

A moving cohesive interface model for brittle fracture propagation

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Abstract. In this work, an advanced numerical model, based on the cohesive zone approach and the moving mesh technique, is proposed to simulate fracture propagation in quasi-brittle materials. In particular, the proposed numerical procedure consists of two stages. In the former one, according to the inter-element crack approach, once a suitable stress criterion for fracture onset is satisfied, a mesh boundary, representing the crack segment, is selected and aligned along the crack propagation direction by using the well-known moving mesh technique. In the latter one a zero-thickness interface cohesive element, equipped with a traction-separation law, is inserted on-the-fly along the previously selected mesh boundary, in order to describe the nonlinear fracture process. Comparisons with available experimental and numerical results have highlighted the effectiveness and the reliability of the proposed model in the prediction of the brittle fracture phenomenon.

Introduction

The accurate prediction of fracture propagation in quasi-brittle materials is one of the most challenging issues in the framework of structural mechanics because of the complex interplay of different irreversible processes such as defects nucleation, micro-cracks growth. In the literature, different numerical models, based on smeared and discrete fracture approaches, have been proposed to analyze fracture phenomena and their influence on the global mechanical response of the structures [1]. In particular, the discrete fracture models simulate the crack as an actual displacement discontinuity within or between the finite elements of a standard computational mesh, adopting a suitable cohesive traction-separation law to describe the crack propagation within the so-called fracture process zone (FPZ) [2,3]. Such a modeling approach results to be very efficient for predicting the fracture process in quasi-brittle materials with respect to the smeared crack approaches that often lead to ill-posed boundary value problems (BVPs) [4]. The discrete crack models are often employed to simulate the interfacial crack in adhesive-bonded ductile sheets by using different phenomenological-type traction-separation laws, to describe the mechanical behavior of the cohesive forces acting along a predefined crack path [5,6]. Recently, to predict unknown crack paths, a diffuse interface model has been introduced and employed to investigate the damage phenomena in quasi-brittle materials and masonry structures [7,8]. However, such a model suffers from the well-known artificial compliance effect and requires a suitable calibration of the cohesive properties of the embedded interfaces to ensure the desired numerical accuracy [9,10]. Moreover, the strongly nonlinear fracture behavior, including plate end debonding phenomena, has been properly simulated by cohesive crack models [11,12]. In some recent works, fracture models, based on discrete and smeared crack approaches, have been incorporated within advanced numerical homogenization to predict micro-cracking and contact evolution in composite materials [13–16]. However, fracture mechanics-based models result to be very efficient for predicting damage phenomena with a-priori known crack path but provide some

divergences with respect to the experimental outcomes, in terms of crack patterns, in the case of unknown crack paths. To overcomes these drawbacks, the moving mesh technique has been recently introduced in the structural mechanics field to easily permit the moving of the computational mesh according to specific conditions [17–19].

In this work, an advanced numerical model, based on the cohesive zone approach and the moving mesh technique, is proposed to simulate fracture propagation in quasi-brittle materials. In particular, the cohesive fracture model, based on an inter-element crack approach, has been employed to describe the nonlinear behavior of the crack while the moving mesh technique allows aligning the crack element along the crack propagation direction.

Adopted numerical framework for cracking simulation

The numerical procedure proposed to simulate the crack propagation in quasi-brittle materials consists of two stages. In the former one, according to the inter-element crack approach, once a suitable stress criterion for fracture onset is satisfied, a mesh boundary, representing the crack segment, is selected and aligned along the crack propagation direction by using the well-known moving mesh technique. In the latter one a zero-thickness interface cohesive element, equipped with a traction-separation law, is inserted on-the-fly along the previously selected mesh boundary, in order to describe the nonlinear fracture process.

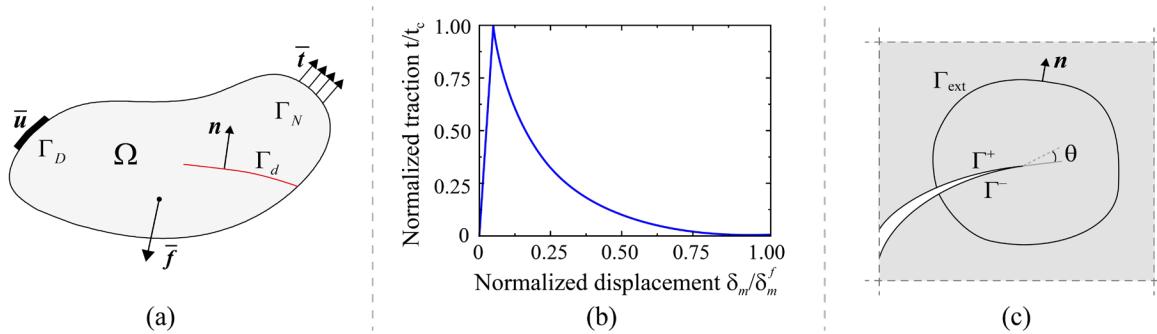


Fig. 1. Equilibrium problem for a 2D fractured body (a), adopted traction-separation law (b), and arbitrary closed path around the crack tip for J-integral approach (c).

