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Optimal Design of Tuned Mass Damper Inerter for Base-Isolated Buildings

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Abstract. Tuned mass dampers (TMD) are installed in base-isolated building to suppress the excessive isolator displacement and acceleration responses of primary structure. By incorporating an inerter element into the original configuration, the seismic performance of TMD is significantly enhanced. In this work, optimal solutions of tuned mass damper inerter (TMDI) for improving the seismic resilience of base-isolated building are proposed. The analytical formulations of optimal design of TMDI are respectively developed to minimize the H_2 norm of the displacement of primary structure relative to the base floor and the isolator displacement. The performance of presented optimal methods are validated by using stationary responses under the stochastic excitations. Additionally, the seismic performance of TMDI with parameters obtained from the proposed method are compared with the established methods.

Introduction

Base isolation (BI) system has been widely employed in public building which is of importance to keep its' functionality [1-2] (e.g. hospital, disaster prevention center) and prevent catastrophic results. The mechanical characteristic of BI is to decouple the resonance of building and ground motions by inserting a very soft base layer between them [3]. Although, BI technique presents attractive benefits in mitigating the earthquake responses of primary structure, some detrimental features, *e.g.* serious concentration of base isolators deformation, need to be carefully taken into account. In order to reduce the excessive displacement of base floor, the so called "hybrid base isolation system", which consists of traditional base isolators (*e.g.* lead rubber bearing, friction peculium slide and natural rubber bearing etc.) and additional damping devices, was developed.

It was mentioned that the effectiveness of displacement dependent damper is less than the viscous damper because the adverse contribution of higher modes. Furthermore, it is worthy to note that a high damping ratio of viscous damper may lead to an excessive inter-story drift and acceleration response [4]. Although an active or semi-active damper may overcome above difficulties, it requires stable power supply and active feedback system might be hardly satisfied during a major earthquake. Alternatively, a tuned mass damper, which is able to simultaneously mitigate the displacement of base floor and acceleration response, are recommended to work with base isolators to suppress the isolator displacement [5].

Although, the effectiveness of TMD has been confirmed in previous studies, it should be noted that a large mass ratio is necessary to achieve the designated performance target. An inerter element, which amplifies the mass effect by using the relative motion of two terminals, can be employed to enhance the seismic performance of traditional TMD. A novel configuration consists of a TMD and an inerter element which is called tuned mass damper inerter (TMDI) has been proposed to mitigate the excessive displacement of a single-degree-of-freedom (SDOF) system [6].

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With an excellent seismic performance for mitigating the excessive displacement responses and suppressing the acceleration responses, TMDI was gradually popularized in suppressing isolator displacement demand in a BI system [7].

For implementation of TMDI in a BI system, the optimal solutions with numerical optimization algorithm were carried out in [7]. Although, the effectiveness of optimal results has been validated by a lot of benchmark models, it should be noted that the numerical results cannot be readily applied to practical design, because most of practicing engineers are unfamiliar with those methods to derive optimal designs. Thus, an analytical formulation of optimal parameters needs to be developed to fit the request of practical design. Marian and Giaralis [6, 8] presented two analytical formulae to minimize the H_2 and H_∞ norms of displacement responses of a SDOF system under white-noise excitation and harmonic excitation. An analytical formula which consider the characteristics of a BI system was derived to suppress the excessive isolator displacement by Matteo *et al*. [9].

In this work, two closed-form formulae for suppressing the acceleration of primary structure and excessive isolator displacement were developed. Moreover, an investigation of the influences of external excitation on the optimal solutions were discussed with a series of numerical calculations. To access the effectiveness of proposed method for improving the seismic performance of a BI system under stationary excitation, a performance comparison is carried out in Section 7.

BI system equipped with a TMDI

On the basis of Kelly's work [4], a BI system can be simplified as a 2-degree-of-freedom (2-DOF) system as shown in Fig. 1(a). m_s and m_b denote the mass of the primary structure and the basement, respectively. The stiffness of primary structure and basement are respectively represented by k_s and k_b. c_s and c_b are used to denote the inherent damping of primary structure and base isolators. As shown in Fig. 1(b), a TMDI can be simplified to a tuned inerter damper (TID) by removing the physical mass. On the other hand, TMDI is an advanced TMD whose control effect is significantly improved by connecting a ground linked inerter. As mentioned above, we expect to develop a comprehensive closed-form formula of TMDI that can represent TID or TMD by setting the physical mass or inertance as zero. Assume the aforementioned 2-DOF system is subjected to an external excitation $\ddot{x}_0$, the governing equations of this hybrid BI system can be expressed as,

