

# Combining pendulum and gyroscopic effects to step-up wave energy extraction in all degrees of freedom

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**Abstract.** The fight against the global threat of climate change requires, among other actions, to increase the penetration of renewable energy technologies and diversify the energy mix in order to support a resilient energy system that can reach net-zero greenhouse gas emissions. Offshore energy is expected to drive the energy transition, with wave energy having the major role to provide a reliable baseload and reduce the need for storage; however, its techno-economic feasibility requires reduction of costs and increase of energy conversion efficiency. This paper tackles a fundamental innovation of a device's working principle which, jointly exploiting pendulum and gyroscopic effects, steps-up the overall conversion efficiency in real operational conditions. A recent patent proposes a technological solution that conveniently combines pendulum and gyroscopic effects in order to effectively exploit motion also outside the plane, namely in the three-dimensional space and from all degrees of freedom (DoFs). This paper tackles the endeavour of the analytical formulation of the electro-mechanical conversion system dynamics, considering at first the fully-nonlinear equation of motion, obtained through a Lagrangian approach. Consequently, incremental simplifications are applied to accommodate practical application, based on the study on the relative importance of each term in the equation of motion. Furthermore, preliminary results are produced and discussed, comparing the behaviour in response to 3-DoF to 6-DoF exploitation.

## Introduction

The European Commission has set ambitious carbon neutrality targets, reaching net-zero greenhouse gas emission by 2050, as defined in the Green Deal Communication [1]; moreover, the key enabling role of ocean energy (tidal and wave energy) and offshore wind energy has been expressively declared by the European Commission [2], including quantitative targets for the envisioned installed capacity: 1 GW installed capacity by 2030 and 40 GW by 2050 in the European Union waters for ocean energy devices, and 60 GW by 2030 and 300 GW by 2050, for fixed and floating wind turbines. A techno-economic improvement is mandatory for both offshore wind and ocean technologies to become viable on a such a large and systemic scale, which requires further developing of floating wind substructure and the related installed turbine, via techno-economic optimization and numerical modelling [3]; likewise, leveraging the complementarity between wave and wind energy (load profiles, low statistical correlation [4], and cost sharing synergies [5]), it is also crucial to flank the offshore wind development with more efficient wave energy converters (WECs) which, albeit a smaller total capacity, provide a more stable and reliable baseload to reduce production variability and need for storage. Increasing WECs efficiency is a complex and multidisciplinary task, relying on a variety of energy-maximisation techniques, e.g.

via model-predictive control [6], extremum-seeking [7], or time-varying control [8], time-effective estimation [9] and forecasting [10] algorithms, data-driven improved accuracy of numerical models of fluid-structure interactions [11] by means of the identification of grey-parameters in low-complexity models [12], mid-fidelity partially-nonlinear models [13], or direct implementation of fully-nonlinear Navier-Stokes fluid-dynamics equations [14]. Nevertheless, such models should preserve the inherent passive characteristics of the system [15], while remaining able to articulate typical instabilities of floating structures, e.g. parametric resonance [16] or yaw instability [17]. Finally, due to the resulting potential complexity of such nonlinear models, it is often convenient to apply model order reduction techniques to achieve real-time computation, required for practical implementation [18].

All of the previous techniques are applicable to arbitrary WEC concepts; however, more fundamental routes to increase energy efficiency is to operate on the underlying conversion principle [19]. A novel device is indeed proposed in this paper and in a recent patent, based on the idea to notionally merge two already successful devices, both using inertial coupling between the motion of a floating hull and the inner electromechanical system: the ISWEC [9] (Inertial Sea Wave Energy Converter) and the PeWEC [20] (Pendulum Wave Energy Converter). Both technologies perform best when the incoming wave direction is along the hull main axis, and/or the spreading factor of the wave is infinite (i.e. all spectral components are travelling in the same direction) [21], so the WEC motion remains in a plane. The herein discussed technological solution conveniently combines gyroscopic and pendulum effects to expand the operational space of the WEC also outside the plane, namely to extract energy from all degrees of freedom (DoFs). The content of the paper is the analytical formulation of the electromechanical conversion system dynamics, starting from the fully-nonlinear equation of motion, obtained through a Lagrangian approach; consequently, a sensitivity analysis explores the system's most influential mechanical parameters. Such results constitute the first stepping stone towards a holistic techno-economic analysis, based on the results presented in this paper.

