Materials Research Proceedings **26** (2023) 97-102 https://doi.org/10.21741/9781644902431-16

Flexural tensegrity: Field applications

Claudio Boni [1,a] * and Gianni Royer-Carfagni [1,2,b]

[1] Department of Engineering and Architecture, University of Parma, Parma, Italy

[2] Construction Technologies Institute - Italian National Research Council (ITC-CNR), San Giuliano Milanese, Milano, Italy

[a] claudio.boni@unipr.it, [b] gianni.royer@unipr.it

Keywords: Kinetic Structure, Segmental Beam, Nonlinear Nonlocal Response

Abstract. Flexural tensegrities represent a class of structural elements composed of segments in unilateral contact along properly-designed pitch profiles and held together by pre-tensioned cables. The cables are constrained in cavities inside the segments and can move within them, while the segments can roll one another along the pitch surfaces, thus straining the cables to an amount dictated by the shape of the contact profiles, and affecting the energy landscape of the assembly. A range of possible field applications, yet to be fully explored, is here presented.

Introduction

The term *tensegrity* was coined by Buckminster Füller [1] to indicate structures whose integrity is granted by tensioned cables, prestressed by matching a few floating compression struts. A novel type of beam-like structures was proposed in [2]. They have been named *flexural tensegrity* to indicate that they are formed by a chain of segments whose integrity under flexure is provided by prestressed tendons anchored at the beam ends and pressing the segments together. The key point is that the contact surfaces of adjacent segments are shaped according to properly designed pitch profiles, such that the relative rotation of the segments opens up the joints. There is a substantial difference with prestressed reinforced concrete segmented beams, for which the contact joints are approximately flat surfaces, designed to remain tight (no opening) at least in the serviceability limit state. Here, on the contrary, the opening of the joints produces the straining of the tendons, free to move in properly shaped cavities inside the segments (unbonded cables), to an amount dictated by the shape of the contact surfaces. This affects the elastic energy of the system, thus characterizing the constitutive bending properties, as a function of the shape of contact profiles.

The flexural-tensegrity concept can be explored in many forms. By changing the shape of the contact profiles, while the cables axially move inside tubular sheaths, either linear, or sub-linear or super-linear constitutive responses can be obtained [2]. As a function of the tendon stiffness, nonlinear Duffing-like vibrations are attained and can be controlled by varying the tendon force [3]. By enhancing the mobility of the tendon in large cavities, the bending energy can be made non-convex in type [4], possibly achieving complex snap-through sequential motions [5]. The limit, when the number of segments goes to infinity and their length to zero, corresponds to a particular type of Euler's elastica with nonlocal response [6], whose bent shape can be computed either analytically (with elliptic integrals), or numerically.

Here, we want to present a possible range of field applications, yet to be fully explored. The segmental beam can be used in archery, to manufacture a new type of bow [7], and in soft robotics, obtaining limbs controlled by internal/external cables. Multi-stable flexural tensegrities can be employed for propulsion in fluids in the form of flagellating tails [8], while larger scale applications in kinetic architecture, yet to be fully appreciated, can be found in the manufacturing of movable skeletons supporting envelopes for water collection and shielding.

The concept of flexural tensegrity

Flexural-tensegrity structures [2] consist of segments, supposed to be stiff, that are pierced and house tubular sheaths, or properly shaped larger cavities, through which an unbonded elastic cable can freely slide while keeping the segments together. Specifically, the contact surfaces are tailor-shaped, so that the segments can rotate one another along the design pitch profiles in pure rolling motion (no sliding). To illustrate, Figs. 1(a) and 1(b) report a theoretical scheme for a simply-supported flexural-tensegrity beam under transversal loads in the reference straight configuration and in the deformed state under applied loads, respectively. The cable (red solid line in Fig. 1) is tensioned and anchored only at the beam ends, where additional springs can be added in series to increase the cable compliance. A detail view of the segments in contact is shown in Fig. 1(c): the segments are pressed together and a certain stiffness against relative rotation is offered, similarly to a spring hinge, because their rotation causes the elongation of the cable. The elongation between segments i and $i+1$ in Fig. 1(c) is indicated as Λ_i , while the total elongation of the cable reads $\Lambda = \sum_{i=1}^{n-1} \Lambda_i$, being n the total number of segments in the chain. Note that, since the cable can freely slide within the segments, the increase of length due to the relative rotation of just one joint causes an increase of tensile force in the cable that is transmitted to *all* the other joints of the segmental beam. This renders the flexural response *nonlocal* in type.

