

Surface error correction of a mesh deployable reflector

MADDIO Pietro Davide^{1, a *}, SALVINI Pietro^{2, b}, SINATRA Rosario^{1, c} and
CAMMARATA Alessandro^{1, d}

¹University of Catania, DICAR, Via S. Sofia 64, 95123 Catania, Italy

²University of Rome "Tor Vergata", Department of Enterprise Engineering, Via del Politecnico 1,
00133 Rome, Italy

^apietro.maddio@unict.it, ^bsalvini@uniroma2.it,
^crosario.sinatra@unict.it, ^dalessandro.cammarata@unict.it

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Abstract. Large Deployable Reflector (LDR) systems are commonly used as mesh reflectors for large aperture space antennas in aerospace applications since they provide affordability while guaranteeing at the same time a high gain and a high directivity. To improve the surface accuracy several methods have been studied, most of which measure the distance between the cable-net system that forms the reflector surface and the desired paraboloid. In this paper we want to improve the reflector's ability to convey a greater concentration of reflected rays in the direction of the feed. To deal with this issue, a numerical optimization algorithm has been proposed.

Introduction

Applications that make use of satellite communications are now widely spread, both in civil and military fields. Bear in mind, for example, about the analysis of images for the study of terrestrial morphology, or in the meteorological field for weather forecasts, or even about studies on climate change. In the civil field, then, almost all mobile devices, thus far, are equipped with a GPS module for tracking the position. Large Deployable Reflectors (LDR) are the main types of reflectors used for most of the above mentioned fields. LDR systems are commonly used as mesh reflectors for large aperture space antennas in aerospace applications since they provide affordability while guaranteeing at the same time a high gain and a high directivity.

The key features that characterize the geometry of LDR systems are closely connected with volume constraints of launch vehicles, mainly because of budget problems [1]. Deployable mesh reflectors are composed of rigid bodies, deformable components, mechanical joints, and control actuators [2-4] which allows for achieving a complete transition between the initial stowed configuration to the final deployed configuration.

The fundamental problems for the correct functioning of an LDR system are, therefore, the proper deployment of the folding mechanism and the form-finding of the cable-net which serves as support for the metal mesh. To maintain excellent reflective qualities and meet the prescribed bandwidth requirements, the reflector surface must be as close as possible to the shape of a paraboloid. Most of the methods used in the literature define the best surface of the reflector as the one passing through the nodes of the cable system of the front net or through the centroid of each triangular facet [5-10]. In this case, the RMS error depends on the distance between the nodes of the front net with respect to the desired working surface. Agrawal et al. [11] examined the RMS error between the best-fit paraboloid with flat facets and a sphere. Deng et al. [12] defined the RMS error as the distance between the desired working surface and the nodes of the front net. A similar interpretation is given by Morterolle et al. [13], where the *z*-direction distance between the desired working surface and a facet of each triangular facet is used to calculate the surface accuracy. In this work, however, we want to focus on the amount of energy that hits the feed, thus

investigating the best topology of the net that guarantees a greater concentration of the incident rays directed towards the focus of the paraboloid. This paper is organized as follows. At first, the receiver deviation error is introduced, both in the two-dimensional and in the three-dimensional model. Hence the optimization model is presented, along with the results. Finally, the conclusions are reported.

Receiver deviation error

Figure 1 shows a polygonal chain in blue, whose vertices pass through the parabola drawn in red and having the following equation:

$$y = \frac{x^2}{4F} \tag{1}$$

where F is the focus of the parabola.

According to what is described in the literature, the polygonal chain would represent the ideal condition for the reflector surface as its vertices are located exactly on the parabola. In fact, if we consider the RMS error as

$$e_{RMS} = \sqrt{\frac{1}{n}(d_1^2 + d_2^2 + \dots + d_n^2)} \tag{2}$$

where d_n represents the distance between each vertex of the polygonal and the parabola, we obtain a value equal to zero. Despite this result, the most representative value that guarantees the reflector a good frequency band is given by the contribution of each reflected ray that intercepts the feed located at the point F . As shown in Fig. 2 (left), taking into consideration a prime-focus antenna, where the focus is positioned in the center of the reflector, the incident rays parallel to the y -axis are reflected on the surface of the reflector and then directed towards the feed.

