

## A simple procedure for the non-linear optimization of cable tension for suspended bridges

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**Abstract.** Long-span suspended bridges rely upon networks of tensed cables that carry the weight of the deck. The networks of cables are generally connected to a number of vertical towers (pylons) that transfer the forces to the foundations as in the case of cable-stayed bridge. Their structural behaviour is highly influenced by the pretension forces on account of the redundancy of the structure. Several methods have been proposed for determining pretension the forces in the cables, such as Load-Balance Method, Iterative Unit Load Method, Force Equilibrium Method, Zero Displacement Method. The present study aims to investigate the influence of geometrical non-linearities on the optimization of the design. To this end, the Force Equilibrium Method is here extended and compared to the use of a Finite Element commercial package, since this is the standard method in everyday engineering practice. The comparison between the Force Equilibrium Method and FEM results shows that the first method, in spite of its simplicity, is able to provide a reasonable and reliable alternative to the more complex non-linear FE approaches.

### Introduction

One of the fundamental problems with cable-sustained bridges is related to the determination of initial cable forces and loading sequence, which are needed to reach the correct geometrical configuration [1-2]. An optimal cable stress control allows for the stress and strain of the whole structure to be controlled both during and after the construction phase. Despite the fact that various approaches have been proposed and analysed, the whole problem still lacks concrete definition. There are no analytical frameworks to determine the stressing sequences necessary to obtain a predetermined final design configuration but a number of iterative algorithms that allow for the progressive adjustment of cable forces during the pre-construction phase have been proposed, some by the present authors, which can apply to both proportional and non-proportional loading [3-6]. Obviously, the cable tightening operations involve technological, structural, and economic problems.

The simple procedure proposed here is an original solution to the problem of cable pretension in cable-stayed structures. Unlike other methods, it makes reference to sensitivity analysis to identify the optimal configuration of stresses. More than a design tool, it can be seen as a structural behaviour optimization method that allows to determine the values of initial cable forces in order to give the structure a certain state of stress under prescribed loads.

Actually the illustrated method is an extension of the Influence Matrix method, already employed for other types of structures and suspended bridges [7-10]. In the present case it is applied to a suspension system consisting of a main cable and secondary hangers.

### The proposed procedure

The procedure is based on a preliminary analysis of the structure subjected to external loads. By means of a sensitivity analysis, in which the stiffness of the load-bearing elements are changed, an optimal distribution of stress is identified, such as the bending moment in the deck.



The chosen optimal configuration constitutes the target solution to be attained by pre-tensioning the cables.

The proposed procedure relies upon the application of the Influence Matrix Method in order to attain a chosen optimum configuration previously identified by means of a suitable sensitivity analysis.

The novelty here with respect to [9] is that the geometry of the structure is updated step by step until the final configuration is reached. The optimization procedure is thus applied to each step. At each pre-tension step the influence matrix is updated according to the actual geometry of the structure.

Once the single step has been solved, the new geometric configuration of the structure is set. A new influence matrix is constructed on the updated configuration, the optimum solution is identified and the pre-tension stresses to be applied to each cable are calculated. The procedure is considered concluded when the variation of the pretension efforts from one step to another becomes negligible.

### A worked example

In order to illustrate the procedure, a very simple system made of a main cable, three secondary cables, and a beam is taken into consideration, see Fig.1. The loading is applied to the horizontal beam and represents the self-weight load of the structure.

The sensitivity analysis showed that for this example reference can be made to a Low-stiffness deck (LSD) - High-stiffness cables model. Actually, the system can be considered as made of two macro systems, i.e. the suspension system and the loaded beam.

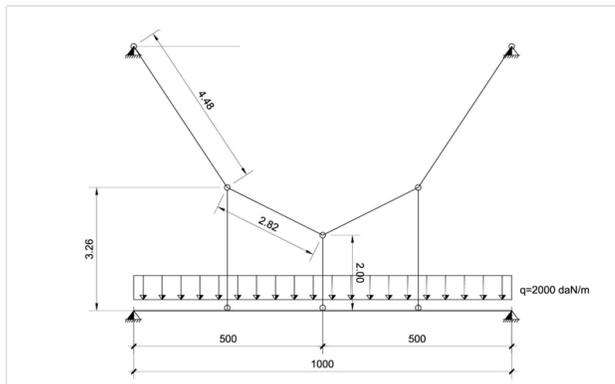


Fig.1. Geometry [m] and loads [daN].

