Materials Research Proceedings **6** (2018) 283-288 doi: http://dx.doi.org/10.21741/9781945291890-45

An Analytical Method for Predicting Residual Stress Distribution in Selective Laser Melted/Sintered Alloys

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Keywords: Residual Stress, Modelling, Selective Laser Melting, Additive Manufacturing

Abstract. Residual stresses that build up during selective laser melting or sintering (SLM/SLS) process can influence the dimensional accuracy, mechanical properties and in-service performance of SLM/SLS parts. Therefore, it is crucial to understand, predict and effectively control residual stresses in a part. The present study aims at developing an analytical model to predict the through-thickness distribution of residual stresses in an SLM part-substrate system. The proposed model demonstrates how residual stresses built up in the substrate and previously deposited layers are related to the stress induced by a newly deposited layer, based on the stress and moment equilibrium requirements. The model has been validated by published experimental measurements and verified with existing analytical/numerical models. The outcomes of the study suggest that the proposed analytical model can be used for quick estimation of residual stress distribution and the order of magnitude.

Nomenclature

a_n , b_n	Constants (related to material, process parameter and part geometry)
h, Δh	Substrate height, layer height, respectively (mm)
k	Residual stress in a newly deposited layer (MPa)
m	An individual deposited layer, m = $1, 2, 3, 4, \dots \dots \dots . (n-1)$
n	Number of deposited layers $(1, 2, 3, 4, \dots \dots \dots . n)$
y	Distance from substrate bottom surface (mm)
$\Delta\sigma_n(y)$	Stress increment in the substrate due to deposition of the nth layer (MPa)
$\Delta\sigma_{TS_{(nL)}}(y)$	Total(T) stress increment in substrate(S) due to deposition of 'n' layers(nL) (MPa)
$\Delta\sigma_{T(Lm)_{(Ln)}}(y)$	Total(T) stress increment in m^{th} layer(Lm) due to depositing the nth layer(Ln) (MPa)

Introduction

Selective laser melting (SLM) and selective laser sintering (SLS) are two commonly employed additive manufacturing processes, belong to the laser powder bed fusion (LPBF) technology, offering great advantages and opportunities compared to traditional subtractive techniques. They are primarily used to build complex geometry, lightweight and customized functional parts directly from CAD data by consolidating successive layers of powder by using a high power-density laser to melt and fuse metallic powders together. The main difference between SLM and SLS is the binding mechanism between the powder particles. In SLS, powder particles are partially molten and requires post treatment to improve part's density and mechanical properties. In SLM, powder particles are fully molten. Since the difference between SLM and SLS is

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somewhat ambiguous, the stress-inducing mechanisms in LPBF are usually described for the case of SLM [1,2].

SLM is known for introducing significant amounts of residual stresses owing to the large thermal gradients inherently present in the process [1]. During the SLM process, the material experiences large localised heat fluctuations in a short period of time. This causes a high-temperature gradient and the resulting residual stress can cause part warpage, crack formation both during laser processing and after cutting parts from the substrate. Process induced residual stresses can affect the functional performance of the parts by undesired strength reduction, which limits the applicability of the process [1,3–5].

Although the experimental methods for measuring residual stresses possess various advantages, modelling of residual stresses is an alternative in many cases as it is quicker, inexpensive and has no restriction on specimen size, surface finish, etc. For successful manufacturing of an SLM part, effective evaluation of residual stress is very important. Therefore a simple analytical model is required. In this study, the original idea of Shiomi, et al. [5] is extended (through changes in assumptions) to calculate the through-thickness distribution and magnitude of residual stresses in an SLM part-substrate assembly due to the deposition of '*n*' number of SLM layers. The mathematical formulation by Shiomi is limited to stress increment in the substrate $\left(\Delta\sigma_1(y)\right)$ due to the deposition of only one layer of SLM. Moreover, the proposed model provides a better explanation of layer by layer building up of residual stresses (with mathematical representations) due to the progressive deposition of additive layers (starting from a single layer to '*n*' SLM layers), taking into account of the force and moment equilibrium.