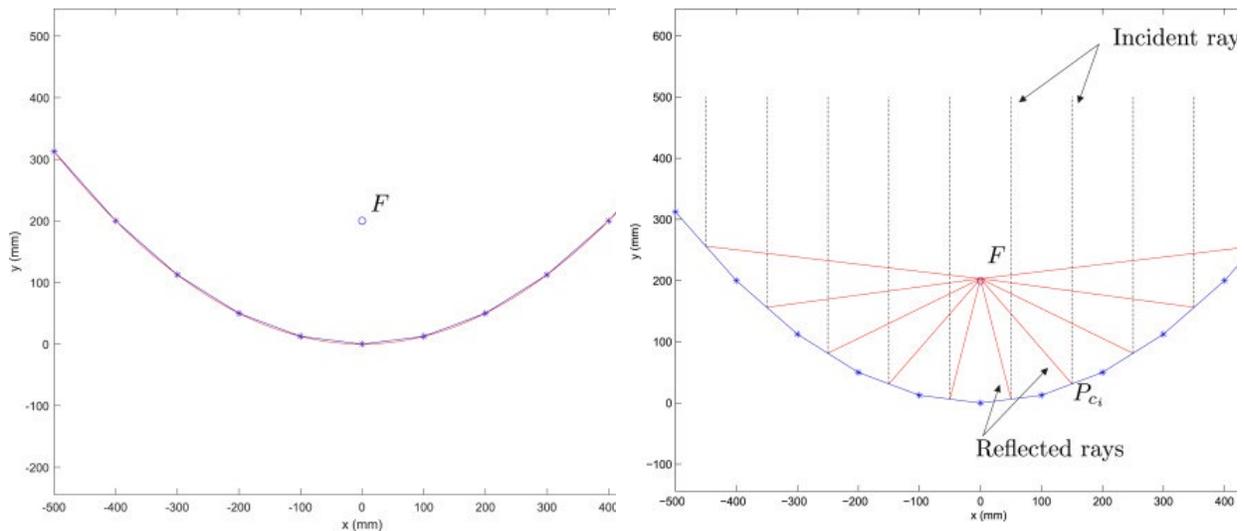


Fig. 1: Polygonal chain and parabola (left); Incident and reflected rays (right).

The point P_{ci} represents the centre of the i -th line segment of the polygonal. Figure 2 shows an enlargement around the feed, which highlights how the incident rays coming from the points are not directed exactly on the focus F , but are distant from it by a quantity d_i . To determine this quantity, we need to calculate the minimum distance between the focus and each reflected ray coming from all line segments of the polygonal chain.

By keeping the aperture width of the parabola fixed, it is easy to understand how by increasing the number of line segments of the polygonal, the RMS error decreases exponentially, as the approximation of the parabola improves. Theoretically, the ideal configuration occurs when the number of line segments tends to infinity, but this information collides with the manufacturing limits, as it is not possible to make a cable-net with an infinitesimally small cable length.

What described for the two-dimensional model can be even extended in the three-dimensional model. Unlike the 2D model, the polygonal chain is replaced by the surface reflector consisting of the cable-net system, whose nodes are located on the paraboloid surface. Here, for the RMS error evaluation, the vertices of the triangular facets and their centroid are considered.

Using the equation to determine the minimum distance of the reflected rays from the focus of the paraboloid and considering a prime-focus antenna, we obtain what is depicted in Fig. 3. This figure shows in blue the points of minimum distance calculated by the reflected rays coming from the vertices of each triangle, while in red those coming from the centroids. As for the 2D model, also in this case it is necessary to associate a weight according to the different contributions of the rays. At the centroid a weight of 1/2 is assigned, while at the three vertices of the triangle a weight of 1/6 is assigned, since their contribution needs to be split equally between them. In the 3D model, by adding the weights, the Eq. (2) changes as follows:

$$e_{RMS} = \sqrt{\frac{(w_1 d_1)^2 + (w_2 d_2)^2 + \dots + (w_n d_n)^2}{3n}} \tag{3}$$

Correction algorithm

To improve the performance of the reflector, the best geometric figure formed by contiguous and flat triangular facets needs to be found.