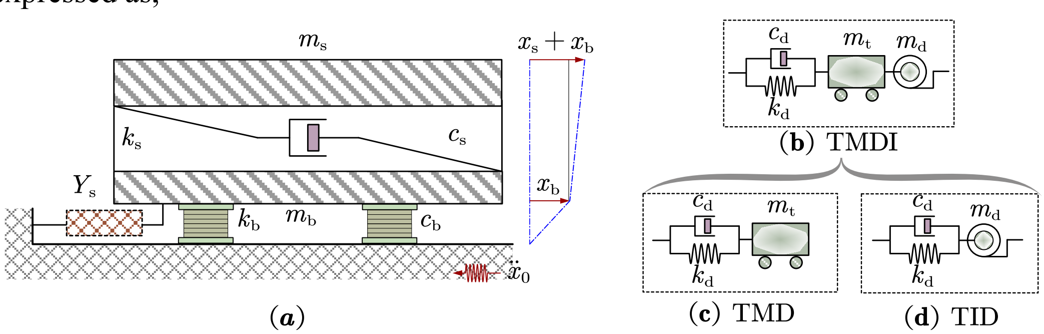

Fig. 1 *(a) Schematic of BI system equipped with (b) TMDI, (c) TMD and (d) TID.*

$$\begin{gathered}
m_\mathrm{s}\ddot{x}_\mathrm{b}+m_\mathrm{s}\ddot{x}_\mathrm{s}+c_\mathrm{s}\dot{x}_\mathrm{s}+k_\mathrm{s}x_\mathrm{s}=-m_\mathrm{s}\ddot{x}_0\\
m_\mathrm{tot}\ddot{x}_\mathrm{b}+m_\mathrm{s}\ddot{x}_\mathrm{s}+c_\mathrm{b}\dot{x}_\mathrm{b}+k_\mathrm{b}x_\mathrm{b}+c_\mathrm{d}(\dot{x}_\mathrm{b}-\dot{x}_\mathrm{d})+k_\mathrm{d}(x_\mathrm{b}-x_\mathrm{d})=-m_\mathrm{tot}\ddot{x}_0\,.\\
m_\mathrm{d}\ddot{x}_\mathrm{d}+m_\mathrm{t}\ddot{x}_\mathrm{d}+c_\mathrm{d}(\dot{x}_\mathrm{d}-\dot{x}_\mathrm{b})+k_\mathrm{d}(x_\mathrm{d}-x_\mathrm{b})=-m_\mathrm{t}\ddot{x}_0
\end{gathered} \tag{1}$$

where, $m_{\mathrm{tot}} = m_{\mathrm{s}} + m_{\mathrm{b}}$ denote the total mass of the BI system, m_{d} and m_{t} denote the apparent mass of the inerter element and the physical mass of the TMD, c_{d} and k_{d} denote the damping coefficient and stiffness of the TMDI, x_{s}, x_{b} and x_{d} denote the relative displacement of the primary structure, isolator and TMDI. The dot over x_{s}, x_{b} and x_{d} denote the differential operation with regard to the time *t*. With introducing the following parameters,

$$\begin{aligned}
&\mu_{\mathrm{s}} = \frac{m_{\mathrm{s}}}{m_{\mathrm{tot}}},\ \mu_{\mathrm{d}} = \frac{m_{\mathrm{d}}}{m_{\mathrm{tot}}},\ \mu_{\mathrm{t}} = \frac{m_{\mathrm{t}}}{m_{\mathrm{tot}}}, \omega_{\mathrm{s}} = \sqrt{\frac{k_{\mathrm{s}}}{m_{\mathrm{s}}}}, \omega_{\mathrm{b}} = \sqrt{\frac{k_{\mathrm{b}}}{m_{\mathrm{tot}}}}, \omega_{\mathrm{d}} = \sqrt{\frac{k_{\mathrm{d}}}{m_{\mathrm{d}} + m_{\mathrm{t}}}} \\
&\mu_{\mathrm{e}} = \mu_{\mathrm{d}} + \mu_{\mathrm{t}}, \beta_{\mathrm{s}} = \frac{\omega_{\mathrm{s}}}{\omega_{\mathrm{b}}}, \beta_{\mathrm{d}} = \frac{\omega_{\mathrm{d}}}{\omega_{\mathrm{b}}}, \zeta_{\mathrm{s}} = \frac{c_{\mathrm{s}}}{2\omega_{\mathrm{s}} m_{\mathrm{s}}}, \zeta_{\mathrm{b}} = \frac{c_{\mathrm{b}}}{2\omega_{\mathrm{b}} m_{\mathrm{tot}}}, \zeta_{\mathrm{d}} = \frac{c_{\mathrm{d}}}{2\omega_{\mathrm{b}} m_{\mathrm{tot}}}
\end{aligned} . \tag{2}$$