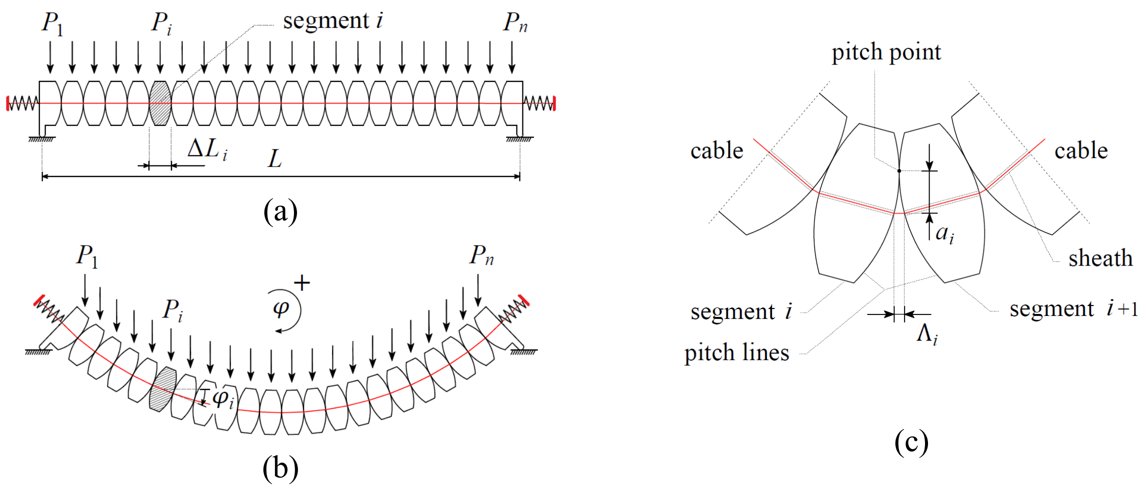


Fig. 1. Theoretical scheme for a simply-supported flexural-tensegrity beam under transversal loads: (a) reference straight configuration; (b) deformation under applied loads; (c) detail of the contact joint, with indication of the lever arm a_i and the local cable elongation Λ_i.

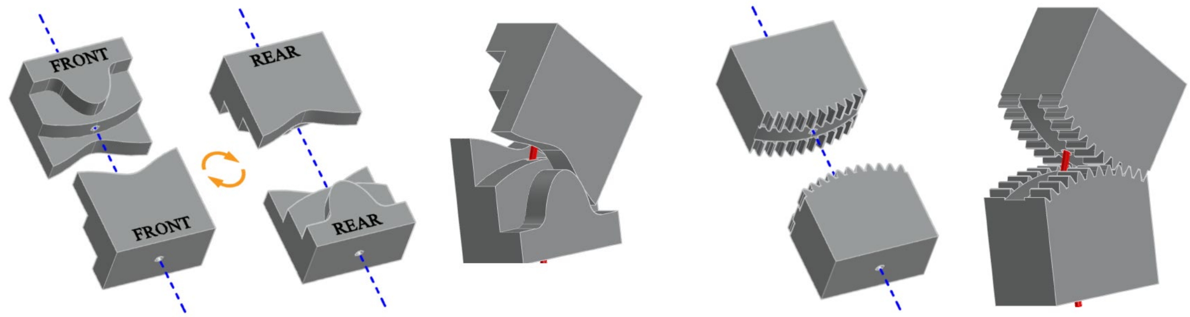
Another noteworthy parameter, reported in Fig. 1(c) is the lever arm a_i of the cable tensile force N with respect to the pitch point. Let N_0 denote the initial prestress in the cable and K its effective axial stiffness, accounting for the springs in series. Hence, the internal bending moment at joint i can be written as $M_i = N a_i = (N_0 + K\Lambda) a_i$. Note that both Λ_i and a_i depend upon the shape of the pitch profiles when the relative rotation of the segments is increased: the response of the joint can be tuned by simply changing the shape of contact profiles, kept fixed the cable properties.

The requirement that the segments are in pure rolling contact is fulfilled by means of the interlocking provided by conjugate profiles associated with the pitch profiles, for which various possibilities are reported in Fig. 2. Each segment is composed of three layers: the central one reproduces the pitch profile, while the two external ones are shaped according to the conjugate profiles. Each one of the external layers can be shaped according to smooth conjugate profiles, as per Fig. 2(a), or manufactured with multiple teeth similarly to spur gears, as in Fig. 2(b).

