack arrest or collapse conditions |

**Results**

Fig. 2-a shows a rectangular pre-cracked plate of length L=10 m, height 2H=4 m, and thickness b=1 m. The plate is subjected to a uniform and distributed traction $\sigma_0$ = 500 MPa at the upper boundary. Externally, line constraints on vertical boundaries limit the horizontal displacements of the plate against horizontal displacements. The Young's Modulus, Poisson's ratio, and mass density are equal to $E$=210 GPa, $\nu$=0.3, $\rho$=8000 kg/m$^3$, respectively. Besides, the dilatational and Rayleigh wave speeds are $c_d$=5944.5 m/s and $c_r$=2942.8 m/s, respectively.

Fig. 2-b depicts the mesh configuration used in the analysis. It comprises 432 triangular elements densely arranged around the crack tip and somewhat coarser in the remaining zones.

The analytical solutions associated with this fracture problem concerning a stationary crack and a moving crack have been developed by Freund [21]. In particular, the mode I DSIF analytical expressions gained by Freund for stationary and moving cracks assume these form:

$$K_I(0,t) = \frac{2\sigma_0}{1-\nu}\sqrt{\frac{c_d(t-t_c)(1-2\nu)}{\pi}} \quad \text{(Stationary crack)}$$
$$K_I(\dot{a},t) = \frac{2\sigma_0}{1-\nu}\sqrt{\frac{c_d(t-t_c)(1-2\nu)}{\pi}}\frac{1-\dot{a}/c_r}{1-\dot{a}/(2c_r)} \quad \text{(Moving crack)} \tag{3}$$

Note that such solutions are valid until the stress wave generated by the action of the external loads on the upper boundary of the plate, and reflected by the bottom boundary, reaches the crack tip. This behavior is experienced in the interval of time estimated to be $0<t<3t_c = 3H/c_d$.

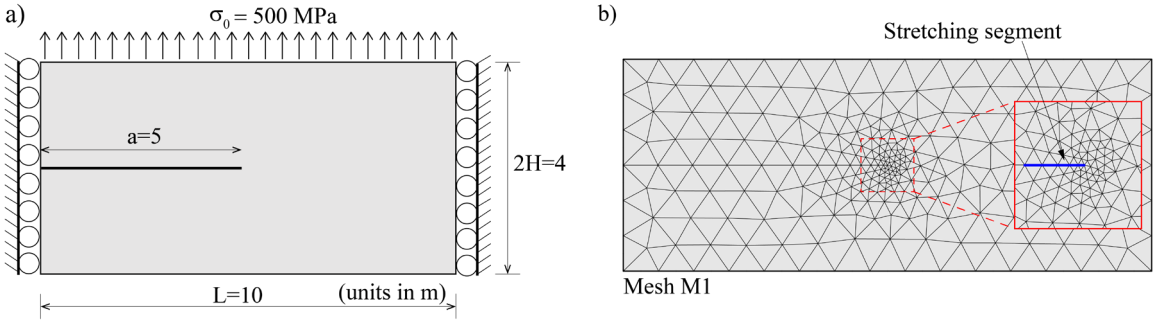


*Fig. 2 – (a) Geometry and boundary conditions; (b) Mesh configuration adopted in numerical simulations.*

Besides analytical solutions developed by Freund, there are several numerical results in the literature. Among these, Menouillard et al. [22] and Chen et al. [23] have investigated the fracture behavior of the plate using an XFEM strategy and an advanced numerical procedure based on the Singular Edge-based Smoothed Finite Element Method (SE-FEM), respectively.

In order to assess the reliability of the proposed model, it is supposed a failure behavior of the plate characterized by an initial stationary crack that starts moving for $t/t_c>1.5$.

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Fig. 3-a compares the normalized Mode-I DSIFs $K_I/K_0$ ( $K_0 = \sigma_0\sqrt{H}$ ) versus normalized time $t/t_c$ predicted by the proposed method with the analytical solution of Freund and numerical results of Menouillard et al. [22] and Chen et al. [23]. The results denote that the proposed method agrees well with both analytical and numerical solutions since no appreciable differences in the curves of $K_I$ are observed.

A parametric study in terms of mesh configurations is now performed to assess the computational efficiency of the numerical model. In particular, the study compares the mesh in Fig. 2-b (Mesh M1), with the Mesh M2 (entirely coarse with 175 triangular elements) and the Mesh M3 (refined configuration of 1113 triangular elements) reported in Fig. 4-b.