the Laplace transformation of Eq. (1) is rewritten as,

$$\begin{aligned}
(\beta_{\mathrm{s}}^2 - \beta_0^2 + 2i\,\beta_0 \beta_{\mathrm{s}} \zeta_{\mathrm{s}})\mu_{\mathrm{s}} \omega_{\mathrm{b}}^2 X_{\mathrm{s}} - \beta_0^2 \mu_{\mathrm{s}} \omega_{\mathrm{b}}^2 X_{\mathrm{b}} + \mu_{\mathrm{s}} X_0 = 0 \\
-\beta_0^2 \mu_{\mathrm{s}} \omega_{\mathrm{b}}^2 X_{\mathrm{s}} + (\beta_{\mathrm{d}}^2 \mu_{\mathrm{e}} - \beta_0^2 + 1 + 2i\,\beta_0 \zeta_{\mathrm{b}} + 2i\,\beta_0 \zeta_{\mathrm{d}})\omega_{\mathrm{b}}^2 X_{\mathrm{b}} - (\beta_{\mathrm{d}}^2 \mu_{\mathrm{e}} + 2i\,\beta_0 \zeta_{\mathrm{d}})\omega_{\mathrm{b}}^2 X_{\mathrm{d}} + X_0 = 0 \\
-(\beta_{\mathrm{d}}^2 \mu_{\mathrm{e}} + 2i\,\beta_0 \zeta_{\mathrm{d}})\omega_{\mathrm{b}}^2 X_{\mathrm{b}} + [2i\,\beta_0 \zeta_{\mathrm{d}} + (\beta_{\mathrm{d}}^2 - \beta_0^2)\mu_{\mathrm{e}}]\omega_{\mathrm{b}}^2 X_{\mathrm{d}} + \mu_{\mathrm{t}} X_0 = 0
\end{aligned} . \tag{3}$$

where, X_{s}, X_{b} and X_{d} denote the Laplace transformation of x_{s}, x_{b} and x_{d}. Frequency ratio of external excitation is defined as $\beta_0 = \omega_0/\omega_{\mathrm{b}}$, where ω_0 is the input frequency of excitation. For simplicity of derivation, inherent damping ratio of primary structure and base isolator are assumed as $\zeta_{\mathrm{s}} = 0$, $\zeta_{\mathrm{b}} = 0$. Thus, the displacement amplification ratio X_{s}/X_0 and X_{b}/X_0 are expressed as,

$$\begin{aligned}
\frac{X_{\mathrm{s}}}{X_0} &= \frac{1}{\omega_{\mathrm{b}}^2} \frac{-2i\,\beta_0^3 \mu_{\mathrm{d}} \zeta_{\mathrm{d}} - \mu_{\mathrm{e}}(\beta_{\mathrm{d}}^2 \mu_{\mathrm{d}} + 1)\beta_0^2 + 2i\,\beta_0 \zeta_{\mathrm{d}} + \beta_t^2 \mu_{\mathrm{e}}}{a_0 \beta_0^6 + a_1 \beta_0^5 + a_2 \beta_0^4 + a_3 \beta_0^3 + a_4 \beta_0^2 + a_5 \beta_0 + a_6} \\
\frac{X_{\mathrm{b}}}{X_0} &= \frac{1}{\omega_{\mathrm{b}}^2} \frac{-\mu_{\mathrm{e}}(\mu_{\mathrm{s}} - 1)\beta_0^4 + 2i\,\zeta_{\mathrm{d}}(\mu_{\mathrm{s}} - b_0)\beta_0^3 + \mu_{\mathrm{e}}(\beta_{\mathrm{d}}^2(\mu_{\mathrm{s}} - b_0) - \beta_{\mathrm{s}}^2)\beta_0^2 + 2i\zeta_{\mathrm{d}} \beta_{\mathrm{s}}^2 b_0 \beta_0 + \beta_{\mathrm{s}}^2 \beta_{\mathrm{d}}^2 \mu_{\mathrm{e}} b_0}{a_0 \beta_0^6 + a_1 \beta_0^5 + a_2 \beta_0^4 + a_3 \beta_0^3 + a_4 \beta_0^2 + a_5 \beta_0 + a_6}
\end{aligned} . \tag{4}$$

In which, all terms of coefficients in Eq. (4) are given as,

$$\begin{cases}
a_6 = \beta_{\mathrm{s}}^2 \beta_{\mathrm{d}}^2 \mu_{\mathrm{e}} & a_5 = 2i\,\beta_{\mathrm{s}}^2 \zeta_{\mathrm{d}} \\
a_4 = -\beta_{\mathrm{s}}^2 \beta_{\mathrm{d}}^2 \mu_{\mathrm{e}}(\mu_{\mathrm{e}} + 1) - \mu_{\mathrm{e}}(\beta_{\mathrm{s}}^2 + \beta_{\mathrm{d}}^2) & a_3 = -2i\zeta_{\mathrm{d}}(\beta_{\mathrm{s}}^2 \mu_{\mathrm{e}} + \beta_{\mathrm{s}}^2 + 1) \\
a_2 = \beta_{\mathrm{d}}^2 \mu_{\mathrm{e}}^2 - \beta_{\mathrm{d}}^2 \mu_{\mathrm{s}} \mu_{\mathrm{e}} + \beta_{\mathrm{s}}^2 \mu_{\mathrm{e}} + \beta_{\mathrm{d}}^2 \mu_{\mathrm{e}} + \mu_{\mathrm{e}} & a_1 = 2i\,\zeta_{\mathrm{d}}(1 + \mu_{\mathrm{e}} - \mu_{\mathrm{s}}) \\
a_0 = \mu_{\mathrm{e}}(\mu_{\mathrm{s}} - 1) & b_0 = \mu_{\mathrm{t}} + 1
\end{cases} . \tag{5}$$