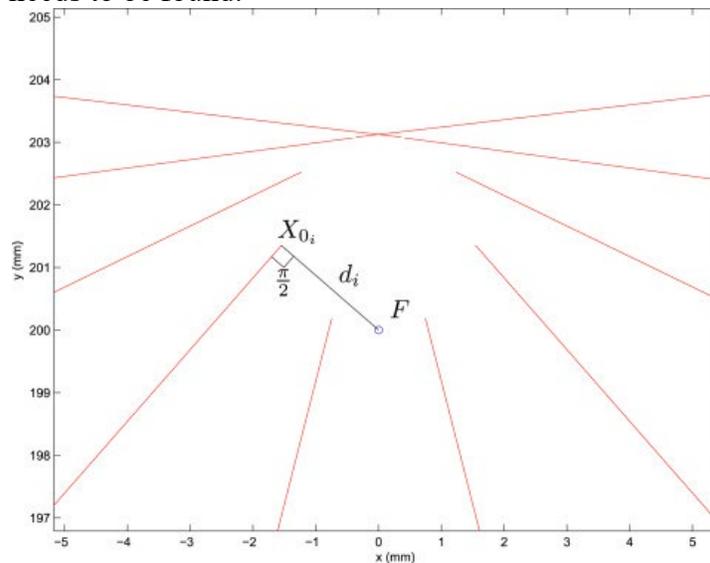


Fig. 2: Reflected rays around the receiver.

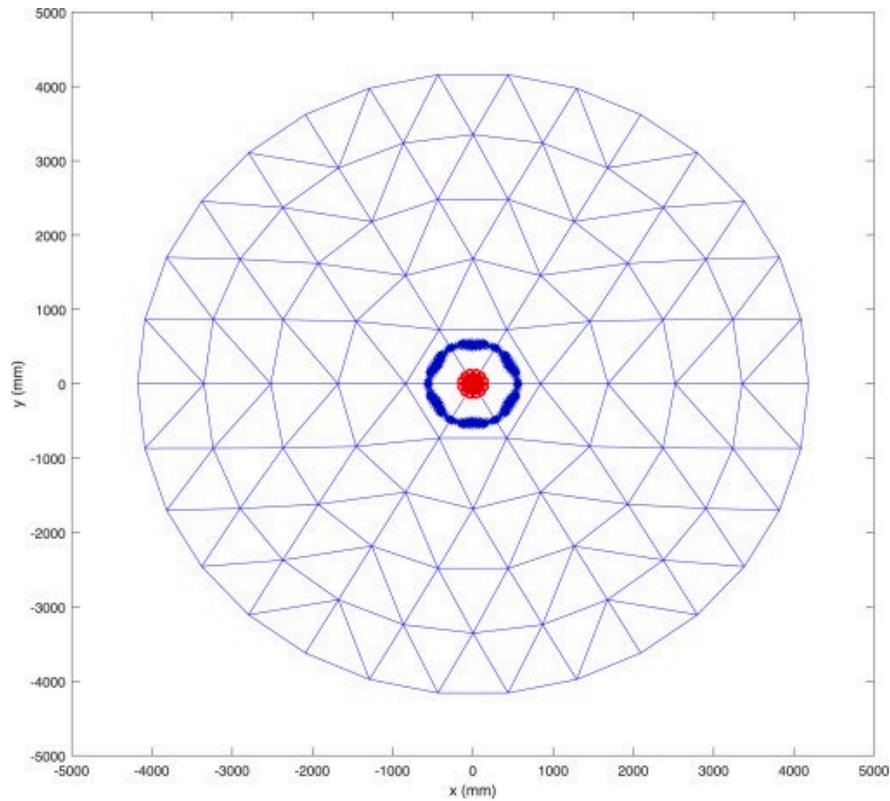


Fig. 3: Top view of the triangular facets of the cable-net and the points of minimum distance of each reflected ray for a prime-focus reflector.

The geometry of the above said paraboloid can be achieved thanks to the formulation of an optimization algorithm which, by varying the position of the nodes of the cable-net, minimizes the distance of each reflected ray with respect to the focus of the paraboloid. The optimization problem can be defined as follows:

$$\begin{aligned} &\text{Find } \mathbf{x}, \mathbf{y}, \mathbf{z} \\ &\text{min } e_{RMS} \end{aligned} \tag{4}$$

where $\mathbf{x}, \mathbf{y}, \mathbf{z}$ represent the coordinate of the internal nodes of the cable-net, since the nodes located on the perimeter remain fixed in their initial position, while e_{RMS} is the RMS error defined by Eq. (3). By applying this algorithm to an offset reflector with a parent paraboloid diameter of 6 meters, it is possible to appreciate how the cable-net topology changes according to the minimum distance between each reflected ray and the receiver.

Figure 4, on the left, shows the initial topology of the cable-net of the offset reflector. On the left side the location of the points of minimum distance of the reflected rays with respect to the focus is shown. In particular, the blue points are closer to the focus as they represent those coming from the centroids of each triangle of the cable-net, while those in red from the three vertices. By applying the optimization problem described above, the result shown in the right side of Fig. 4 is obtained. The greater concentration of points around the focus demonstrates the effectiveness of the algorithm, and this confirms how the optimized net topology concentrates the rays reflected by the reflector in a smaller area of the illuminator.