Formulation of the proposed analytical model

Mechanism of residual stress development. The mechanism of residual stress development in SLM can be distinguished into two descriptive models: (i) Temperature gradient mechanism (TGM) model, resulting from the large thermal gradients that occur around the laser spot owing to the rapid heating of the upper surface (substrate or a previously deposited layer) by the laser beam. Since the thermal expansion of the heated top layer is restricted by the underlying colder material, yielding a compressive stress-strain condition in the irradiated zone [1,3]. (ii) The cool-down phase model, which represents a cooling stage of the irradiated zone after the laser beam leaves that area, as a result, the material tends to shrink. The shrinkage is partially inhibited by the underlying material layers or substrate, yielding a residual tensile stress condition in the newly added layer and compressive stress in below [1,3].

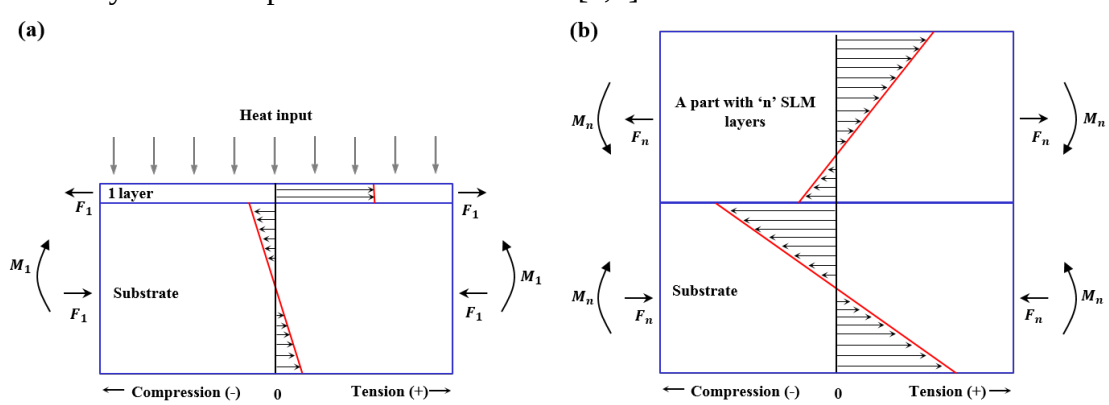

Fig. 1: Conceptual model showing the mechanism of building up of residual stresses during SLM process: (a) stress distribution after deposition of the first layer, (b) stress distribution after deposition of 'n' layers

A conceptual model has been developed to interpret the mechanism of building up of residual stresses in the SLM process as illustrated in Fig 1. Fig. 1 (a) shows the generation of an upward bending moment (M_1) in the SLM part-substrate assembly by a pair of equal and opposite forces to balance the process induced tensile residual stress (k) while depositing 1st layer. This process gets more complicated as more layers build up, and the part height becomes significant as compared to the substrate. Each successive SLM layer induces the same amount of misfit strain each time on the building part of changing height. Therefore, the final stress distribution in a multilayer SLM part-substrate assembly is significantly different from the starting system with a single layer (Fig. 1a), which can be determined by a succession of force and moment balance calculations. Fig. 1 (b) represents the final state of residual stress distribution in an SLM part-substrate assembly after deposition of 'n' SLM layers, having an upward moment in the substrate and a downward moment in the part to maintain the force and moment equilibrium condition. As a result, residual stresses at the free surface of the newly deposited material and substrate are tensile in nature, at the interface residual stresses are compressive in nature [1,3].

Assumptions. Based on the known distribution of residual stresses in [1,3–5] and the aforementioned conceptual model, the following assumptions are made:

1) The part is being built at room temperature on top of a substrate of height 'h'.
2) The substrate is free from residual stresses prior to material deposition, no external forces are applied to the part-substrate assembly, and the general beam theory is valid.
3) The substrate and deposited layers have the same length and same width.
4) Residual stress of a newly deposited layer has a value 'k', which is tensile in nature and constant throughout the layer height (Δh).
5) The height of each newly deposited layer (i.e. Δh) and the resulting stress induced by each newly deposited layer is the same.
6) The increment of residual stress '$\Delta\sigma$' owing to deposition of a new layer distributes linearly in the substrate following a linear equation.