Consider the stochastic properties of earthquake ground motions, external excitation can be directly expressed as a sequence of white-noise. According to the basic concepts of stochastic vibration theory, the covariance of system response is obtained from an integrated power spectral density function (PSDF) over entire frequency domain. Therefore, the H_2 norm of above two performance indicators can be expressed as an integral of PSDF of a white-noise S_{w} in frequency domain.

$$PI(X_{\mathrm{s}}/X_0) = S_{\mathrm{w}} \omega_{\mathrm{b}} \int_{-\infty}^{\infty} |X_{\mathrm{s}}/X_0|^2 \mathrm{d}\beta_0,\ \ PI(X_{\mathrm{b}}/X_0) = S_{\mathrm{w}} \omega_{\mathrm{b}} \int_{-\infty}^{\infty} |X_{\mathrm{b}}/X_0|^2 \mathrm{d}\beta_0 . \tag{6}$$

In H_2 norm-based optimization, the objective function is the covariance of system responses. The optimization procedure of frequency and damping ratios of TMDI with respect to above two performance indicators are illustrated in the following sections.

Optimal solution for acceleration response of primary structure
On the basis of Spanos' pioneering works, the first integral in Eq. (6) can be expressed with following analytical formula,

$$PI\left(\frac{X_{\mathrm{s}}}{X_0}\right)=\frac{S_{\mathrm{w}}}{\omega_{\mathrm{b}}^3}\frac{\beta_{\mathrm{s}}^4\mu_{\mathrm{e}}^2\phi_{\mathrm{s},1}+\beta_{\mathrm{s}}^2\mu_{\mathrm{e}}^2\phi_{\mathrm{s},2}\beta_{\mathrm{d}}^2+\mu_{\mathrm{e}}^2\phi_{\mathrm{s},3}\beta_{\mathrm{d}}^4+4\,\beta_{\mathrm{s}}^2\phi_{\mathrm{s},4}\zeta_{\mathrm{d}}^2}{2\beta_{\mathrm{s}}^{10}\,\mu_{\mathrm{s}}\mu_{\mathrm{e}}^2\zeta_{\mathrm{d}}}\,. \tag{7}$$

where, the related coefficients are,

$$\begin{aligned}
\phi_{\mathrm{s},1}&=\beta_{\mathrm{s}}^2\mu_{\mathrm{s}}+1\\
\phi_{\mathrm{s},2}&=\beta_{\mathrm{s}}^4\mu_{\mathrm{s}}(\mu_{\mathrm{d}}-2-\mu_{\mathrm{t}})+2\beta_{\mathrm{s}}^2(\mu_{\mathrm{d}}-2\mu_{\mathrm{s}})-2\\
\phi_{\mathrm{s},3}&=\beta_{\mathrm{s}}^6\mu_{\mathrm{s}}b_0^2+\beta_{s}^4\left((\mu_{\mathrm{d}}-\mu_{\mathrm{s}})^2+2\mu_{\mathrm{s}}b_0\right)+\beta_{\mathrm{s}}^2(3\mu_{\mathrm{s}}-2\,\mu_{\mathrm{d}})+1\\
\phi_{\mathrm{s},4}&=\beta_{\mathrm{s}}^4\mu_{\mathrm{d}}(\mu_{\mathrm{d}}-\mu_{\mathrm{s}})+\beta_{\mathrm{s}}^4\mu_{\mathrm{s}}b_0+2\beta_{\mathrm{s}}^2(\mu_{\mathrm{s}}-\mu_{\mathrm{d}})+1
\end{aligned}\,. \tag{8}$$

To obtain the minimization of above performance indicators, the following equations hold,

$$\partial PI(X_{\mathrm{s}}/X_0)/\partial\beta_{\mathrm{d}}^2=0\,,\quad \partial PI(X_{\mathrm{s}}/X_0)/\partial\zeta_{\mathrm{d}}=0\,. \tag{9}$$

Then, the optimal solution is obtained as,

$$\beta_{\mathrm{d}}=\beta_{\mathrm{s}}\sqrt{\frac{-\phi_{\mathrm{s},2}}{2\,\phi_{\mathrm{s},3}}}\quad \zeta_{\mathrm{d}}=\frac{\beta_{\mathrm{s}}\mu_{\mathrm{e}}}{4}\sqrt{\frac{4\,\phi_{\mathrm{s},1}\phi_{\mathrm{s},3}-\phi_{\mathrm{s},2}^2}{\phi_{\mathrm{s},3}\phi_{\mathrm{s},4}}}\,. \tag{10}$$