Under the hypothesis of stiff segments (rigid in the limit case), one can overlook their deformation energy, so that the only contribution remains that of the (compliant) tendon. In this

Materials Research Proceedings **26** (2023) 97-102 https://doi.org/10.21741/9781644902431-16

case, with respect to the reference state under null external actions, the variation of the strain energy of the structure reads $\Delta U = N_0\Lambda + \mathrm{K}\Lambda^2/2$, which depends upon the cable initial prestress N_0 and, specifically, the shape of contact profiles through the total cable elongation Λ. The Lagrangian variables for the bending problem can be set equal to the rotations φ_i of the segment i, $i = 1\dots n$ (Fig. 1(b)). A variational approach, via Hamilton principle [9], can be used to find the set of nonlinear equations governing the static and dynamic response of the structural system.



(a) (b)

Fig. 2. Three-dimensional drawing of the joint. Exploded view and assembled joint in the rotated state for: example (a) with one pair of smooth conjugate profiles (external layers) and pitch profiles (central layer) and (b) with toothed profiles (external layers) and pitch profiles (central layer). Blue dashed lines indicate the cable axis; exposed portion of the cable is drawn in red in the rotated configurations.

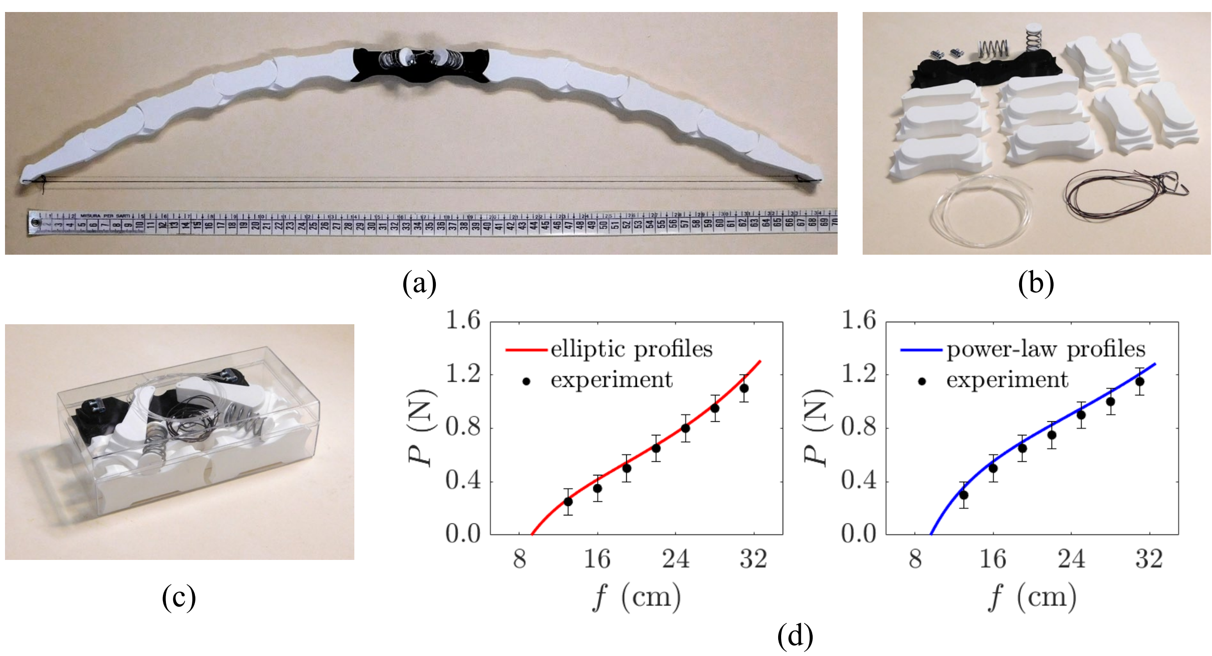


Fig. 3. Flexural-tensegrity bow: (a) photograph of the assembled bow; (b) components of the bow, including segments, bow string, prestressing tendons and additional springs; (c) the bow can be packed in a prismatic box for transportation. (d) Influence of the shape of pitch profiles on drawing force P as a function of drawing height f.