**Mechanical device and its mathematical modelling**

This section introduces the underlying mechanical innovation of the analysed system and presents the related mathematical model. The technology is composed of a flywheel rotating around an axis  $\varphi$ , hosted within a gimbal and vertical at static equilibrium. The gimbal is supported by a structure able to rotate around an axis  $\varepsilon$ , normal to  $\varphi$ . The main innovation with respect to the ISWEC system is to displace the middle symmetry plane of the flywheel by a distance  $\lambda$  from the axis  $\varepsilon$ ; in this way, the arm  $\lambda$  introduces the pendulum behaviour while the gyroscopic action is obtained by the flywheel rotation  $\dot{\varphi}$ . Both torques are parallel to the  $\varepsilon$  axis, so a potentially constructive interference may be obtained. A power take-off unit converts the mechanical energy into electrical

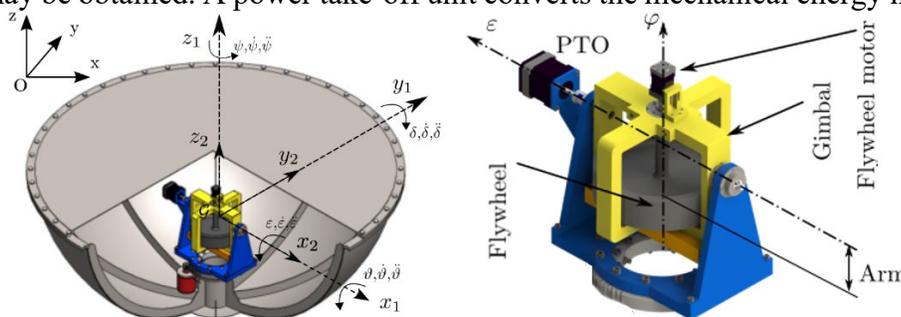


Figure 1. Whole WEC (on the left) and inner electromechanical components (right)

Figure 1 shows a CAD impression of such technology, and a possible installation within a sealed floating hull, loosely moored to the seabed to enable rotations around the horizontal axis. Considering the inertial frame  $O(x, y, z)$ , the pose of both the gimbal and the flywheel are computed by means of a set of linear differential equations, assuming small oscillations to make

the use of linear mapping reasonable. The system motion is described with  $n = 6$  degrees of freedom (DoFs), so that the vector of generalized coordinates  $X(t) \in R^n$  is defined as:

$$X(t) = [x(t), y(t), z(t), \theta(t), \delta(t), \varepsilon(t)]^T \tag{1}$$

which refer to the surge, sway, and heave displacements, and the roll and pitch rotations of the floater; in addition,  $\varepsilon(t)$  is the pendulum rotation with respect to the precession axis. Note that no yaw motion is considered, since the axisymmetric floater is not affected by yaw hydrodynamic torques. Thanks to the linear assumptions, equation of motion is derived through the superposition principles, combing both the elastic effect due to the flywheel mass  $m_f$  and the gyroscopic effect. It is further assumed that the centres of mass of the floater and the gimbal coincide, both placed on the precession axis. The corresponding inertia matrix are defined as  $I_{g,g} = \text{diag}(I_{g,x}, I_{g,y}, I_{g,z})$  and  $I_{f,g} = \text{diag}(I_{f,x}, I_{f,y}, I_{f,z})$ , where  $\{I_g, I_f\} \subset R^{3 \times 3}$  are the inertia matrices of the gimbal and flywheel, respectively. Then, the system of differential equations describing the mechanical coupling between pendulum and hull become  $M_w \ddot{X}(t) + C_w \dot{X}(t) + K_w X(t) = \mathcal{F}_{ext}(t)$ , with:

$$C_w = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -I_{f_z} \dot{\phi} \\ 0 & 0 & 0 & I_{f_z} \dot{\phi} & 0 & 0 \end{bmatrix}, K_w = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_f g l_f & 0 & m_f g l_f \\ 0 & 0 & 0 & 0 & m_f g l_f & 0 \\ 0 & 0 & 0 & m_f g l_f & 0 & m_f g l_f \end{bmatrix} \tag{2}$$

$$M_w = \begin{bmatrix} m_f + m_g & 0 & 0 & 0 & -m_f l_f & 0 \\ 0 & m_f + m_g & 0 & m_f l_f & 0 & m_f l_f \\ 0 & 0 & m_f + m_g & 0 & 0 & 0 \\ 0 & m_f l_f & 0 & I_{f_x} + I_{g_x} & 0 & I_{f_x} + I_{g_x} \\ -m_f l_f & 0 & 0 & 0 & I_{f_y} + I_{g_y} & 0 \\ 0 & m_f l_f & 0 & I_{f_x} + I_{g_x} & 0 & I_{f_x} + I_{g_x} \end{bmatrix}$$

