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Interface reduction in flexible multibody systems

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Abstract. The problem of imposing the reference conditions in a floating frame of reference formulation is coupled with the necessity to reduce the interfaces to virtual nodes required to define the multibody joints. Two methods are implemented for rigid and interpolation multipoint constraints, and the reference condition matrix is derived employing all the interface dofs. The case study of a slider-crank mechanism is discussed to show how different sets of reference conditions can modify the system's dynamics.

Introduction

Interface reduction is a recurring problem in substructuring and model reduction theory [1]. In flexible multibody dynamics, the interfaces are primitive geometric features employed to form a joint. In practice, what is done is to individually reduce each interface to a single virtual node, usually a not collocated node outside the volume of the body. This reduction occurs through two types of multipoint constraints (MPCs): the rigid multipoint constraint, usually referred to as the RBE2 element, and the interpolation multipoint constraint, usually referred to as the RBE3 element [2]. Subsequently, the virtual nodes of the two interfaces are linked through kinematic constraints necessary to define a joint. In [3], the authors raised the problem of the scarce use of RBE3 in multibody simulations and identified the disappearance of the dependent coordinates, operated by FE software after eliminating the multipoint constraints, as one of the possible causes. The method proposed in [3] has practical implications to be used in commercial FE software. Still, it neglects essential aspects related to the presence of MPCs and generic reference conditions (RCs) within the Floating Frame of Reference Formulation (FFRF) [4].

Here, a different approach is presented that is perfectly integrated inside the FFRF working with every RCs. Both types of MPC are treated, and the interpolation MPC exploits all interface DOFs without the need to select dependent nodes or to introduce selection criteria. MPCs are directly connected to the reference conditions necessary to define the floating frame correctly. This issue has only been marginally addressed in the literature without providing a general treatment for any RC.

The paper is organized as follows. First, the mathematical background of the FFRF and the role of the RCs are introduced. Then, the method to apply the reference conditions to the virtual nodes of rigid and interpolation MPCs is provided. The reference condition matrix is obtained in explicit form for both cases. The case study of a flexible crank is analyzed, and three different sets of RCs have been applied to the interface virtual nodes of the component. Finally, the crank is assembled with a flexible connecting rod and a rigid piston to simulate a single cylinder of an internal combustion engine.

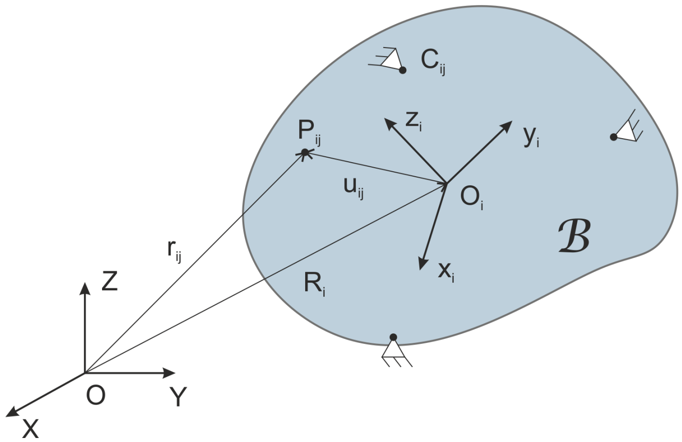

Figure 1: Floating frame of reference nomenclature and reference conditions

Background

The Floating Frame of Reference formulation describes the motion of a flexible body composing the gross motion of a particular frame, i.e., the floating frame and the local deformation with respect to the floating frame. In the finite element theory, the shape function necessary to describe the elastic behavior of a deformable body must be able to represent the rigid-body motion. Otherwise, the body cannot satisfy the *objectivity* property. The rigid-body motion provided by the shape function can be a duplicate or interfere with the gross motion of the floating frame. A set of linear constraints must be applied at some points C_{ij} to remove the rigid-body motion. These constraints are referred to as the *reference conditions* (RCs) and define the nature of the floating frame.