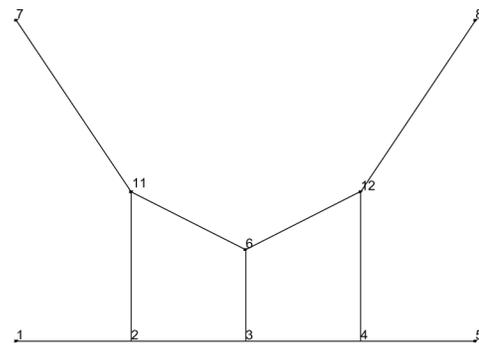
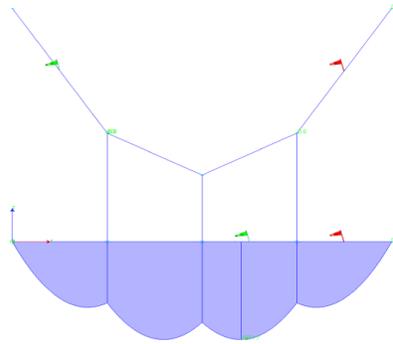
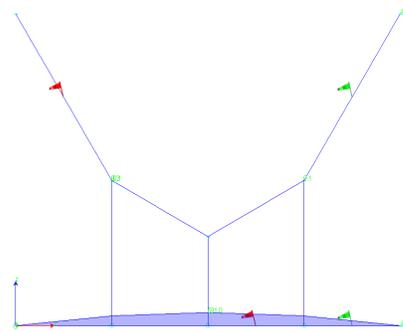


Fig.2. Point numbering.

In absence of any pre-tensioning of the cables, the bending moment diagram shown in Fig.2 is found. Fig.3 shows the diagram of the bending moment following the application of the first pre-stress loading in the cables and Figs. 4 and 5 the resulting optimised bending moment diagram and the deformed configuration at the end of the first step.



*Fig.3. Bending moment diagram in absence of any cable pre-stress.*

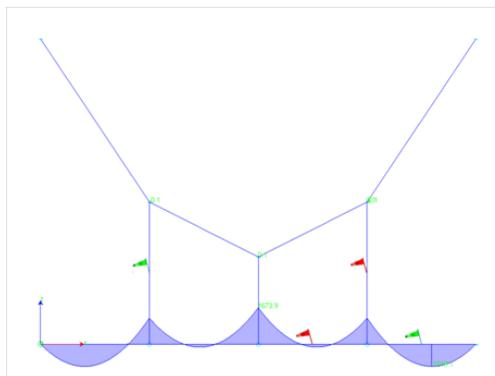


*Fig.4. Bending moment diagram due to the first pre-stress step application.*

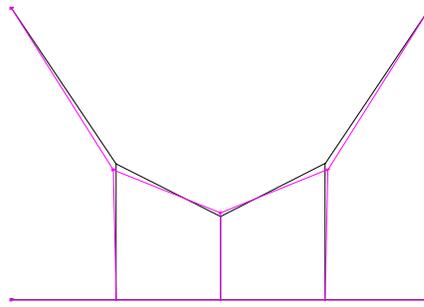
Rather obviously, the configuration of the structure changes at the end of the first step, thus triggering a geometric non-linearity in the problem. Therefore, the optimization procedure needs to be repeated for the displaced structure.

By applying the optimization procedure iteratively, for each step a new influence matrix is calculated and updated pre-tension loads are found.

The optimization goal can be considered attained when the configuration differences between two subsequent steps become negligible.



*Fig.5. Optimized bending moment (first step).*



*Fig.6. Deformed configuration after the first step optimization.*

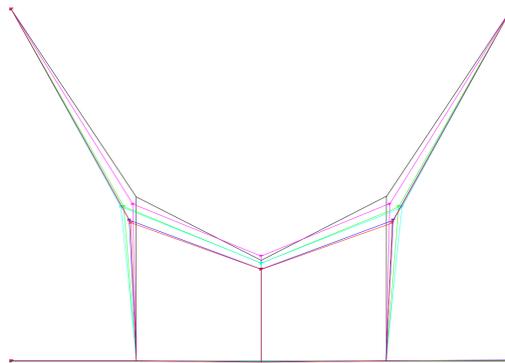
Rather intuitively, points 11 and 12 move outwards on account of the pretensions applied to the cables and elements (2-11-7) and (4-12-8) tend to align following the pre-tensioning. Overall, points 11 and 12 tend to line up with points 2-7 and 4-8. For a similar reason point 6 tends to move upwards. In the shift table, already in the first step, this phenomenon is found.

In the following steps, a new matrix of influence on the deformed configuration is defined and the forces to be applied to the cables in order to optimize the status of stress on the beam are calculated iteratively.