Formulations. Fig. 2 shows the building up of residual stresses in the substrate and the SLM part due to the deposition of the first to n^{th} layer. A summary of the key equations of the proposed analytical method is presented in Eqs. 1-5.

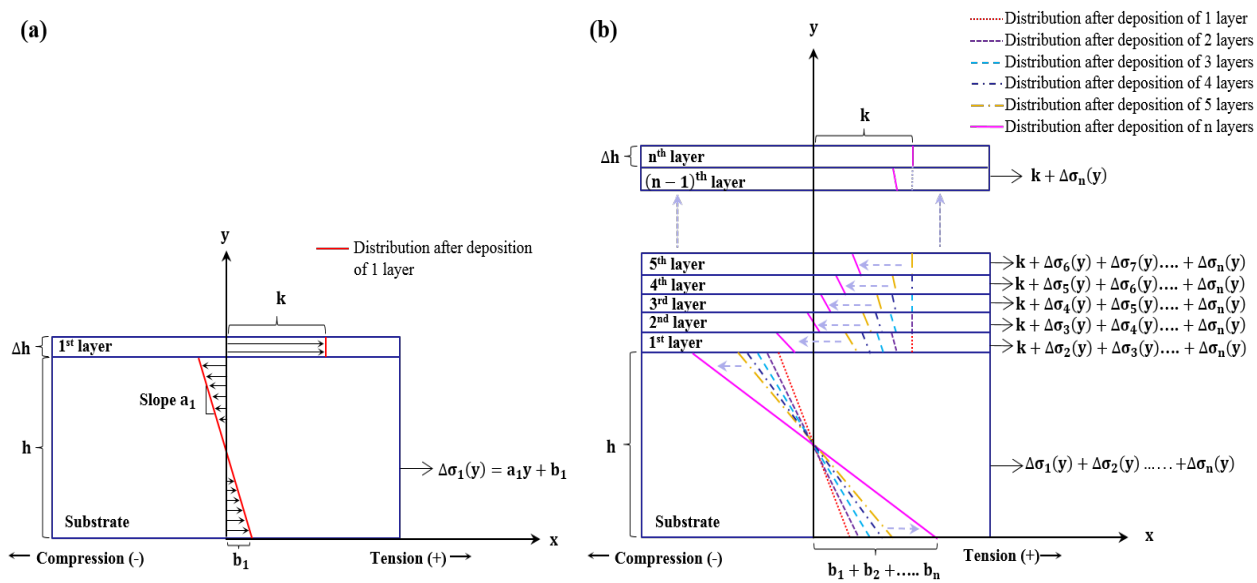

Fig. 2: Building up of residual stresses by SLM process: residual stress distribution due to the stress induced by deposition of (a) the first layer, (b) 'n' layers additively.

Assume that stress increment distributes linearly in the substrate due to the addition of $1^{st}, 2^{nd}, \ldots\ldots\ldots n^{th}$ layer respectively as follows:

$$\Delta\sigma_1(y) = a_1 y + b_1,\ \ \Delta\sigma_2(y) = a_2 y + b_2, \ldots\ldots\ldots\ldots\ldots, \Delta\sigma_n(y) = a_n y + b_n \tag{1}$$

After solving the force and moment equilibrium equations, the total stress increment in the substrate due to the deposition of $1, 2, \ldots\ldots n$ layers, respectively, can be expressed by Eq. 2:

$$\left.\begin{aligned}
&\Delta\sigma_{TS_{(1L)}}(y) = -6k\Delta h y\left(\frac{h+\Delta h}{h^3}\right) + k\Delta h\left(\frac{2h+3\Delta h}{h^2}\right)\\
&\Delta\sigma_{TS_{(2L)}}(y) = -6ky\Delta h\left\{\frac{h+\Delta h}{h^3} + \frac{h+2\Delta h}{(h+\Delta h)^3}\right\} + k\Delta h\left\{\frac{2h+3\Delta h}{h^2} + \frac{2h+5\Delta h}{(h+\Delta h)^2}\right\}\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&\Delta\sigma_{TS(nL)}(y) = -6ky\Delta h\left\{\frac{h+\Delta h}{h^3} + \frac{h+2\Delta h}{(h+\Delta h)^3} + \frac{h+3\Delta h}{(h+2\Delta h)^3}\ldots\ldots\ldots + \frac{h+n\Delta h}{\{h+(n-1)\Delta h\}^3}\right\} + k\Delta h\left\{\frac{2h+3\Delta h}{h^2} + \right.\\
&\qquad\left.\frac{2h+5\Delta h}{(h+\Delta h)^2} + \frac{2h+7\Delta h}{(h+2\Delta h)^2}\ldots\ldots\ldots + \frac{2h+(2n+1)\Delta h}{\{h+(n-1)\Delta h\}^2}\right\}
\end{aligned}\right\} \tag{2}$$