where  $m_g$  is the gimbal mass. Considering that the gyropendulum motion is controlled by the torque  $\tau_\varepsilon(t)$ , the forcing vector  $\mathcal{F}_{ext}(t) \in R^n$  is defined as  $\mathcal{F}_{ext}(t) = [\mathbf{0}_{1 \times n}, \tau_\varepsilon(t)]^T$ . The changing in time of position and orientation of the body-fixed reference frame and the potential energy variation result in inertial, elastic and gyroscopic forces that induce the gyropendulum oscillation. Therefore, energy from the wave forces induces the movement of the floater, hence the oscillation of the PTO axis via the gyroscopic effect and/or the restoring force due do the eccentric mass. Fluid-structure interactions are modelled using linear potential flow theory, based on the assumptions of inviscid fluid and incompressible and irrotational flow. The floater-only response with respect to its generalized coordinates in Fourier transform form  $X_f(\omega) = [\hat{x}(\omega), \hat{y}(\omega), \hat{z}(\omega), \hat{\theta}(\omega), \hat{\delta}(\omega)]^T$  is defined by the following frequency-domain equation:

$$-\omega^2 M_f X_f(\omega) = F_r(\omega) + F_{exc}(\omega) + F_h(\omega) + F_p(\omega) \tag{3}$$

where  $M_f \in R^{5 \times 5}$  is the floater inertia matrix,  $F_r(\omega) \in R^{5 \times 1}$  is the radiation force,  $F_h = K_h X_f(\omega) \in R^{5 \times 1}$  is the hydrostatic restoring force, proportional to the hydrostatic stiffness matrix  $K_h \in R^{5 \times 5}$ ,  $F_{exc}(\omega) \in R^{5 \times 1}$  is the wave exciting force and  $F_c(\omega) \in R^{5 \times 1}$  is the reaction force generated by the dynamic coupling of the mechanical system. Finally, it is now possible to combine the two sub-systems (inner mechanics and wave-structure system) into the WEC entire device dynamics:

$$-\omega^2 (M_w + \tilde{M}_f + \tilde{A}(\omega)) X(\omega) + j\omega \tilde{B}(\omega) X(\omega) + (K_w + \tilde{K}_h) X(\omega) = F_e(\omega) \tag{4}$$

$$\widetilde{M}_f = \begin{bmatrix} M_f & \mathbf{0}_{5 \times 1} \\ \mathbf{0}_{1 \times 5} & \mathbf{0}_{1 \times 1} \end{bmatrix}, \quad \widetilde{K}_h = \begin{bmatrix} K_h & \mathbf{0}_{5 \times 1} \\ \mathbf{0}_{1 \times 5} & \mathbf{0}_{1 \times 1} \end{bmatrix} \quad (5)$$

where,  $X(\omega)$  is the vector containing the Fourier transforms of the generalized displacements  $X(t)$ ,  $\widetilde{M}_f$  and  $\widetilde{K}_h$  are the mass and stiffness matrices defined as block matrices, and  $\widetilde{A}(\omega)$  and  $\widetilde{B}(\omega)$  are the added mass and the radiation damping matrices with respect to the generalized coordinates, and  $F_e(\omega)$  is the excitation forces vector. The excitation forces vector  $F_e(\omega)$  can be defined as  $F_e(\omega) = [F_{exc}(\omega), \tau_\varepsilon(\omega)]^T$ , where  $\tau_\varepsilon(\omega)$  is the actual control action on the gyropendulum axis  $\varepsilon$ . The control is actuated by the PTO system that exerts a damping torque  $\tau_\varepsilon(t)$  proportional to the pendulum swinging velocity  $\dot{\varepsilon}(t)$ . In this work, the simulations are performed considering a frequency domain model, whose steady-state response under stochastic loads can be generally determined on the basis of the transfer function of the system. The related complex form of transfer function is:

$$H(\omega) = [-\omega^2 [M_w + \widetilde{M}_f + \widetilde{A}(\omega)] + j\omega \widetilde{B}(\omega) + K_w + \widetilde{K}_h]^{-1} \quad (6)$$

The frequency domain representation of eq. (6) is used as a basis for a spectral-domain description of the entire energy conversion process, which considers a probabilistic representation of the waves and of the model response as input and output respectively. The free surface elevation is defined as a discretised signal with  $N$  fixed time steps ( $\Delta t$ ). According to [22], a sea wave can be modelled using a Random Amplitude Scheme: considering the finite realization length  $T = N\Delta t$ , the discrete sequence of simulated free surface elevation  $\eta(t_i)$  with  $t_i = i\Delta t$  can be represented as:  $\eta(t_i) = \sum_{k=1}^N a_k \cos(\omega_k t_i + \phi_k)$ , with randomly chosen phases ( $\phi_k$ ) from a uniform distribution  $\mathcal{U}(0, 2\pi)$ , while the amplitudes ( $a_k$ ) follow a Rayleigh distribution  $\mathcal{R}(\sigma)$  with variance  $\sigma = 2S_\eta(\omega_k)\Delta\omega$ . According to the Gaussian closure assumption, the WEC can be simulated as a linear Gaussian process, as the Gaussian process that drives the system: being the system linear, its steady-state response to a sum of orthogonal frequency components is the sum of the responses to each of the frequency components, with phase shift  $\Delta\phi_k$ . The calculation of the mean absorbed power ( $P_{abs,avg}$ ), which is the main variable of interest, is non-deterministic and is a function of the sea-state conditions and of the control action  $c_{PTO}$  applied on the WEC. Under the hypothesis of a large number of realizations, the expected value of  $P_{abs,avg}$ , denoted by  $E(P_{abs,avg})$ , is defined as  $E(P_{abs,avg}) = -c_{PTO} \sum_{k=1}^N S_{\dot{\varepsilon}\dot{\varepsilon}}(\omega_k)\Delta\omega$  [22], where  $S_{\dot{\varepsilon}\dot{\varepsilon}}$  refers to the power spectral density function of the DoF related to power extraction. It is worth noticing that, having considered the power output a Gaussian process,  $E(P_{abs,avg})$  is also the mean value  $\mu_{P_{abs,avg}}$  of the distribution.