Additionally, the optimized performance indicator of relative displacement amplitude is given as,

$$PI_{\mathrm{opt}}\left(\frac{X_{\mathrm{s}}}{X_0}\right)=\frac{S_{\mathrm{w}}}{\omega_{\mathrm{b}}^3}\frac{\sqrt{4\,\phi_{\mathrm{s},1}\phi_{\mathrm{s},3}-\phi_{\mathrm{s},2}^2}\sqrt{\phi_{\mathrm{s},3}\phi_{\mathrm{s},4}}}{\beta_{\mathrm{s}}^7\,\mu_{\mathrm{s}}\mu_{\mathrm{e}}\phi_{\mathrm{s},3}}\,. \tag{11}$$

As indicated in Eq. (11), we have obtained the optimal performance indicator of relative displacement of primary structure. Furthermore, the optimized $PI(X_{\mathrm{s}}/X_0)$ lead to an optimal formula of absolute acceleration responses of primary structure,

$$\ddot{x}_{\mathrm{s},0}=-2\zeta_{\mathrm{s}}\omega_{\mathrm{s}}\dot{x}_{\mathrm{s}}-\omega_{\mathrm{s}}^2x_{\mathrm{s}}\,. \tag{12}$$

Because the value of damping ratio of primary structure ζ_{s} always locates in the range of [0.02, 0.05], which means that the contribution of x_{s} dominate the acceleration responses $\ddot{x}_{\mathrm{s},0}$.

Optimal solution for isolator displacement minimization
According to the second formulation in Eq. (6), the closed-form formula of performance indicator related to the basement displacement is expressed as,

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$$PI\left(\frac{X_\mathrm{b}}{X_0}\right)=\frac{S_\mathrm{w}}{\omega_\mathrm{b}^3}\frac{\beta_\mathrm{s}^4\mu_\mathrm{e}^2\phi_{\mathrm{b},1}+\beta_\mathrm{s}^2\mu_\mathrm{e}^2\phi_{\mathrm{b},2}\beta_\mathrm{d}^2+\mu_\mathrm{e}^2\phi_{\mathrm{b},3}\beta_\mathrm{d}^4+4\,\beta_\mathrm{s}^2\phi_{\mathrm{b},4}\zeta_\mathrm{d}^2}{2\,\beta_s^6\mu_\mathrm{e}^2\zeta_\mathrm{d}}. \tag{13}$$

where, all terms of coefficients of Eq. (13) are,

$$\begin{aligned}
\phi_{\mathrm{b},1}&=\beta_\mathrm{s}^2+\mu_\mathrm{s}\\
\phi_{\mathrm{b},2}&=\beta_\mathrm{s}^4(\mu_\mathrm{t}^2-1)\mu_\mathrm{e}-2\,\beta_\mathrm{s}^2(\beta_\mathrm{s}^2+\mu_\mathrm{s})b_0-2\,\mu_\mathrm{s}(\beta_\mathrm{s}^2+1)\\
\phi_{\mathrm{b},3}&=\beta_\mathrm{s}^6b_0^2(\mu_\mathrm{e}+1)^2+\beta_\mathrm{s}^4\,\mu_\mathrm{s}(2\,\mu_\mathrm{d}b_0+3b_0^2)+\beta_\mathrm{s}^2\mu_\mathrm{s}(\mu_\mathrm{s}+2b_0)+\mu_\mathrm{s}\\
\phi_{\mathrm{b},4}&=\beta_\mathrm{s}^4(\mu_\mathrm{e}+1)b_0^2+2\,\beta_\mathrm{s}^2\mu_\mathrm{s}b_0+\mu_\mathrm{s}
\end{aligned}. \tag{14}$$

To obtain the minimization of above performance indicator, following conditions are obtained,

$$\partial PI(X_\mathrm{b}/X_0)/\partial\beta_\mathrm{d}^2=0\quad \partial PI(X_\mathrm{b}/X_0)/\partial\zeta_\mathrm{d}=0\,. \tag{15}$$

The analytical formulations of optimal parameters are expressed as following,

$$\beta_\mathrm{d}=\beta_\mathrm{s}\sqrt{\frac{-\phi_{\mathrm{b},2}}{2\,\phi_{\mathrm{b},3}}}\quad \zeta_\mathrm{d}=\frac{\beta_\mathrm{s}\mu_\mathrm{e}}{4}\sqrt{\frac{4\,\phi_{\mathrm{b},1}\phi_{\mathrm{b},3}-\phi_{\mathrm{b},2}^2}{\phi_{\mathrm{b},3}\phi_{\mathrm{b},4}}}\,. \tag{16}$$