Therefore, the total increment of residual stresses in the substrate due to the deposition of 'n' layers of SLM can be expressed by a generalised Eq. 3:

$$\Delta\sigma_{TS(nL)}(y) = -6ky\Delta h\sum_{n=1}^{n}\left\{\frac{h+n\Delta h}{\{h+(n-1)\Delta h\}^3}\right\} + k\Delta h\sum_{n=1}^{n}\left\{\frac{2h+(2n+1)\Delta h}{\{h+(n-1)\Delta h\}^2}\right\} \tag{3}$$

The total stress increment in the $1^{st}, 2^{nd}, \ldots\ldots\ldots (n-1)^{th}$ layers respectively due to the deposition of the nth layer of SLM can be expressed by Eq. 4:

$$\left.\begin{aligned}
&\Delta\sigma_{TL1_{(Ln)}}(y) = k - 6ky\Delta h\left\{\frac{h+2\Delta h}{(h+\Delta h)^3} + \frac{h+3\Delta h}{(h+2\Delta h)^3}\ldots\ldots\ldots + \frac{h+n\Delta h}{\{h+(n-1)\Delta h\}^3}\right\} + k\Delta h\left\{\frac{2h+5\Delta h}{(h+\Delta h)^2} + \right.\\
&\qquad\left.\frac{2h+7\Delta h}{(h+2\Delta h)^2}\ldots\ldots\ldots + \frac{2h+(2n+1)\Delta h}{\{h+(n-1)\Delta h\}^2}\right\}\\
&\Delta\sigma_{TL2_{(Ln)}}(y) = k - 6ky\Delta h\left\{\frac{h+3\Delta h}{(h+2\Delta h)^3} + \frac{t+4\Delta t}{(t+3\Delta t)^3}\ldots\ldots\ldots + \frac{h+n\Delta h}{\{h+(n-1)\Delta h\}^3}\right\} + k\Delta h\left\{\frac{2h+7\Delta h}{(h+2\Delta h)^2} + \right.\\
&\qquad\left.\frac{2h+9\Delta h}{(h+3\Delta h)^2}\ldots\ldots\ldots + \frac{2h+(2n+1)\Delta h}{\{h+(n-1)\Delta h\}^2}\right\}\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&\Delta\sigma_{T\{L(n-1)\}_{(Ln)}}(y) = k - 6ky\Delta h\left\{\frac{(h+n\Delta h)}{\{h+(n-1)\Delta h\}^3}\right\} + k\Delta h\left\{\frac{2h+(2n+1)\Delta h}{\{(h+(n-1)\Delta h)\}^2}\right\}
\end{aligned}\right\} \tag{4}$$

Eq. 4 can be written in a generalised form as Eq. 5 for the stress in the mth layer:

$$\Delta\sigma_{T(Lm)_{(Ln)}}(y) = k - 6ky\Delta h\sum_{m+1}^{n}\left\{\frac{h+n\Delta h}{\{h+(n-1)\Delta h\}^3}\right\} + k\Delta h\sum_{m+1}^{n}\left\{\frac{2h+(2n+1)\Delta h}{\{h+(n-1)\Delta h\}^2}\right\} \tag{5}$$

Eqs. 3 and 5 can be used to calculate the distribution of residual stresses in an SLM part-substrate assembly if the residual stress of a newly deposited layer (k) is known. In SLM, the value of 'k' can be considered as the yield strength of the material [1,4,5]. However, it is recommended to calibrate the 'k' by experimentally measuring the near-surface residual stress of the SLM part.