In particular, the adopted interface cohesive model relies on a variational formulation referring to the problem of a fractured body containing a discontinuity Γ_d , representing an existing crack (Fig. 1a). The associated BVP is expressed in the following weak form: find $\mathbf{u} \in U$ such that

$$\int_{\Omega \setminus \Gamma_d} \mathbf{C} \boldsymbol{\varepsilon}(\mathbf{u}) \cdot \boldsymbol{\varepsilon}(\mathbf{v}) \, d\Omega + \int_{\Gamma_d} \mathbf{K}([\mathbf{u}]) [\mathbf{u}] \cdot [\mathbf{v}] \, d\Gamma = \int_{\Omega \setminus \Gamma_d} \bar{\mathbf{f}} \cdot \mathbf{v} \, d\Omega + \int_{\Gamma_N} \bar{\mathbf{t}} \cdot \mathbf{v} \, d\Gamma \quad \forall \mathbf{v} \in V, \quad (1)$$

where \mathbf{C} and $\boldsymbol{\varepsilon}$ are the fourth-order elasticity tensor and the linear strain operator, respectively, whereas \mathbf{u} and \mathbf{v} are the unknown displacement field and arbitrary virtual displacement field, respectively, which belong to the set of kinematically admissible displacements U and to set of kinematically admissible variations of the approximated displacement field V , respectively. The second term in Eq. 1, represents the nonlinear constitutive behavior of the cohesive interfaces expressed by a traction-separation law, reported in Fig. 1b, which involves the following effective displacement jump:

$$\delta_m = \sqrt{\langle \delta_n \rangle^2 + \delta_s^2}. \quad (2)$$

A stress-based criterion has been employed to detect the onset of the fracture and to insert a cohesive crack element. In particular, when the normal stress along a mesh boundary exceeds the critical tensile strength of the material, such an element is selected and aligned with the theoretical crack propagation direction. The direction of the crack extension has been found by the maximum

energy release rate (ERR). ERR is computed by using the J-integral-based approach with respect to a closed path containing the crack tip and a local coordinate system aligned with the crack direction:

$$J_k = \int_{\Gamma} \left[W n_k - \sigma_y \frac{\partial u_i}{\partial x_k} n_j \right] d\Gamma, \quad (3)$$

with $\Gamma = \Gamma_{ext} + \Gamma^+ + \Gamma^-$ an arbitrary closed path around the crack tip singularity (see Fig. 1c). Is worth noting that, the Γ^+ and Γ^- represent the faces of the stress-free crack. The ERR can be expressed as a function of J vector components, J_1 and J_2 , as follows: $G = J_1 \cos(\theta) + J_2 \sin(\theta)$. Maximization of G yields to evaluation of the kinking angle:

$$\theta = \arctan \left(\frac{J_2}{J_1} \right). \quad (4)$$

After that, the selected crack element is aligned according to the computed kinking angle (Eq. 4). The crack alignment procedure is accomplished by relocating nodes performed by the Arbitrary Lagrangian-Eulerian (ALE) description. To this end, a referential configuration Ω_χ , whose points are identified by the reference coordinates χ , is introduced. The corresponding mesh displacement field $\mathbf{u}_{mesh} = \mathbf{X} - \chi$ represents a structural change, which, in the context of this work, consists in the selected element rotation of the small kink emanated from the current crack tip.

Numerical results

In order to validate the proposed numerical fracture model, a numerical simulation of the three-point bending test, experimentally tested by Rots in [20], is performed, involving a plain concrete specimen. Both geometry and boundary conditions of the notched concrete beam are depicted in Fig. 2a. The Young's modulus and the Poisson's ratio of the bulk material are equal to 20 GPa and 0.20, respectively. The cohesive parameters, i.e. critical tensile stress, mode-I fracture energy, and initial cohesive stiffness, required by the adopted traction-separation law, are set equal to $t_c = 2.4$ MPa, $G = 113$ N/m, and $K = 1E15$ N/m³, respectively.