## Results and conclusions

Based upon the linear model presented in the previous section, the dynamic of the system can be studied directly at the PTO DoF applying the impedance matching theory introduced in [20]. Let us recall the principle of maximum power transfer that is obtained applying a control action equal to the inverse of the impedance of the equivalent circuit related to the model stated in Eq (2).

Based upon the device characteristic, a sensitivity analysis of the precession axis equivalent force is performed, varying both the flywheel speed and the wave direction. The analysis is computed considering an irregular input wave, with a unitary amplitude and varying the energetic period of the input wave, selected between 3s and 10s. Figure 2 presents an appraisal of the equivalent force at the PTO axis  $\hat{F}_{w,swingo}$  varying the flywheel angular velocity. The extremum scenario are considered, where the wave impacts the floater at  $0^\circ$  and  $90^\circ$ , with a third intermediate operating condition of  $45^\circ$ . We can notice that the gyropendulum mechanism, mounted in the SWINGO device, allows the increment of the total force acting on the power take off DoF with respect to the gyroscope mechanism. Such an improvement ranges from the 70% to the 10% considering the overall set of operating condition. Moreover, we notice that the greatest

improvement happens when the flywheel velocity  $\dot{\varphi} = 0$ , then the percentage gradually decreases, increasing the flywheel velocity. We state that even if the gyropendulum system can improve the equivalent force acting on the PTO DoF, for a wide set of operating condition, the highest advantages set up in term of input force is achieved for zero velocity of the flywheel.

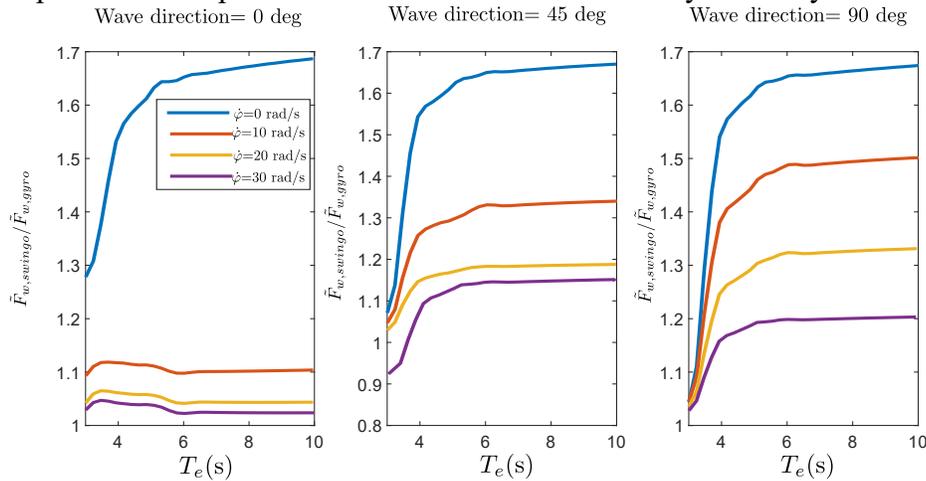


Figure 2 Equivalent force at the PTO: comparison between the gyropendulum and simple gyroscope.

In conclusion, in this work we have introduced a new concept of wave energy converter based on both pendulum and gyroscopic device. The resulting mechanism is the gyropendulum system, that because of its characteristic can be activated with respect to the  $\varepsilon$ -axis independently from the incoming wave direction. Such a device has the peculiarity of an eccentric flywheel mounted at a distance  $l_f$  from the precession axis. Applying the impedance matching principle, the dynamic can be mapped to the PTO axis and then the equivalent system depends on an equivalent force  $\widetilde{F}_w$  and impedance  $I(j\omega)$ . Then we have demonstrated that the gyropendulum system allows to project more input force on the PTO axis than a simple gyroscopic configuration. Such an improvement of input power ranges from 70 to 20%, accordingly to the flywheel speed.

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