Denoting with $\boldsymbol{R}_i$ and $\boldsymbol{A}_i$ the position and the rotation matrix of the floating frame ($x_i y_i z_i$) with respect to the inertial frame (XYZ), and with $\boldsymbol{u}_{ij}$ the position of a generic point P_{ij} with respect to the floating frame, we write

$$\boldsymbol{r}_{ij} = \boldsymbol{R}_i + \boldsymbol{A}_i \bar{\boldsymbol{u}}_{ij} = \boldsymbol{R}_i + \boldsymbol{A}_i \boldsymbol{S}_{ij}(\boldsymbol{q}_0 + \boldsymbol{B}_2 \boldsymbol{q}_f) \tag{1}$$

where $\boldsymbol{S}_{ij}$ is the shape matrix, $\boldsymbol{q}_0$ and $\boldsymbol{q}_f$ respectively are the vectors of elastic coordinates in the undeformed and deformed configuration and $\boldsymbol{B}_2$ is the reference condition matrix that removes the rigid-body motion from the elastic displacements. It is noteworthy that the gross motion defined by $\boldsymbol{R}_i$ and $\boldsymbol{A}_i$ is not, in general, a rigid-body motion. Only when the RCs impose a fixed constraint at point O_i the gross motion becomes a rigid-body motion. The RCs can be applied at any point of the body, respecting that the final structure is isostatic or hyperstatic. Although this is the only prescription imposed on the RCs, however, further advice based on the experience is advisable:

1. the RCs should be a subset of the multibody joints, meaning that the body should deform following shapes allowed by the actual joints.
2. different RCs yield different results and should be experimentally validated.

The first advice comes from a recent paper [5] in which it is demonstrated that the free-free or mean-axis RCs, in particular system layouts, do not satisfy the mechanical joints. If the RCs must respect the mechanical joints, they must be applied on the same nodes used to define those joints. These nodes can be points of the structure, as in the case of beam or plate elements, or they can be virtual points, as in most cases where three-dimensional elements are used.

The second piece of advice comes from the evidence that different RCs create different component modes of the reduced system and eventually modify the elastic response of the system.

This probably stems from the finiteness of the component mode set employed to obtain the reduced-order model. In [6-10], the authors investigated this problem by providing different planar and spatial examples based on beams.

In summary, it would be convenient to use different sets of RCs. Advisable FFR-based methods must be able to apply the RCs to any physical or virtual point of the structure to create the component modes necessary for the dynamic analysis [11]. In FEA, this issue is solved using the rigid and the interpolation multipoint constraints (MPC), often referred to as RBE2 and RBE3 elements, respectively. In flexible multibody systems, as for structures, the RBE elements are also employed to create multibody joints. In RBE2, a group of dependent nodes follows the rigid body displacements of a single independent node. In RBE3, the displacement of a given dependent node is calculated using the displacements of a group of independent nodes.

Methodology

Given a component discretized into FE, let B, I, and V be the sets of boundary, internal, and virtual nodes, respectively. All interface nodes belong to B, while V contains the virtual nodes necessary to create an MPC; the remaining nodes belong to I. The virtual nodes are essential creating an MPC and can be collocated, that is, physical nodes of the mesh or non-collocated nodes, i.e., nodes not belonging to the body's volume. Then, the RBE2 element can be described in terms of the mentioned sets:

$$\boldsymbol{F}: B \longrightarrow V, \boldsymbol{q}_B = \boldsymbol{F}\boldsymbol{q}_V \text{ (RBE2)} \tag{1}$$

Fort he and RBE3 element, it follows

$$\boldsymbol{G}: V \longrightarrow B, \boldsymbol{q}_V = \boldsymbol{G}\boldsymbol{q}_B \text{ (RBE3)} \tag{2}$$

where $\boldsymbol{F}$ and $\boldsymbol{G}$ are linear functions of the independent nodes whose expressions are reported in [2, 3]. The vectors $\boldsymbol{q}_B$ and $\boldsymbol{q}_V$ contain the displacements of nodes belonging to B and V, respectively. Usually, $\dim(V) < \dim(B)$, and this explains the limited use of RBE3 in multibody applications. While imposing the RCs on the virtual nodes of the RBE2 elements is often immediate, doing the same with the RBE3 element needs some tricks. Applying the RCs on the virtual nodes of V, the RCs can be expressed through the following linear constraint equations

$$\boldsymbol{D}\boldsymbol{q}_V = \boldsymbol{0} \tag{3}$$

where $\boldsymbol{D}$ is a matrix containing the coefficients of these equations.