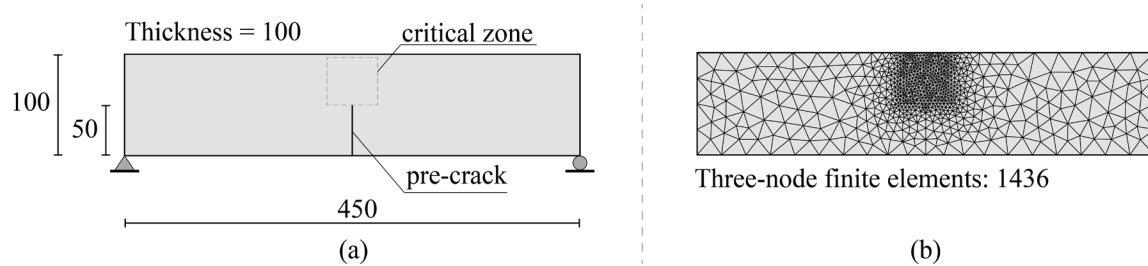


Fig. 2. Geometry and boundary conditions (a) and adopted numerical discretization (b).

The numerical discretization adopted for the analysis has been reported in Fig. 2b. In particular, within the critical zone ahead of the pre-crack, a suitable mesh refinement has been performed, using a uniform (isotropic) Delaunay tessellation and imposing a maximum edge length of 3.00 mm, which results in an average mesh size of about 1.46 mm.

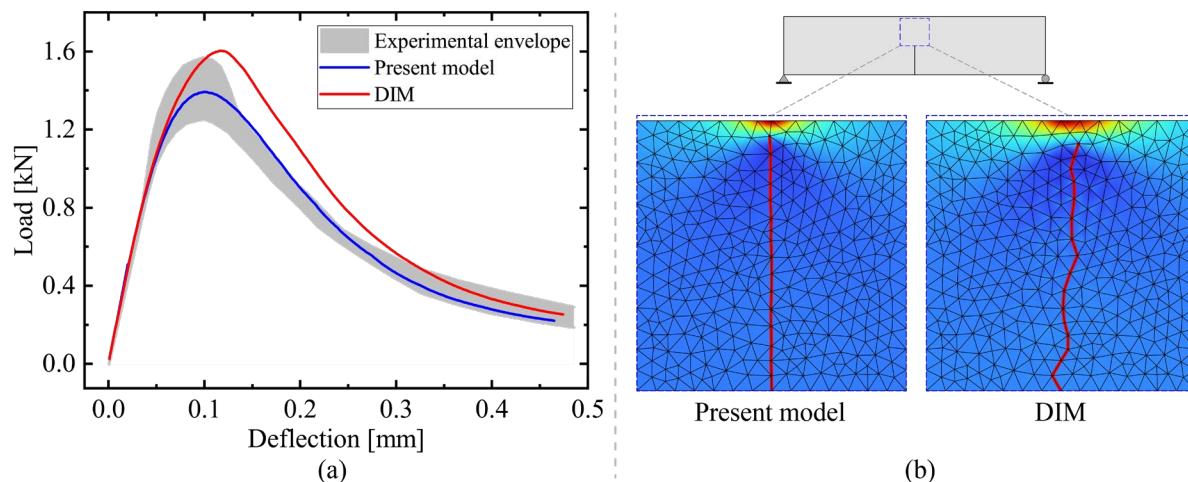


Fig. 3. Numerical loading curves together with the experimental envelope (a) and crack pattern obtained by the proposed model and the DIM model (b).

The numerical results, in terms of load carrying capacity and crack pattern, are reported in Fig. 3. In particular, Fig. 3a shows a comparison of the loading curves predicted by the proposed model, the Diffuse Interface Model (DIM), and the experimental test. The numerical model (DIM), taken for comparison purposes, has been recently introduced by some of the authors in [21,22] and relies on an inter-element fracture approach according to which cohesive interfaces are inserted at the beginning of the simulation along all mesh boundaries inside the critical zone. We can note that the proposed model predicts a loading curve within the experimental envelope providing a numerical prediction, in terms of peak load, better than the DIM model. This result is confirmed by the crack pattern (see Fig. 3b) predicted by the proposed model which results to be coincident with the exact crack path, assuming self-similar propagation.

Conclusions

In this work, a numerical model, based on the cohesive fracture approach and the moving mesh technique, has been proposed to simulate the crack propagation in quasi-brittle materials. In particular, once a suitable stress criterion for fracture onset is satisfied, a boundary of the adopted mesh is selected and aligned with the theoretical crack direction. This alignment procedure relies on the J-integral approach, to compute the kinking angle, and the moving mesh technique, to relocate nodes of the mesh due to the crack update. Mode-I fracture has been simulated by the proposed model involving quasi-brittle materials like concrete. Comparisons with available experimental and numerical results have highlighted the effectiveness and the reliability of the proposed model to predict the cracking behavior in concrete structures. As a future perspective of this work, the proposed model could be incorporated into a multiscale approach to improve the related computational performances, similarly to what has been suggested by [23–25].

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