

# Residual Stress Redistribution During Elastic Shakedown in Fillet Welded Plate

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**Abstract.** Welding residual stresses exist in various welded structures such as ships and offshore structures. According to the load levels during operation, the as-welded residual stresses can be relaxed or redistributed. The elastic shakedown phenomenon can be considered as one of the reasons for the stress relaxation or redistribution. This work studies the redistribution of welding residual stresses during different levels of shakedown in a fillet-welded plate manufactured in line with ship design and welding procedures. Fillet welding was performed on a ship structural steel, DH36. The fillet welds were subjected to different levels of shakedown under tensile cyclic load. Neutron diffraction was used to measure residual stresses in these plates in the as-welded state and after elastic shakedown. A mixed hardening model in line with the Chaboche model was determined for both weld and base material. A shakedown limit analysis based on plastic work dissipation was developed as the shakedown criterion to estimate the shakedown limit on the component. Further, the redistribution of residual stresses due to elastic shakedown was quantified through experimental measurements.

## Introduction

Complex residual stresses are induced in load bearing members because of welding in offshore structures. For example, longitudinally stiffened plates which resist bending stresses consist of weld residual stresses due to fabrication [1]. Pre-existing residual stresses in load bearing members may be relieved or redistributed depending on the load levels during operation [2]. Elastic shakedown is one of the main phenomena contributing to this change in residual stress. Elastic shakedown is defined as a plastic deformation during the first few load cycles, followed by an elastic response which is associated with a limit called the shakedown limit [3]. The change in residual stresses is a result of plasticity induced during the first few cycles. The elastic shakedown limit in a component lies between the first yield and the plastic collapse load [4]. An applied load above the elastic shakedown limit will result in either plastic shakedown or ratcheting.

There are a number of research activities on the shakedown of residual stresses in welded structures [1,5-7]. However, the focus has been on the influence of shakedown on fatigue behaviour of the plate and hence on the relaxation of residual stresses at the surface of the plate. Paik *et al.* [6] conducted experiments to study the shakedown of residual stresses in butt-welded aluminium plates subjected to 3-point bending. Using hole drilling technique for residual stress

measurements, they measured a 36% relaxation in the tensile longitudinal component of residual stresses after 3 load cycles with a magnitude equal to 88% of the yield strength. Lautrou *et al.* [5] studied a numerical model of a rectangular plate under plane stress conditions where initial residual stress was introduced using a non-uniform displacement at one side. The results show elastic behaviour within a few load cycles. Liangbi *et al.* [7] concluded that under constant amplitude load conditions, stress relaxation was limited to the first load cycle.

Fillet welds are more common in offshore applications and are considered more critical in fatigue due to the stress concentration at the weld toe. However, experimental research on the shakedown of residual stresses in fillet welds is very rare. Our recent study conducted on butt welded steel plates [8] found that the residual stresses, both transverse and longitudinal components, redistributed through the thickness depending on the relaxation at the top surface. This present paper is a continuation of that previous study conducted on butt-welded specimens [8]. The shakedown limit analysis was implemented in the fillet joints initially to determine the elastic shakedown limit. Absorbed plastic deformation energy at the end of each load cycle was used to determine the elastic shakedown limit [9]. Since linear or nonlinear isotropic and linear kinematic classics cannot represent realistic structural materials subjected to cyclic loadings, a mixed hardening model in line with Chaboche [10,11] was used for all numerical simulations. Experimental testing was used to study the effect of elastic shakedown in the redistribution of pre-existing residual stress fields in a fillet weld manufactured using DH36, a shipbuilding steel.

**Methodology for estimating shakedown limit**

Constitutive model: An elastic-plastic model with a combination of one isotropic hardening  $R$  and three kinematic hardenings  $\alpha_1, \alpha_2$  and  $\alpha_3$ , initially developed by Lemaitre and Chaboche [10] was implemented in the numerical model. Total strain,  $\epsilon_{ij}^t$  is divided into elastic strains,  $\epsilon_{ij}^e$  and plastic strains  $\epsilon_{ij}^p$ . The elastic domain is described by a typical von Mises yield criterion  $f=0$ , where  $f$  is defined as:

$$f = J(\sigma - \alpha_1 - \alpha_2 - \alpha_3) - R - \sigma_y \tag{1}$$

where  $\sigma$  is the Cauchy stress tensor,  $J$  is the von Mises equivalent stress and  $\sigma_y$  is the yield strength of the material. The isotropic hardening  $R$  and the kinematic hardening  $\alpha_i$  are defined as below:

$$\dot{R} = b(Q - R) (\dot{p}) \tag{2}$$

$$\dot{\alpha}_i = \frac{2}{3} C_i \dot{\epsilon}_p - \gamma_i \dot{\alpha}_i \dot{p} \tag{3}$$

where  $b, Q, C_1, C_2, C_3, \gamma_1, \gamma_2$  and  $\gamma_3$  are material parameters and  $\dot{p}$  is the rate of accumulated plasticity. The material parameters obtained for DH36 base material and weld material in previous work [8] are used in this study which are given in Table 1.