Then, the related optimal performance indicator is estimated as,

$$PI_\mathrm{opt}\left(\frac{X_\mathrm{b}}{X_0}\right)=\frac{S_\mathrm{w}}{\omega_\mathrm{b}^3}\frac{\sqrt{4\,\phi_{\mathrm{b},1}\phi_{\mathrm{b},3}-\phi_{\mathrm{b},2}^2}\sqrt{\phi_{\mathrm{b},3}\phi_{\mathrm{b},4}}}{\beta_\mathrm{s}^3\mu_\mathrm{e}\phi_{\mathrm{b},3}}\,. \tag{17}$$

Validation of the effectiveness of proposed optimal design method

As mentioned in Sections 3 and 4, the optimal solution of frequency ratio β_d and damping ratio ζ_d were approximated by minimizing the H_2 norm of two performance indicators of a BI system subjected to a white-noise excitation. Herein, the intensity of the power spectral density (PSD) of the white-noise is assumed as $5\times10^{-4}\ \mathrm{m/s^3}$, and a benchmark model with analytical parameters namely, fundamental frequency $\omega_\mathrm{b}=2.53$, frequency ratio, mass ratio and inherent damping ratio of primary structure $\beta_\mathrm{s}=7.95$, $\mu_\mathrm{s}=0.813$, $\zeta_\mathrm{s}=0.02$, and a low damping ratio of base isolator $\zeta_\mathrm{b}=0.02$ was analyzed. The mass ratio is assumed as $\mu_t=0.05$, and $\mu_\mathrm{d}=0.45$, the total mass ratio of TMDI $\mu_\mathrm{e}=0.5$. The contour plots of two performance indicators with respect to the variations of $\beta_\mathrm{d}\in[0.1,2]$ and $\zeta_\mathrm{d}\in[0.1,1]$ are given in Fig. 2.

As shown in Fig. 2 (a) and (b), the proposed optimal solutions lie in the global minimum point in the contour plots of two performance indicators, which validates the effectiveness of proposed method. As shown in above formulations, the optimal parameters and performance indicators are estimated under the assumption that the excitation is a white-noise, it is worth investigating the influences of the variation of filtered excitation models on the related performance.

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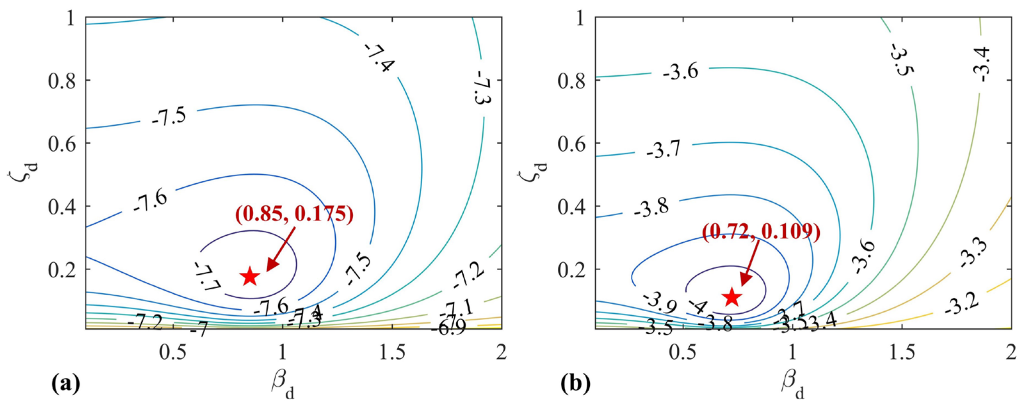

***Fig. 2** Contour plots of (a) $lg(PI(X_s/X_0))$ and (b) $lg(PI(X_b/X_0))$ of BI system.*

Comparison of optimal design with different power spectral density model.

In this part, the parameters related to the stochastic excitation model is studied. The one side Clough-Penzien model is used to act the different soil conditions related to the external excitation.

$$S_{\ddot{x}_0\ddot{x}_0} = S_{\mathrm{w}} \frac{\omega_{\mathrm{g}}^4 + 4\zeta_{\mathrm{g}}^2\omega_{\mathrm{g}}^2\omega^2}{(\omega^2-\omega_{\mathrm{g}}^2)^2 + 4\zeta_{\mathrm{g}}^2\omega_{\mathrm{g}}^2\omega^2} \frac{\omega^4}{(\omega^2-\omega_f^2)^2 + 4\zeta_f^2\omega_f^2\omega^2}. \tag{18}$$

where, ζ_{g}, ω_{g}, ω_f and ζ_f are parameters related to filter model which are given in Table 1.