Examples

The beam-like segmental structure, with just one cable passing through the segments inside a tubular sheath, as per the schematic of Fig. 1, has been used to manufacture the elastic limbs of a

Materials Research Proceedings **26** (2023) 97-102 https://doi.org/10.21741/9781644902431-16

new type of bow. The assembled bow is displayed in Fig. 3(a); the various components (segments, bow string, one prestressed tendon for each limb, and additional springs) are shown in Fig. 3(b). The segmental assembly permits to fold the bow by slackening the tendons; it can be totally disassembled and stored in a small box for easy transportation (Fig. 3(c)). The design is focused on the shape of contact profiles to attain different responses for the bow. Fig. 3(d) compares the static drawing of two bows in terms of drawing force P as a function of the drawing height f for two different shapes of the contact profiles; the theoretical description is corroborated by experimental dots in the chart. Note that power-law profiles of the type $y \propto x^4$ provide a response more concave than circular/elliptic profiles, for the same prestress $N_0 \approx 16.5$ N, so that a greater amount of energy is stored in the deformed limbs and can be later transmitted to the fired arrow.

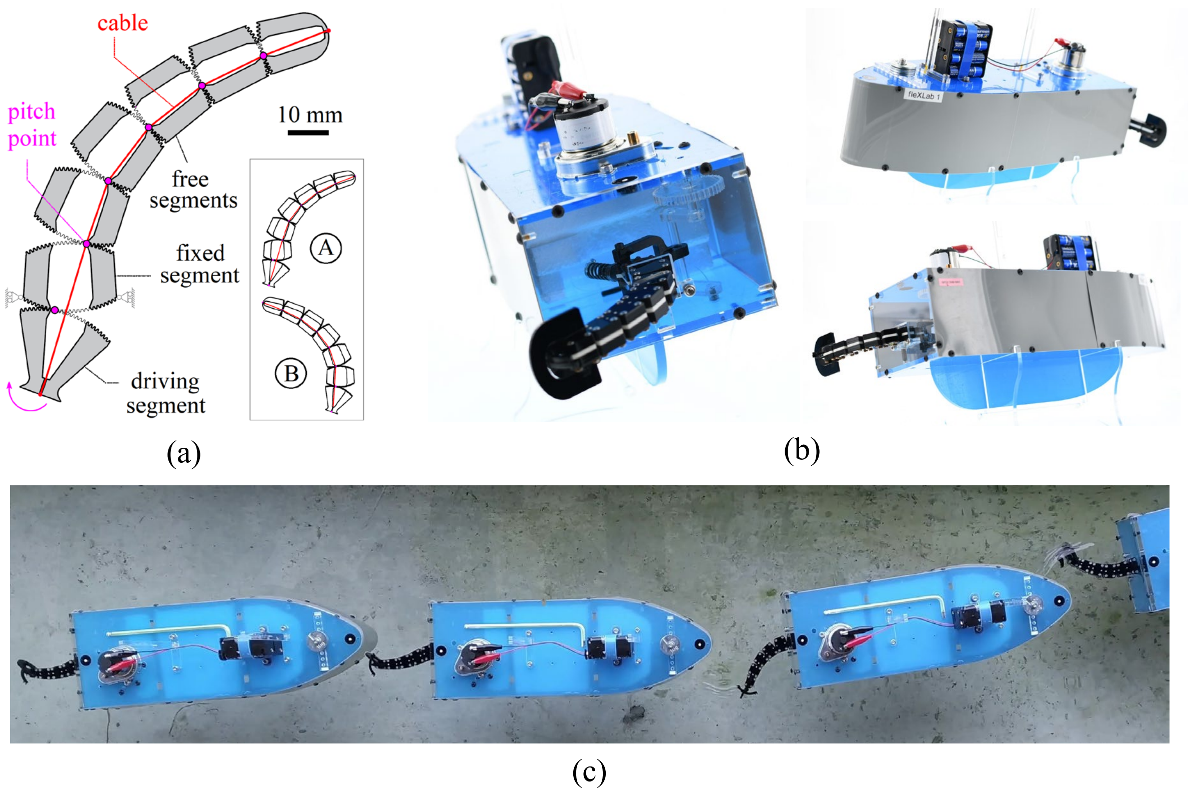
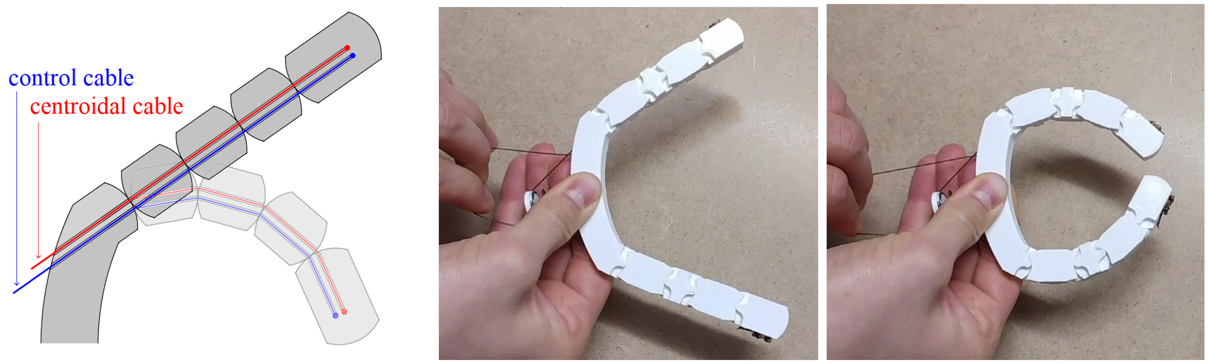