In Fig. 5 it is possible to better evaluate a comparison between the two obtained results.

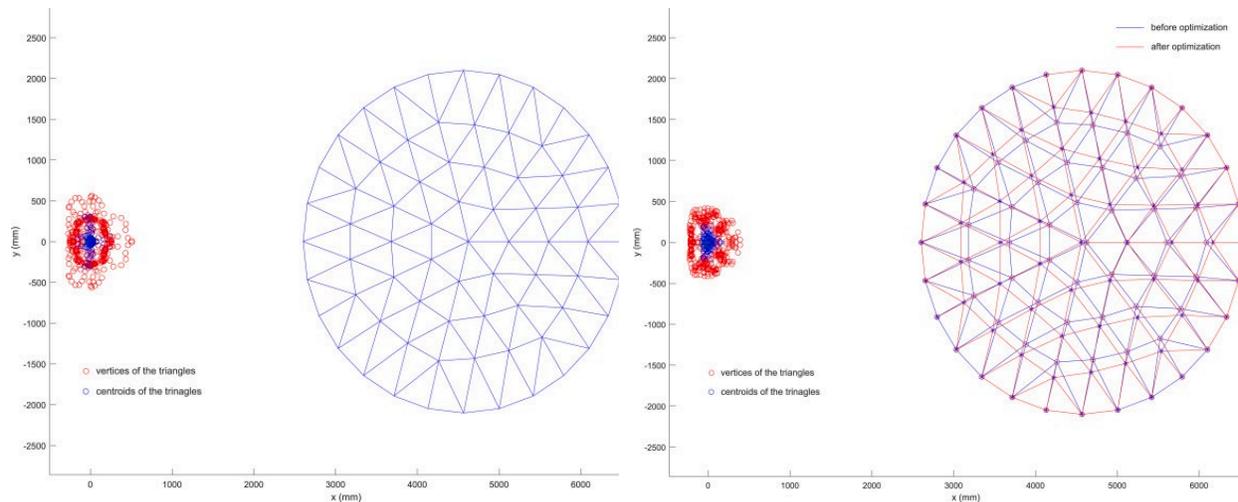


Fig. 4: The initial (left side) and final (right side) topology of the cable-net of an offset reflector and the points of minimum distance with respect to the focus.

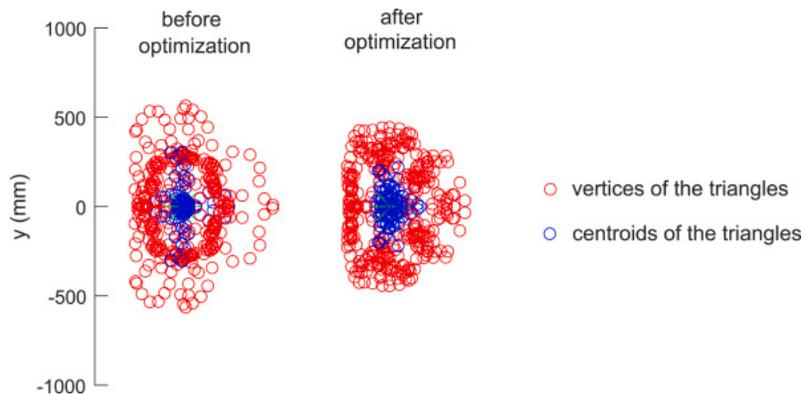


Fig. 5: An enlargement of the points of minimum distance with respect to the focus before and after optimization.

Conclusions

The surface accuracy of mesh reflectors is the main objective to be achieved in order to guarantee the prescribed band-width requirements. In this work, a numerical optimization algorithm able to ensure a greater concentration of reflected rays around the focus has been developed. The novelty introduced in this paper concerns the ability to optimize the reflecting surface according to the quantity of reflected rays directed towards the illuminator, unlike the most used method which measures the distance of the nodes of the net with respect to the ideal paraboloid. This approach provides a direct measure of the error of all incoming electromagnetic rays missing the receiver. One operating condition has been addressed: the case of the offset reflector. In this case, an unconstrained numerical optimization problem was adopted as the nodes were fixed on the external circumference and the only variables were represented by the internal nodes, which are free to move over the entire surface of the reflector. The numerical results confirm the goodness of the proposed method, modifying the topology of the cable-net in order to reduce the mean square error around the feed of the reflector.

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