***Table 1** PSDF filter parameters for model soil conditions*

Soil type	$\omega_{\mathrm{g}}(\mathrm{rad}/s)$	ζ_{g}	$\omega_f(\mathrm{rad}/s)$	ζ_f
Firm	15.0	0.6	1.5	0.6
Medium	10.0	0.4	1.0	0.6
Soft	5.0	0.2	0.5	0.6

Herein, the intensity of one side PSD $S_w = 1 \times 10^{-3}$ is employed for the excitation. Then, the performance indicators $PI(X_{\mathrm{s}}/X_0)$ and $PI(X_{\mathrm{b}}/X_0)$ are estimated by using numerical integration. To investigate the feasibility and effectiveness of proposed method, we also compared the performance indicators obtained from numerical results and analytical results with aforementioned benchmark model. Figs. 3 and 4 depict the optimal design parameters and related performance indicators obtained from analytical formula and numerical optimization procedure.

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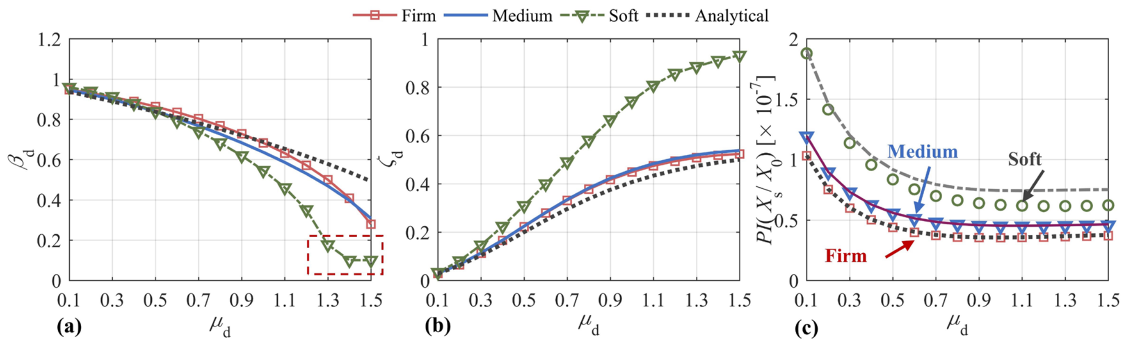

Fig. 3 *Comparison of optimal (a) β_d, (b) ζ_d and (c) the related performance indicators $PI(X_s/X_0)$ with varying mass ratio of inerter and different soil conditions.*

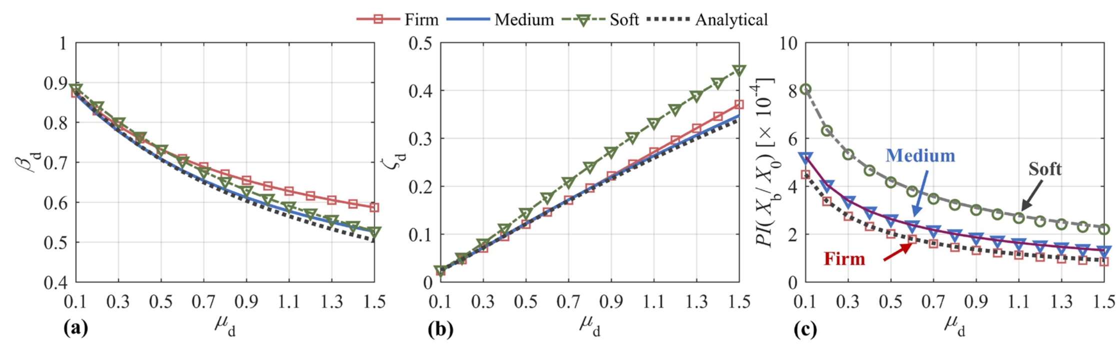

Fig. 4 *Comparison of optimal (a) β_d, (b) ζ_d and (c) the related performance indicators $PI(X_b/X_0)$ with varying mass ratio of inerter and different soil conditions.*

As indicated in Fig. 3, the analytical formula coincides with the numerical results for the Firm and Medium soil conditions. However, the numerical results for soft soil condition shows a large difference with analytical formula. Notably, the optimal value of β_{d} approaches to the lower bound of design parameters for $\mu_{\mathrm{d}} > 1.4$ with the soft soil condition, that means the mechanical behavior of TMDI is close to the simplified mass damper inerter (MDI). Despite of such a large difference in optimal results, it is worthy to note that the analytical solutions presented a comparable control effect as shown in Fig 3(c). As shown in Fig. 4 (a) and (b), the optimal results obtained from numerical results is slightly larger than those by the analytical formulae. As expected, analytical parameters yield similar performance level with the numerical results as shown in Fig 4 (c).

Investigation of different optimal formulae developed for TMDI.

In the previous studies, analytical formulae of optimal parameters were developed with various criteria [6,8,9]. Herein, the benchmark model given in Section 5 is employed to investigate the control effect of each model. Given the inertance ratio $\mu_{\mathrm{d}} = 0.45$, mass ratio of TMD $\mu_{\mathrm{t}} = 0.05$, the transfer function related to basement displacement x_{b} and absolute acceleration of primary structure $\ddot{x}_{\mathrm{s},0}$ are employed to illustrate the seismic response control performance of each method.