Fig. 4. Application to propulsion in fluids: flexural-tensegrity snapping flagellum. (a) Schematics of the structure with large cavities inside the segments where the cable can move; the snap occurs between configurations A and B. (b) Views of a toy boat propelled by the device, with evidence of the actuation via crank and crankshaft connected to a motor; the tail is equipped with a fin to enhance propulsion. (c) Toy boat sailing in a water tank.

The presence of large cavities inside the segments, instead of a tubular sheath, can provoke the snapping of a segmental cantilever, in response to a localized perturbation, represented by the relative rotation of the end segments close to the clamp restraint [5]. Hence, the cantilever can function as a tail that flagellates under a cyclic relative rotation of these segments, finding specific applications in propulsion devices for locomotion in fluids. Fig. 4(a) reports a schematic section of the cantilever segmental beam. Here, the internal cavities are represented by the non-hatched regions inside the segments. The cable (red solid line) can move inside these cavities: while rotating the driving segment, at a certain point the configuration becomes unstable and the beam reverses its shape, passing from state A to state B. By changing the sign of the driving rotation, the beam can snap back to the original state. Fig. 4(b) shows the application to a toy boat, where

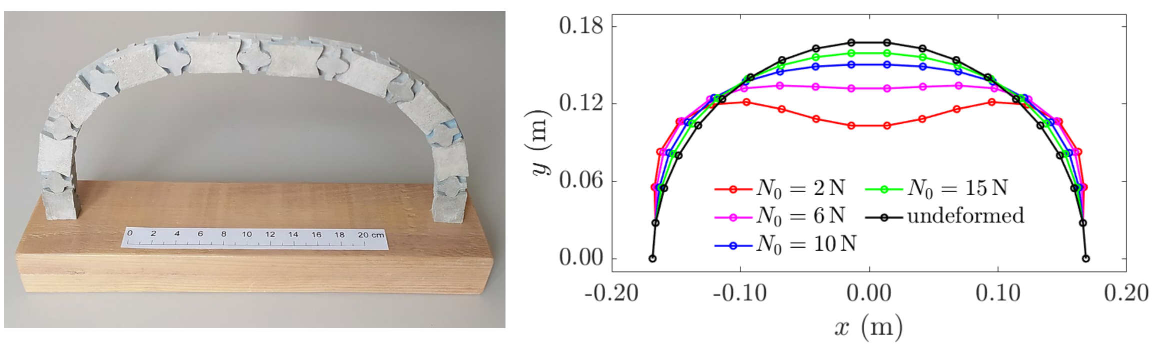
Materials Research Proceedings **26** (2023) 97-102 https://doi.org/10.21741/9781644902431-16

the actuation is obtained through a simple crank and crankshaft mechanism connected to an electric motor. In this example, the terminal segment is equipped with a fin, to enhance the propulsion [8]. Fig. 4(c) reports the final field experiment where the toy boat is sailing in a water tank.

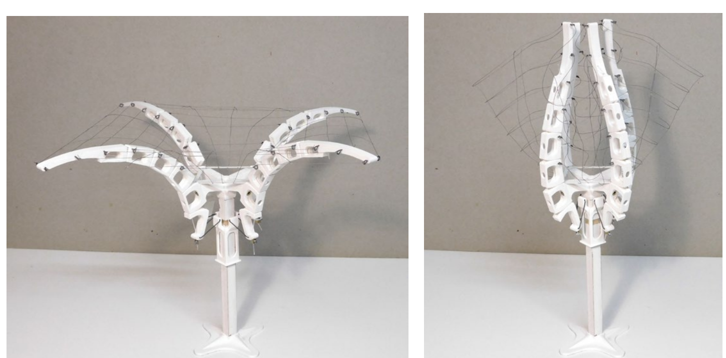


(a) (b)

Fig. 5. Applications to robotics. (a) Schematics of a cable-actuated limb: the centroidal tendon gives integrity to the assembly, while the eccentric one controls grasping when pulled. (b) Corresponding robotic picker manufactured according to this principle.