*Table 1: Chaboche constitutive parameters*

DH36	$E / \text{GPa}$	$\nu$	$\sigma_y / \text{MPa}$	$Q / \text{MPa}$	$b$	$C_1 / \text{MPa}$	$\gamma_1$	$C_2 / \text{MPa}$	$\gamma_2$	$C_3 / \text{MPa}$	$\gamma_3$
BM	200	0.3	350	-48.6	87.5	4360	16.4	38520	116	8000	40
WM	200	0.3	400	-102	14	8912	29.65	102300	400	8000	40

Shakedown limit analysis: The elastic shakedown limit of a material is between the first yield and the plastic collapse limit. Above the elastic shakedown limit, the material fails due to low-cycle fatigue in the presence of cyclic loading. In the previous work [8], it was shown that the plastic work dissipation at the end of each cycle can be considered as a shakedown criterion. On

a component under cyclic loading with load cycles of  $nt$  to  $(n+1)t$  time interval, the plastic strain increment after each load cycle can be expressed as:

$$\Delta \varepsilon_{ij}^p = \int_{nt}^{(n+1)t} \dot{\varepsilon}_{ij}^p dt \quad (4)$$

Work done due to total strain can be decomposed into work done due to elastic strain,  $W^e$  and work done due to plastic strain,  $W^p$ . In the initial few load cycles, where plastic strains are active, the plastic work done is greater than zero. After a few initial load cycles, if the structure achieves elastic shakedown or the response is completely elastic, there will exist a time-independent residual stress field and the plastic work done will be equal to zero [8]. Since the structural response is elastic, it satisfies the lower bound theorem and hence it can be concluded that plastic work done at the end of individual load cycles,  $W^p$  defined in Eq .5 can be used as a shakedown criterion.

$$W^p = \sum_{n=1}^N \int_{V^e} \sigma_{ij} \Delta \varepsilon_{ij}^p dV^e \quad (5)$$

where  $V^e$  is the volume of an element and  $N$  is the number of elements.

### Numerical analysis

**Weld model:** A sequentially coupled thermo-mechanical analysis of the fillet welding was performed. An equivalent static heat source was used to introduce arc heat into the model. Filler deposition was simulated using a ‘chunking’ method as explained in R6 section III [12]. The model was initially developed as a T-joint with dimensions shown in Fig. 1a. The numerical fillet weld model was then cut to the dimension of a mechanical load model as shown in Fig.1b. Element activation/deactivation technique was implemented to simulate the cutting process.

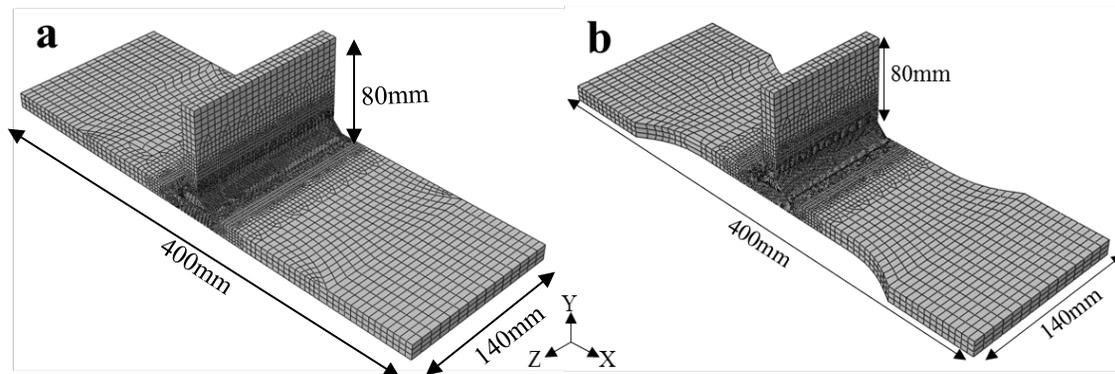


Fig. 1: a) Initial weld model, b) test specimen model following cutting

**Shakedown limit:** A half numerical model of the actual weld plate after cutting was developed to estimate the shakedown limit of the fillet weld under study. The procedure implemented and the detailed description of the procedure is explained in the previous work [8]. The elastic shakedown limit of the fillet weld geometry was determined using a step-by-step iterative procedure detailed in [8].