Parametric study

A parametric study has been performed in terms of the influence of number of deposited layers, layer height and substrate height on residual stress distribution. For all cases, residual stress in the new SLM layer (k) is assumed as 300 MPa (yield strength of steel) [1,5]. Residual stress distribution due to the variation in number of deposited layers is shown in Fig. 3 (a). Four different cases were studied with number of layers being 20, 40, 60, and 80 respectively, with layer height 0.1 mm. For all cases, the substrate height was 10 mm. Residual stress distribution due to the variation of layer height with a substrate of 10 mm height is shown in Fig. 3 (b). The SLM part height was 8 mm with four different layer heights: 0.05 mm (160 layers), 0.1 mm (80 layers), 0.2 mm (40 layers), and 0.32 mm (25 layers) respectively. Fig. 3 (c) demonstrates residual stress distribution variations due to the different substrate heights: 9 mm, 10 mm, 11 mm and 12 mm, respectively, keeping same individual layer height (0.1 mm) and total part height (8 mm) for all.

The parametric study has revealed that: (i) the larger the number of deposited layers, the higher the resulting residual stresses, (ii) variation of individual layer height has no significant effect on stress distribution, and (iii) the lower the substrate height, the higher the resulting stress in it.

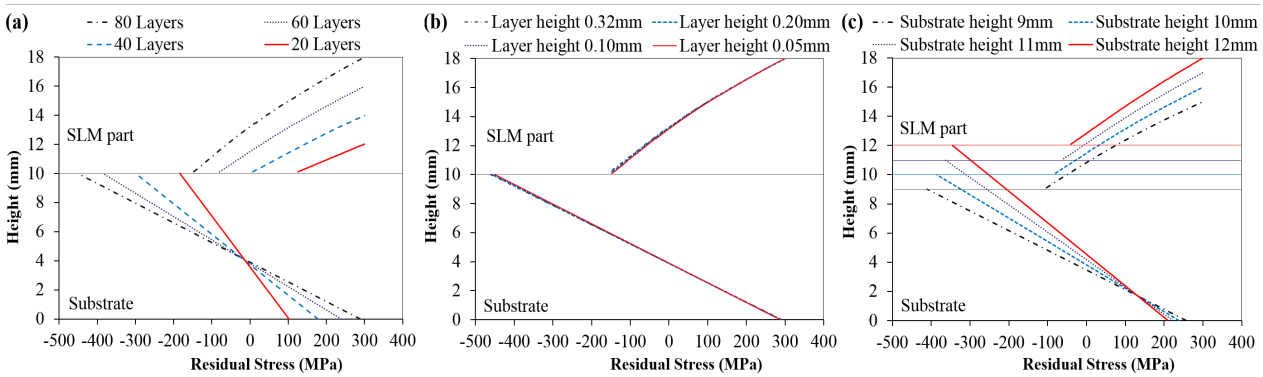

Fig. 3: Residual stress distributions with the variation of (a) number of layers, (b) individual layer height, (c) substrate height by keeping same individual layer height and total part height.

Validation and verification of the proposed analytical model

The proposed analytical model has been validated with experimental measurements in [5], and verified with predictions based on [1,4,5]. Validation and verification examples are presented as three cases of different combinations of part-substrate assemblies. Table 1 represents the parameters used in the models. For all cases, residual stress value in the newly deposited layer was considered as the yield strength of the material. Fig. 4 shows calculated residual stress distributions in this study and comparison with published values.

Table 1: Details of published results used for the validation/verification

SLM part/substrate	Stress evaluation method	k (MPa)	h (mm)	Δh (μm)	n
Chrome molybdenum steel (JIS SCM440) on stainless steel [5]	Layer removal method and analytical modelling	300	8	100	60
Stainless steel 316L on stainless steel [1]	Analytical modelling	300	20	50	50
Steel on steel [4]	Finite element modelling	410	1	150	1