RBE2 element

By introducing Eq.(1) into Eq.(3), we derive

$$\boldsymbol{D}\boldsymbol{F}^{\dagger}\boldsymbol{F}\boldsymbol{q}_V = \boldsymbol{0} \rightarrow \boldsymbol{D}\boldsymbol{F}^{\dagger}\boldsymbol{q}_B = \boldsymbol{0} \rightarrow \boldsymbol{B}_2 = null(\boldsymbol{D}\boldsymbol{F}^{\dagger}) \tag{4}$$

where $\boldsymbol{F}^{\dagger}$ is the generalized inverse of $\boldsymbol{F}$ and $\boldsymbol{B}_2$ is the matrix of the RCs such that $\boldsymbol{q}_B = \boldsymbol{B}_2\boldsymbol{\gamma}$ being $\boldsymbol{\gamma}$ a reduced set of independent elastic parameters.

Usually, this procedure is not needed as $\boldsymbol{B}_2$ can be directly found by removing the constrained dof from $\boldsymbol{q}_V$, i.e.

$$\boldsymbol{q}_V = \boldsymbol{B}_2\boldsymbol{q}_V^* \tag{5}$$

where $\boldsymbol{q}_V^*$ is the reduced set of independent elastic coordinates. Exploiting Eq.(5), it is derived that

$$\boldsymbol{q}_{\mathrm{B}} = \boldsymbol{F}\boldsymbol{B}_2\boldsymbol{q}_{\mathrm{V}}^* \rightarrow \boldsymbol{T} = \boldsymbol{F}\boldsymbol{B}_2 \tag{6}$$

in which $\boldsymbol{T}$ is the transformation matrix that contains both the RBE2 and RCs.

RBE3 element
By introducing Eq.(2) into Eq.(3), we have

$$\boldsymbol{D}\boldsymbol{G}\boldsymbol{q}_B = \boldsymbol{0} \rightarrow \boldsymbol{q}_B = \boldsymbol{B}_2\boldsymbol{\rho},\ \boldsymbol{B}_2 = null(\boldsymbol{D}\boldsymbol{G}) \quad (7)$$

By substituting into Eq.(2), we obtain $\boldsymbol{q}_V = \boldsymbol{G}\boldsymbol{B}_2\boldsymbol{\rho}$. In this case, the direct substitution carried out in the RBE2 element is not possible, and the final transformation matrix $\boldsymbol{T} = \boldsymbol{G}\boldsymbol{B}_2$ maps a reduced set of independent elastic parameters into the virtual node displacement vector $\boldsymbol{q}_V$.

Numerical simulation
The method presented in the previous sections is applied to a slider-crank mechanism. In Figure 2, the crank is modeled considering tetrahedrons. The interface nodes of set *B* are highlighted using red dots. The three interfaces refer to the revolute joints employed to connect the crankshaft to the frame (B_1 and B_3) and the revolute joint between the crank and the connecting rod (B_2). Only RBE3 elements are employed, and a virtual node is defined at the geometric center of each interface. Then, the three virtual nodes are used to determine the revolute joints. To define $\boldsymbol{B}_2$ as in Eq.(7), a set of RCs must be imposed by means of the matrix $\boldsymbol{D}$ of Eq.(3). Here, we consider three sets of RCs as reported in Tab. 1. All sets lead to an isostatic structure.