(a) (b)



(c)

Fig 6. Applications in architecture. Flexural-tensegrity arch: (a) photograph of a small-scale concrete arch and (b) corresponding mobility as the tendon prestress is varied. Flexural-tensegrity umbrella: (c) photograph of a small-scale polylactide structure in the open/closed configuration.

Using two cables a robotic segmental limb can be manufactured. As represented in Fig. 5(a), one tendon (red color) passes through the segments in a centroidal tubular sheath to provide integrity to the assembly, while another tubular sheath, eccentric with respect to the segment centroid, hosts the control cable (blue color): by pulling this second tendon, the limb folds.

Combining two of such limbs, a cable-actuated picker of the type shown in Fig. 5(b) can be obtained, which is capable of grasping and collecting also very soft objects.

Applications in kinetic architecture can be found in the manufacturing of movable skeletons supporting envelopes for water collection and shielding. In Fig. 6(a) a concrete flexural-tensegrity arch is proposed, with the mobility shown in Fig. 6(b) as the tension force in the cable is varied. With a functioning similar to that of the flagellating tail of Fig. 4(a), an umbrella can be designed that opens and closes like a flower, as shown in Fig. 6(c). Note that the structural bending stiffness of the segmental assembly can be tuned by pulling/releasing the prestressing tendons [3][6]; therefore, the force in the cable can be controlled to reduce structural vibrations. This can be very useful to manufacture seismic-resistant bracing systems, amenable to actively respond to earthquakes, especially where the seismic hazard is particularly high.

Conclusive remarks

A range of possible field applications has been presented within the class of flexural-tensegrity segmental structures, to show the versatility and the potentials of this recently proposed structural concept, yet to be fully explored and appreciated. Although the examples were limited to 2D planar mobility and small-scale structures, these represent a noteworthy rich scenario. The extension to full 3D mobility is the subject of current research.

References

[1] S.H. Juan, and J.M. Mirats Tur, Tensegrity frameworks: static analysis review, Mechanism and Machine Theory 43.7 (2008) 859-881. https://doi.org/10.1016/j.mechmachtheory.2007.06.010

[2] C. Boni, M. Silvestri, and G. Royer-Carfagni, Flexural tensegrity of segmental beams, Proceedings of the Royal Society A, 476.2237 (2020) 20200062. https://doi.org/10.1098/rspa.2020.0062

[3] C. Boni, and G. Royer-Carfagni, Nonlinear effects in the vibrations of flexural tensegrity beams, International Journal of Non-Linear Mechanics, 128 (2021) 103616. https://doi.org/10.1016/j.ijnonlinmec.2020.103616

[4] C. Boni, and G. Royer-Carfagni, Equilibrium of bi-stable flexural-tensegrity segmental beams, Journal of the Mechanics and Physics of Solids, 152 (2021) 104411. https://doi.org/10.1016/j.jmps.2021.104411

[5] C. Boni, and G. Royer-Carfagni, Energy harnessing in the snap-through motion of a flexural-tensegrity flagellum, Mechanism and Machine Theory, 173 (2022) 104845. https://doi.org/10.1016/j.mechmachtheory.2022.104845

[6] C. Boni, and G. Royer-Carfagni, A nonlocal elastica inspired by flexural tensegrity, International Journal of Engineering Science, 158 (2021) 103421. https://doi.org/10.1016/j.ijengsci.2020.103421

[7] C. Boni, and G. Royer-Carfagni, A new flexural-tensegrity bow, Mechanism and Machine Theory, 164 (2021) 104398. https://doi.org/10.1016/j.mechmachtheory.2021.104398

[8] C. Boni, P.M. Reis, and G. Royer-Carfagni, Flexural-tensegrity snapping tails for bio-inspired propulsion in fluids, submitted for publication (2022). https://doi.org/10.1016/j.eml.2022.101853

[9] R.W. Clough, and J. Penzien. Dynamics of structures, third ed., Computers and Structures, Berkeley, 2003.