### Experimental studies

**Specimen manufacturing:** Fillet weld was manufactured using Gas Metal Arc Welding process on DH36 steel plates. The dimensions of the base plate and the web were respectively  $400 \times 140 \times 12.7 \text{ mm}^3$  and  $140 \times 80 \times 12.7 \text{ mm}^3$ , before cutting. The welding procedure specification was in accordance with the Lloyd’s Register classification. Full restraint was implemented during welding using strongbacks to represent welding conditions in the ship and offshore structures. Thermocouples were used to monitor transient heat during welding. The welding set-up for the fillet weld is shown in Fig.2a. Thermocouple data were later used to

validate the heat transfer model in the sequentially coupled welding simulation. Following welding, the weld plate was cut using EDM to prepare a mechanical test specimen.

**Cyclic loading:** Three tensile load cycles ( $R$ -ratio = 0) with 0.25 Hz frequency were applied on the plate. The maximum applied stress used on the plate was equivalent to achieving 68% of the yield strength of the parent material (350 MPa). The load was applied across the weld along the transverse direction of the specimen shown in Fig.2b.

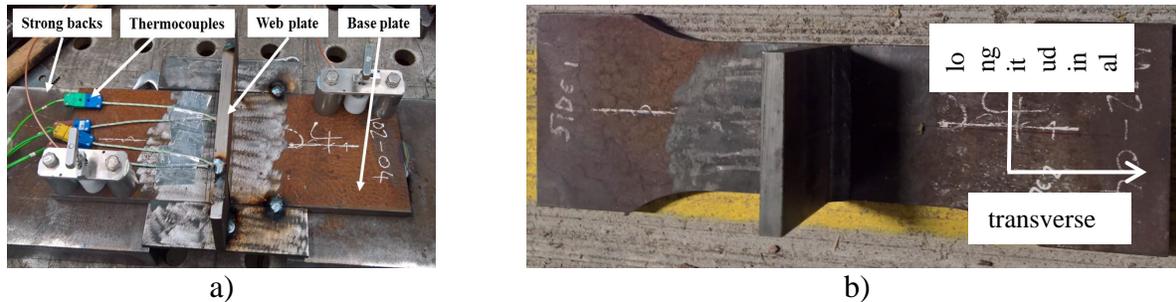


Fig. 2: a) Welding set-up b) Cyclic load specimen after EDM cutting

**Neutron Diffraction:** Neutron diffraction is a non-destructive technique which can measure strains through the thickness of a component. The ENGIN-X instrument at the UK's ISIS pulsed neutron source was used to determine residual stresses in the as-welded state and after different numbers of load cycles, at 30 locations in the plane shown in Fig.3.

### Results and discussions

**Elastic shakedown limit:** The limit analysis FE model is subjected to increasing loads starting from 80% of yield strength of the parent material. The loading is gradually increased until a constant residual stress ceases to exist in the plate. The shakedown limit load factor  $\lambda$  estimated for the fillet weld is 1.2. This implies that in the absence of residual stress field any cyclic load within 1.2 times the yield strength applied in this set-up will achieve elastic-shakedown state.

**As-welded residual stresses:** Only the transverse residual stress distribution on the base plate is discussed in this paper for conciseness. Fig.4 compares the as-welded transverse residual stress component in the base plate predicted using finite element analysis and that measured using neutron diffraction. The stress values are taken from the plane across the weld at the mid-thickness. The comparison is drawn across the weld at 2.5mm below the top surface and 2.5mm above the bottom surface. High tensile transverse residual stresses are observed in both FE and experimental measurements at 2.5mm below the top surface of the base plate. The transverse residual stresses at 2.5mm above the bottom surface are significantly lower than those at 2.5mm below the top surface. The welding simulation captures the trend of residual stress distribution well. The minor difference between the predicted using welding simulation and experimental measurement may be due to experimental errors, including those arising from the EDM cutting, and material assumptions in the FE model. The maximum tensile transverse residual stress is almost identical for both numerical and experimental results, i.e., 250MPa. This value is equivalent to 71% of the yield strength of the base plate. It is demonstrated that the assumption of as-welded residual stress in structural integrity assessment guidance, BS 7910 is conservative.

**Residual stress redistribution:** Redistribution of transverse residual stresses following three tensile load cycles, measured using neutron diffraction is shown in Fig 5. The stresses at the top part of the base plate shown in Fig. 5a have a higher magnitude compared with the stress at the bottom part of the plate, shown in Fig. 5b. The redistribution of transverse stresses is minimum after the application of three load cycles. One of the reasons could be that the combination of pre-existing residual stress after EDM cuts and the applied load are not high enough to induce elastic shakedown in the transverse direction. Interestingly, it is found that after three loading cycles, the maximum value of tensile transverse residual stress at 2.5mm below the top surface is

greater than that in the as-welded condition. However, the maximum tensile stress at 2.5mm above the bottom surface is similar in the as-welded condition and after one and three loading cycles. Based on this experimental evidence, the conservative level of the residual stress relaxation rule in BS 7910 may require re-investigation.