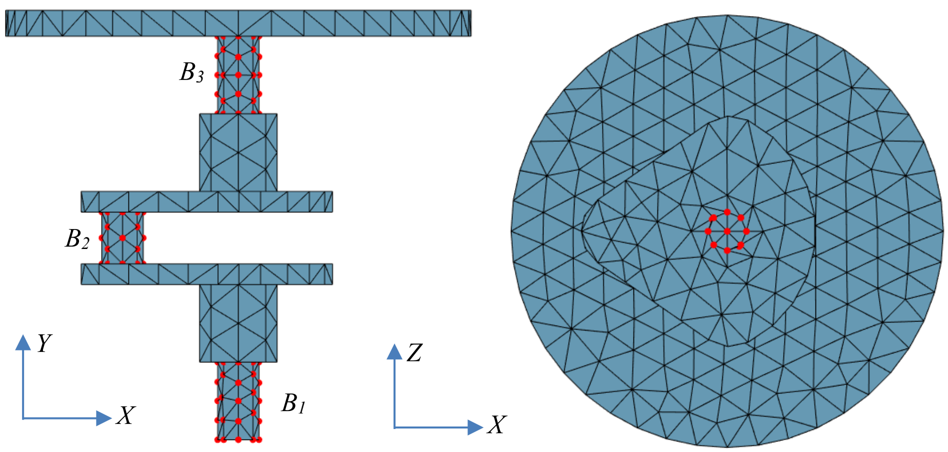

Figure 2: Crank layout and interface nodes highlighted using red dots.

Table 1: Three sets of reference conditions. T and R stand for forbidden translations and rotations.

RCs	Interface B_1	Interface B_2	Interface B_3
Set 1	Spherical (TxTyTz)	-	Universal (TxTyTzRy)
Set 2	Spherical (TxTyTz)	Translation (Tz)	Translations (TxTz)
Set 3	-	Fixed (TxTyTzRxRyRz)	-

The three sets of RCs are used to define the normal modes, i.e., eigenmodes of the component constrained using the RCs [4]. Retained sets of normal modes serve as a basis for reducing the elastic coordinates. The final set of coordinates includes only gross motion coordinates and the modal amplitudes corresponding to the retained normal modes. As can be shown in Fig. 3, the modes and their corresponding frequencies change, passing from one set to another. As already remarked, this modifies the dynamics of the system.

To observe how the elastic field modifies the dynamics of a system, we assembled a slider-crank mechanism simulating an internal combustion engine with a single-cylinder. The connecting rod has been modeled using tetrahedrons and simply supported RCs, while the cylinder has been modeled as a rigid body. An external force arising from the in-cylinder pressure shown in Fig. 4

has been applied to the piston, as detailed in [12]. Two torques simulating the starter electric motor and the external load applied to the crankshaft have also been included in the model. All components and the dynamics simulation have been performed using the commercial software Matlab®. The implicit *generalized-alpha method* has been employed to integrate the equations of motion.