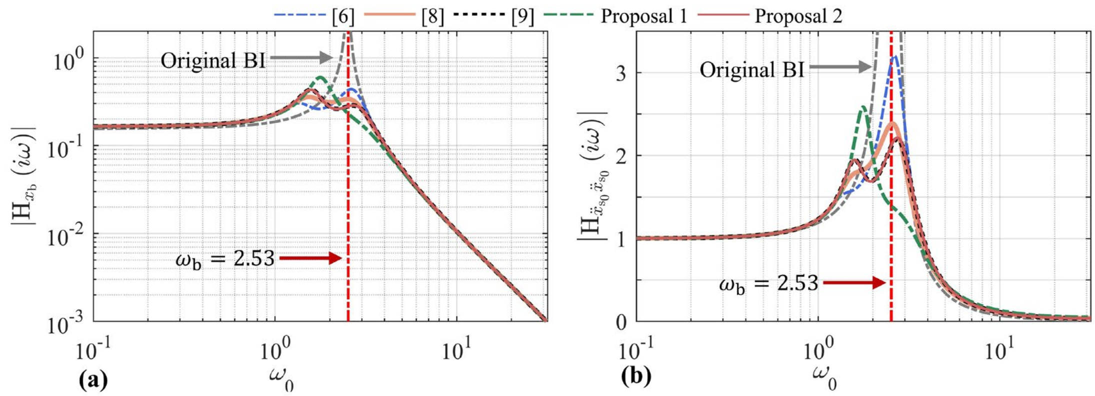

***Fig. 5** Comparison of transfer function of (a) x_b, (b) $\ddot{x}_{s,0}$ obtained from different design formulae.*

As indicated in Fig. 5, the optimal design formula developed by Matteo [9] presents a result that coincides with Proposal 2. Fig. 5 (a) also shows that two fixed-points were observed in the transfer function curve as developed in [8], which is compatible with its assumption of fixed-point method. For the case of transfer function of $\ddot{x}_{s,0}$, it is found that the proposed optimal strategy 1 gives an excellent control effect, when the input frequency is larger than the fundamental frequency ω_b. In order to take a further step to illustrate the seismic performance of the designed TMD with optimal parameters presented by the proposed strategies, a stationary analysis of benchmark model excited by the filtered-white-noise with the power spectral density defined in Section 6. The related results are listed in Table 2.

***Table 2** Comparison of stationary responses with different design formulae*

Proposals	σ_{x_b}[mm]			$\sigma_{\ddot{x}_{s,0}}$[m/s^2]		
	Firm	Medium	Soft	Firm	Medium	Soft
Original BI	52	54	65	0.33	0.34	0.42
Marian 2014[6]	16(-68%)	18(-66%)	23(-65%)	0.12(-65%)	0.13(-64%)	0.16(-61%)
Marian 2017[8]	15(-71%)	17(-68%)	21(-67%)	0.10(-69%)	0.11(-68%)	0.14(-66%)
Matteo 2019[9]	**15(-72%)**	**17(-69%)**	**21(-68%)**	0.10(-68%)	0.11(-68%)	0.14(-66%)
Proposal 1	16(-69%)	18(-66%)	22(-67%)	**0.09(-74%)**	**0.10(-72%)**	**0.13(-70%)**
Proposal 2	**15(-72%)**	**17(-69%)**	**21(-68%)**	0.10(-70%)	0.11(-68%)	0.15(-65%)

As indicated in Table 2, the proposed methods provide excellent performance for suppressing the mean square root of isolator displacement as well as absolute acceleration of primary structure. Specially, the mitigation level provided in [9] almost equals to the proposal 2, which have been validated in above discussions. For the suppression of absolute acceleration response of primary structure, it is observed that the first proposal attains a superior performance than other four strategies.

Conclusions

In this work, two design formulae related to the minimization of H_2 norm of performance indicator of TMDI were developed. The effectiveness and accuracy of proposed method were validated by using the numerical simulation results. As illustrated in Section 5, the optimal parameters related to the analytical formula coincides with the numerical results for firm and medium soil conditions. However, the comparison results related to the soft soil condition shows that the numerical results approach to MDI for the mass ratio larger than 1.4, which implies that the MDI is more effect than the TMDI in suppressing the acceleration responses of a BI system when the mass ratio is larger than the total mass of the BI system. However, it is worthy to note that the analytical result presents a comparable performance in suppressing the performance indicator. This implies that the analytical solution is an effective alternative for searching the optimal parameters. The comparison results in Section 6 elucidate that the proposed two design formulae present excellent seismic response control performance in suppressing the absolute acceleration of primary structure as well as the isolator displacement. To this end, the proposed methods are effective for improving the seismic performance of a BI system with a TMDI.

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