Fig. 4-a shows that the mesh discretization does not influence Mode-I DSIFs results as there are no significant differences between the curves. In addition, Fig. 4-a reports the percentage error (ERR) between numerical results and analytical solutions defined as follows:

$$\mathrm{ERR}(\%) = \frac{\left|\mathrm{K_I^{Analytical}} - \mathrm{K_I^{Numerical}}\right|}{\mathrm{K_I^{Analytical}}} \tag{4}$$

From the curves of ERR, it is possible to note that the proposed approach extracts $K_I$ quite well for each mesh configuration because the percentage variation is less than 10%, even if peaks that exceed this threshold just before a re-meshing event occurs.

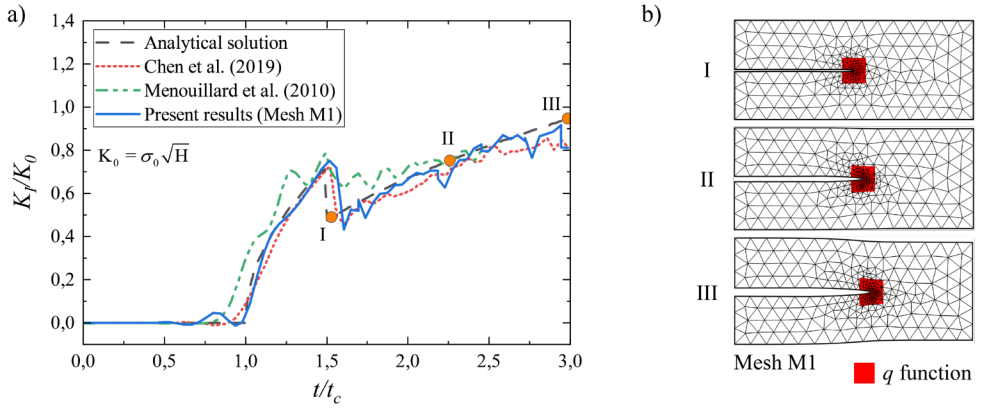


*Fig. 3 – (a) Comparison in terms of normalized Mode-I DSIF between the analytical solutions proposed by Freund [21], the numerical results obtained by Chen et al. [23] and Menouillard et al. [22] and the present approach; (b) A schematic of the crack propagation process simulated by the proposed approach.*

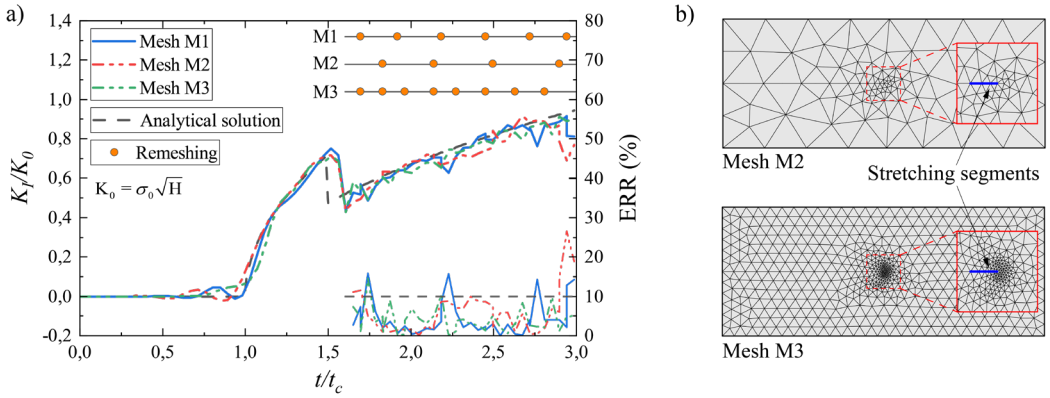


*Fig. 4 – (a) Comparison in terms of normalized Mode-I DSIF and percentage error between numerical results and analytical solutions; (b) Mesh configurations used in the parametric study.*

## Conclusions

This work has proposed a new FE modeling strategy for reproducing dynamic crack propagation phenomena in quasi-brittle materials. The results denote that the proposed model ensures accurate fracture variable prediction, thus representing a powerful numerical tool for analyzing the failure behavior of quasi-brittle materials under the action of dynamic loadings.

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