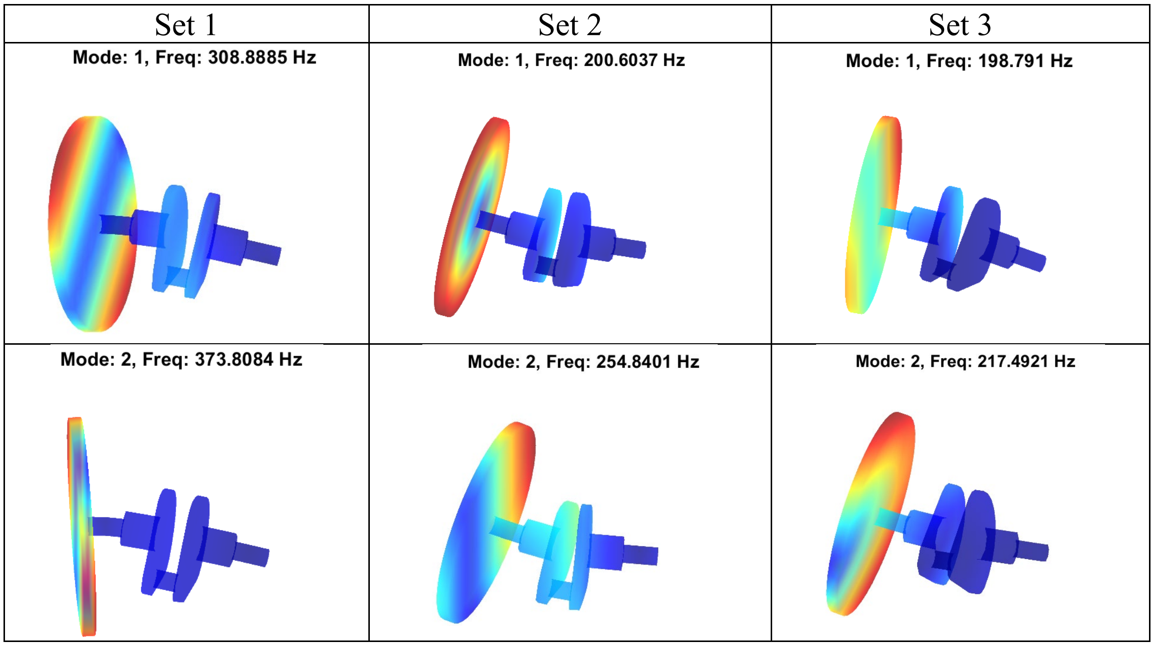

Figure 3: The first two normal modes for each set of reference conditions.

As can be observed in Fig. 4, after a transitory phase in which the system accelerates, the angular speed of the crankshaft stabilizes. The oscillations around the regime speed are typical of a single-cylinder system due to unbalanced forces during the rotation. Comparing the three sets of RCs, the angular velocity of the crankshaft starts to differ for high speeds. This effect comes from the different frequency ranges of the three normal mode sets employed in the reduction process.

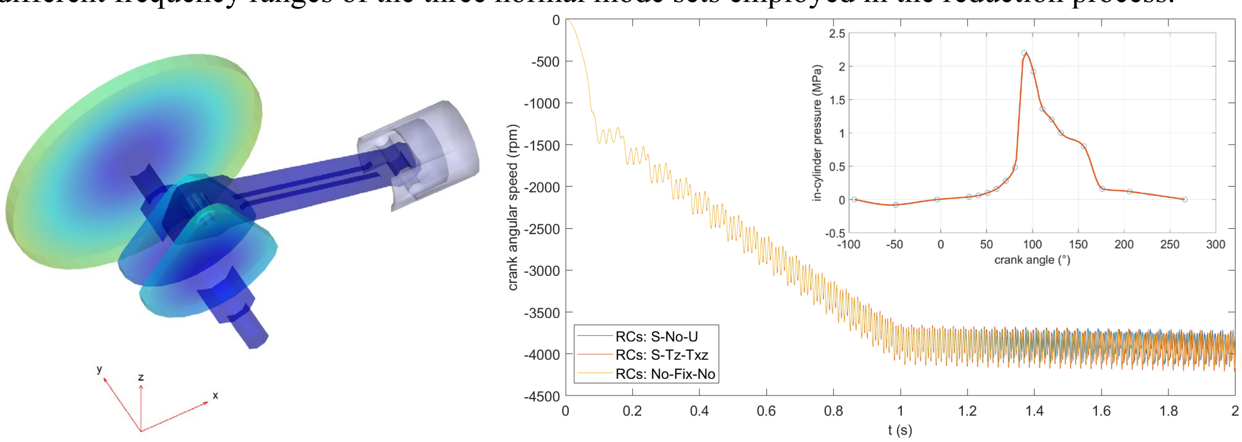

Figure 4: Simulation snapshot of the slider-crank system, in-cylinder pressure, and angular speed of the crankshaft considering the three sets of RCs of Table 1.

Conclusions

This paper presents a method to combine interface reduction and reference conditions to create component modes to be used for the dynamics of flexible multibody systems. The procedure is applied to both rigid and interpolation multipoint constraints. For the latter, all interface dofs are employed without resorting to any selection criteria of the independent dofs. The case study of a slider-crank mechanism simulating an internal combustion engine with a single cylinder is provided to demonstrate how different sets of reference conditions can influence the dynamics of a complex multibody system.

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