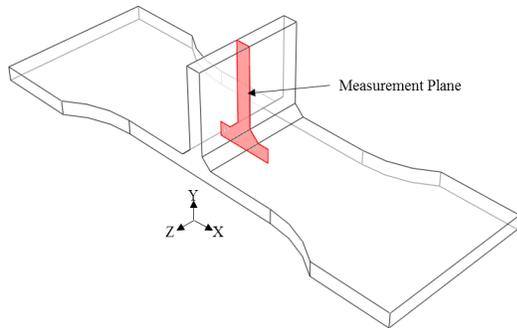


Fig. 3: Neutron diffraction measurement plane

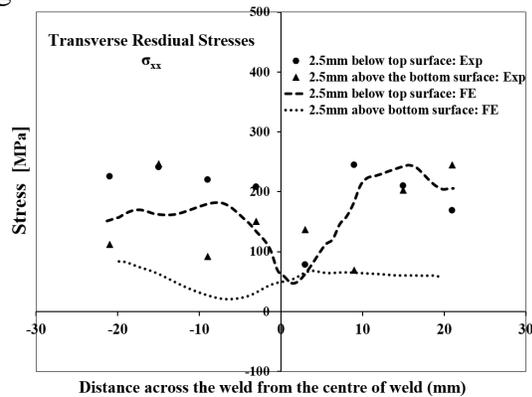


Fig. 4: RS redistribution across the weld at 2.5mm above and below the bottom and top surfaces respectively

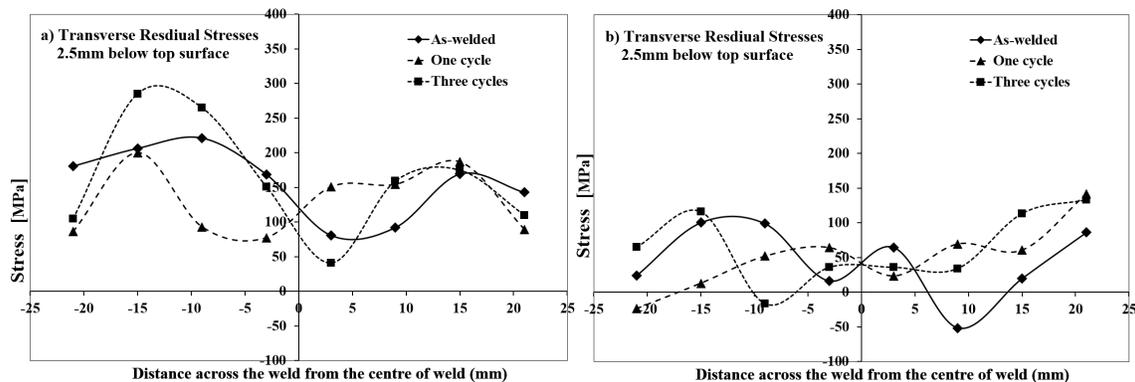


Fig. 5: Transverse residual stress redistribution by neutron measurement: a) 2.5mm below top surface, b) 2.5mm above the bottom surface

**Effect of elastic shakedown:** In the case of simple plates and weld joints like butt welds, if the combination of residual stresses along the load direction after a few cycles and the applied load is within the elastic shakedown limit, it can be said that the structure is stable and will achieve an elastic-shakedown state. However, in the case of fillet welds, since the residual stress component perpendicular to the loading direction is undergoing relaxation, both stress components parallel and perpendicular to the load direction should be considered to confirm elastic shakedown or steady state. If the applied load does not relax the residual stress components in the first few cycles, the structure is expected to follow cyclic plasticity or ratcheting over each load cycle.

It is noted that the relaxation or redistribution of residual stresses in fillet welds are very complex when compared to simple weld joints like butt welds [8]. This calls for further investigation of residual stresses in fillet welds under cyclic loading. The authors are currently investigating the effect of tensile load cycles on a plate with a fillet weld along the specimen's longer axis with a load application along with the longitudinal residual stress component.

### Conclusions

The conclusions drawn from this study are:

- Shakedown limit of fillet welded geometry can be estimated based on a simplified method using plastic work done as a shakedown criterion.

- Experimental measurement of residual stress redistribution after three load cycles shows that there was only a minimal redistribution/relaxation in the transverse residual stress component even though the load applied was along this component.

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