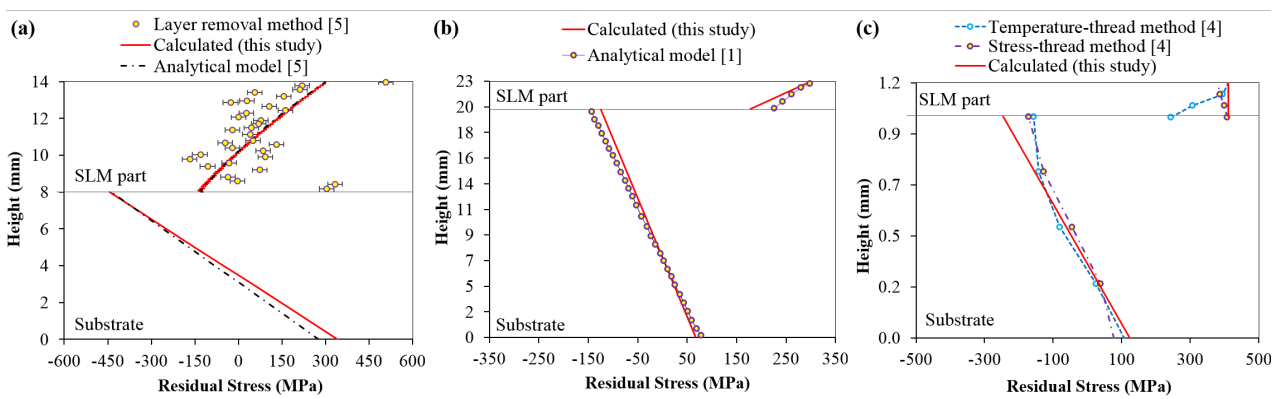

Fig. 4: Comparison of calculated residual stress distributions with experimental measurements and predicted values from references: (a) Chrome molybdenum steel (JIS SCM440) on stainless steel [5], (b) Stainless steel 316L on stainless steel [1], (c) Steel on steel [4]

Discussion

A good agreement was achieved by comparison with literature experimental measurements, numerical [4] and analytical [1,5] models. Predictions by the proposed analytical model and existing models [1,4,5] are very close. However, the mathematical representation by Shoimi [5] is limited to a single SLM layer. Also, they have neglected higher order terms for layer height (Δh^2), which will apparently induce error to the final stress distribution. Likewise, Mercelis's

analytical model [1] doesn't provide a final mathematical expression for calculating the through-thickness residual stresses distribution for a part-substrate assembly having '*n*' number of SLM layers. Moreover, conventional numerical analysis for several single scans with a fine mesh model is very complicated and requires large computational time [4]. Therefore, numerical analyses are limited to a few SLM layers. Conversely, the proposed analytical model is much simpler compared to the existing modelling techniques and can be used for quick prediction of through-thickness residual stress distribution in an SML part-substrate assembly with '*n*' number of layers.

Limitations of the proposed method are: (i) It does not explain the phenomenon of relaxation and redistribution of residual stress once the part is removed from the base plate. However, formulation for stress redistribution after baseplate removal can be found in [1], or by performing FEA; (ii) There is a discontinuity in the stress value on either side of the interface due to the differences in the elastic modulus of the two materials (SLM and substrate), although process induced residual strains are continuous at the interface. (iii) The parametric study using the proposed model is limited to the geometrical variables of an SLM part-substrate assembly. Further investigation is required to link '*k*' to the SLM process parameters (laser power, scan speed, scan strategy, etc.), so that it can be used as an evaluation tool for SLM process parameter design.

Conclusions

An analytical model is presented for predicting residual stress distribution in SLM parts. The model is based on the force and moment equilibrium of induced stresses by progressive deposition of material layers. The model has been validated with experimental measurements, verified with predictive models from the literature. Based on the study, the following conclusions can be drawn:

1) To calculate the through-thickness distribution of residual stresses in an SLM part-substrate system, the proposed model requires only four parameters: the layer height (Δh), substrate height (h), number of deposited layers (n), and residual stress in newly deposited layer (k).
2) Compared to other analytical and numerical methods, this approach is simpler and can give a quick estimation of through-thickness residual stress distribution and magnitude.
3) With good calibration of stress value in a newly deposited layer (k), this method can be used to predict residual stress profile in an SLM part-substrate system with much less cost and time.

Acknowledgement

This publication was made possible by the sponsorship and support of the Lloyd's Register Foundation, which is a charitable organization that helps to protect life and property by supporting engineering-related education, public engagement and the application of research.

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