### **Discussion of results**

The iterations were carried on until the configuration differences between two subsequent steps became close to zero. In the case at hand six steps were required in order to attain the final configuration of the structure. Figs. 6 and 7 show the evolution of the configuration of the structure along with the optimization steps and the initial and final ones, respectively.

With reference to Fig.8, which shows the point numbering, Table 1 collects the absolute and incremental displacements of the point at each step. It appears evident that after a few iterations, the configuration of the structure stabilises quite abruptly. This fact is typical of structures involving cables and is discussed in details in [11]. The plots relative to the displacements of points 6, 11 and 12 are shown in Figs. 8 and 9.



*Fig.7. Deformed configuration evolution from step 0 to step 6.*

Table 1. Point displacements [cm] vs. steps: absolute (left) and incremental (right).

Step	3	6	11	12
0	0	0	0	0
1	0,09	9,47	15,35	-15,35
2	0,36	15,02	18,95	-18,95
3	0,52	0,04	6,45	-6,45
4	0,64	12,12	31,49	-31,49
5	0,61	0,41	6,16	-6,16
6	0,85	0,68	6,22	-6,22

Step	3	6	11	12
0	0	0	0	0
1	0,09	9,47	15,35	-15,35
2	0,27	5,55	3,60	-3,60
3	0,16	-14,98	-12,50	12,50
4	0,12	12,08	25,04	-25,04
5	-0,03	-11,71	-25,34	25,34
6	0,24	0,27	0,06	-0,06

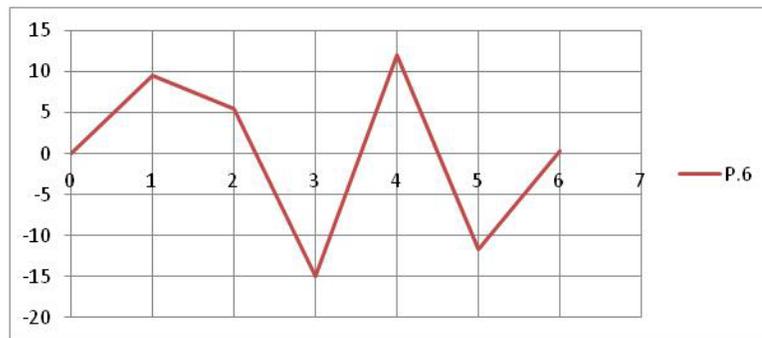


Fig.8. Displacement point n.6 [cm] from step 0 to step 6.

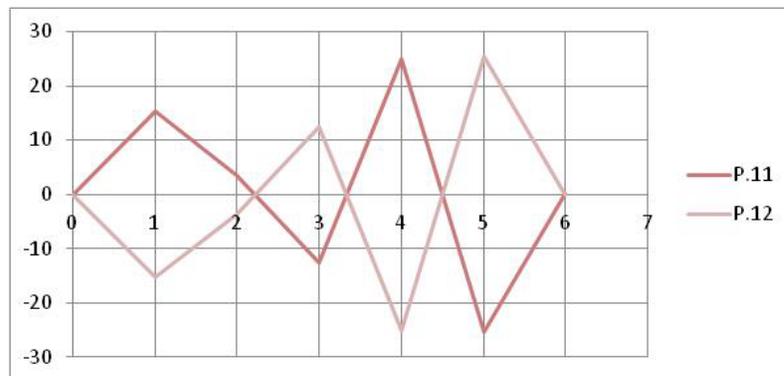


Fig.9. Displacement points n.11 and 12 [cm] from step 0 to step 6.

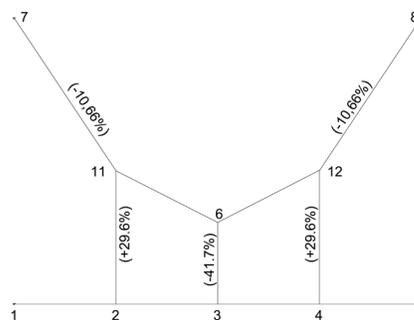


Fig.10. Different pretension between linear and non-linear optimization.

## Conclusions

The proposed procedure is concerned with the application of the Influence Matrix Method to a simple suspended bridge scheme under large displacements. The iterative application of the method, taking into account the geometric non-linearity, allows to calculate quite easily the optimal pre-stress in the cables loading in order to minimise the bending moment in the lower deck. The procedure seems susceptible to be extended to real examples of suspension bridges with reasonable computational effort.

It is worth pointing out that, with respect to the linear optimization procedure, as the one performed at step 1, the main cable at the end of the process results tensioned on average by about 10% less than in the linear procedure. The side hangers, on the other hand, show a pretension increment of 29%, while the central hanger results less pretensioned than in the linear procedure.

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